Mössbauer neutrinos

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Outline

1. The Mössbauer neutrino experiment
2. Oscillations of Mössbauer neutrinos: Qualitative arguments
3. Mössbauer neutrinos in QFT
4. Conclusions
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A similar effect should exist for neutrino emission/absorption in bound state $\beta$ decay and induced electron capture processes.


Production:

$^3\text{H} \rightarrow ^3\text{He} + + \bar{\nu}_e + e^-$

Detection:

$^3\text{He} + + e^- (\text{bound}) + \bar{\nu}_e \rightarrow ^3\text{H}$

$^3\text{H}$ and $^3\text{He}$ are embedded in metal crystals (metal hydrides).

Physics goals:

Neutrino oscillations on a laboratory scale:

$E = 18.6$ keV, $L_{\text{osc}} \approx 20$ m.

Gravitational interactions of neutrinos

Study of solid state effects with unprecedented precision
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Physics goals:

- Neutrino oscillations on a laboratory scale: $E = 18.6$ keV, $L_{\text{osc}}^{\text{atm}} \sim 20$ m.
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Mössbauer neutrinos have very special properties:

- Neutrino receives full decay energy: \( Q = 18.6 \text{ keV} \)
- Natural line width: \( \gamma \sim 1.17 \times 10^{-24} \text{ eV} \)
- Atucal line width: \( \gamma \gtrsim 10^{-11} \text{ eV} \)
  - Inhomogeneous broadening (Impurities, lattice defects)
  - Homogeneous broadening (Spin interactions)
Mössbauer neutrinos (2)

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Experimental challenges:
- Is the Lamb-Mössbauer factor (fraction of recoil-free emissions/absorptions) large enough?
- Can a linewidth $\gamma \gtrsim 10^{-11}$ eV be achieved?
- Can the resonance condition be fulfilled?
Mössbauer neutrinos (3)

Recent controversy:

- **Does the small energy uncertainty prohibit oscillations of Mössbauer neutrinos?**
- **Do oscillating neutrinos need to have equal energies resp. equal momenta?**


- **Does the time-energy uncertainty relation prevent oscillations?**


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Recent controversy:
- Does the small energy uncertainty prohibit oscillations of Mössbauer neutrinos?
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⇒ Careful treatment with as few assumptions as possible is needed
⇒ Answer to the above questions will be No.
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Textbook derivation of the oscillation formula

Diagonalization of the mass terms of the charged leptons and neutrinos gives

\[ \mathcal{L} \supset -\frac{g}{\sqrt{2}} (\bar{e}_\alpha \gamma^\mu U_{\alpha j} \nu_{jL}) W^-_\mu + \text{diag. mass terms} + h.c. \]

(flavour eigenstates: \( \alpha = e, \mu, \tau \), mass eigenstates: \( j = 1, 2, 3 \))
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Assume, at time $t = 0$ and location $\vec{x} = 0$, a flavour eigenstate

$$|\nu(0, 0)\rangle = |\nu_\alpha\rangle = \sum_i U_{\alpha j}^* |\nu_j\rangle$$

is produced. At time $t$ and position $\vec{x}$, it has evolved into

$$|\nu(t, \vec{x})\rangle = \sum_i U_{\alpha j}^* e^{-iE_j t + i\vec{p}_j \vec{x}} |\nu_i\rangle$$
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Oscillation probability:

\[ P(\nu_\alpha \rightarrow \nu_\beta) = \left| \langle \nu_\beta | \nu(t, \vec{x}) \rangle \right|^2 = \sum_{j,k} U^*_{\alpha j} U_{\beta j} U_{\alpha k} U^*_{\beta k} e^{-i(E_j - E_k) t + i(\vec{p}_j - \vec{p}_k) \vec{x}} \]
Equal energies or equal momenta?

Typical *assumptions* in the “textbook derivation” of the oscillation formula:

- Different mass eigenstates have equal energies: $E_j = E_k \equiv E$
  
  (“Evolution only in space”, “Stationary evolution”)

These are assumptions or approximations, not fundamental principles!
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P(\nu_\alpha \rightarrow \nu_\beta) = \sum_{j,k} U_{\alpha j}^* U_{\beta j} U_{\alpha k} U_{\beta k} e^{-i \frac{\Delta m_{jk}^2 L}{2E}}
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Problems with the textbook derivation

In general, neither the equal energy assumption nor the equal momentum assumption is physically justified because both violate energy-momentum conservation in the production and detection processes.


Example: Pion decay at rest:

\[ \pi^+ \rightarrow \mu^+ + \nu_\mu, \quad \pi^- \rightarrow \mu^- + \bar{\nu}_\mu. \]
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**Energy-momentum conservation for emission of mass eigenstate $|\nu_i\rangle$:**

\[
E_i^2 = \frac{m^2_{\pi}}{4} \left( 1 - \frac{m^2_{\mu}}{m^2_{\pi}} \right)^2 + \frac{m^2_i}{2} \left( 1 - \frac{m^2_{\mu}}{m^2_{\pi}} \right) + \frac{m^4_i}{4m^2_{\pi}}
\]

\[
p_i^2 = \frac{m^2_{\pi}}{4} \left( 1 - \frac{m^2_{\mu}}{m^2_{\pi}} \right)^2 - \frac{m^2_i}{2} \left( 1 - \frac{m^2_{\mu}}{m^2_{\pi}} \right) + \frac{m^4_i}{4m^2_{\pi}}
\]

For massless neutrinos: $E_i = p_i = E \equiv \frac{m_{\pi}}{2} \left( 1 - \frac{m^2_{\mu}}{m^2_{\pi}} \right) \approx 30$ MeV.

To first order in $m_i^2$:

\[
E_i \approx E + \xi \frac{m^2_i}{2E}, \quad p_i \approx E - (1 - \xi) \frac{m^2_i}{2E}, \quad \xi \approx \frac{1}{2} \left( 1 - \frac{m^2_{\mu}}{m^2_{\pi}} \right) \approx 0.2
\]
Mössbauer neutrinos are the *only* realistic case, where $E_j \approx E_k$ holds approximately, due to the tiny energy uncertainty, $\sigma_E \sim 10^{-11}$ eV.

More realistic treatment desirable: Wave packet model

$\Rightarrow$

$P(\nu_\alpha \rightarrow \nu_\beta) = \sum_{j,k} U^*_{\alpha j} U_{\beta j} U_{\alpha k} U^*_{\beta k} e^{-i \Delta m^2_{jk} L \frac{E_j}{2}}$
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$\Rightarrow$ We thus expect:

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  - Takes into account finite resolutions of the source and the detector
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Beuthe, Giunti, Grimus, Kiers, Kim, Lee, Mohanty, Nussinov, Stockinger, Weiss, ...
Conditions for oscillations in a wave packet approach
Conditions for oscillations in a wave packet approach

- Coherence in production and detection processes

\[ \sigma^2_m = \sqrt{(2E \sigma^2_E)^2 + (2p \sigma^2_p)^2} > \Delta m^2 \]


This is easily fulfilled for Mössbauer neutrinos, since \( \sigma_E \approx 10^{-11} \) eV, \( \sigma_p = \frac{1}{2} \sigma_x \approx 1/\text{interatomic distance} \approx 10 \) keV, \( E = p = \frac{1}{18.6} \) keV.
Conditions for oscillations in a wave packet approach

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  Neutrino oscillations are caused by the superposition of different mass eigenstates.
  \[ \Rightarrow \text{If an experiment can distinguish different mass eigenstates, oscillations will vanish.} \]
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$$E = p = 18.6 \text{ keV}$$
Conditions for oscillations in a wave packet approach

- Coherence in production and detection processes
- Coherence maintained during propagation

\[ \sigma_x \]

It can be shown that, for Mössbauer neutrinos, \( \sigma_p \) is small enough, so that \( L_{osc} \ll L_{coh} \).

\[ P_{ee} = \sum_{j,k} |U_{ej}|^2 |U_{ek}|^2 \exp\left[ -2\pi i L_{jk} \right] \]

\[ L_{jk} = \frac{4\pi E \Delta m^2}{jk} \]
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\[ L_{\text{osc}} \ll L_{\text{coh}} \]

\[ P_{\text{ee}} = \left| \sum_{j,k} U_{ej} U_{ek} \right|^2 \exp \left[ -2\pi i L_{\text{osc}} \right] \]

\[ L_{\text{osc}} = \frac{4\pi E}{\Delta m^2} \]

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  Decoherence could be caused by wave packet separation

It can be shown that, for Mössbauer neutrinos, $\sigma_p$ is small enough, so that

$$L^{\text{osc}} \ll L^{\text{coh}}.$$ 

$\Rightarrow$ Stanard oscillation formula is approximately recovered:

$$P_{ee} = \sum_{j,k} |U_{ej}|^2 |U_{ek}|^2 \exp \left[ -2\pi i \frac{L}{L^{\text{osc}}_{jk}} \right]$$

$$L^{\text{osc}}_{jk} = \frac{4\pi E}{\Delta m^2_{jk}}$$
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Quantum field theoretical treatment

**Aim**: Properties of the neutrino should be automatically determined from properties of the source and the detector.
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Idea: Treat neutrino as an internal line in a tree level Feynman diagram:
Quantum field theoretical treatment

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Idea: Treat neutrino as an internal line in a tree level Feynman diagram:

External particles reside in harmonic oscillator potentials. E.g. for $^3$H atoms in the source:

$$\psi_{H,S}(\vec{x}, t) = \left[ \frac{m_H \omega_{H,S}}{\pi} \right]^{3/4} \exp \left[ -\frac{1}{2} m_H \omega_{H,S} |\vec{x} - \vec{x}_S|^2 \right] \cdot e^{-iE_{H,S}t}$$
Oscillation amplitude

\[ iA = \int d^3x_1 \, dt_1 \int d^3x_2 \, dt_2 \left( \frac{m_{H\omega_H, S}}{\pi} \right)^{\frac{3}{4}} \exp \left[ -\frac{1}{2} m_{H\omega_H, S} |\vec{x}_1 - \vec{x}_S|^2 \right] e^{-iE_{H, S}t_1} \]

\[ \cdot \left( \frac{m_{He\omega_{He}, S}}{\pi} \right)^{\frac{3}{4}} \exp \left[ -\frac{1}{2} m_{He\omega_{He}, S} |\vec{x}_1 - \vec{x}_S|^2 \right] e^{+iE_{He, S}t_1} \]

\[ \cdot \left( \frac{m_{He\omega_{He}, D}}{\pi} \right)^{\frac{3}{4}} \exp \left[ -\frac{1}{2} m_{He\omega_{He}, D} |\vec{x}_2 - \vec{x}_D|^2 \right] e^{-iE_{He, D}t_2} \]

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\[ \cdot \sum_j M^\mu M^{\nu*} |U_{ej}|^2 \int \frac{d^4p}{(2\pi)^4} e^{-ip_0(t_2-t_1)+i\vec{p}(\vec{x}_2-\vec{x}_1)} \]

\[ \cdot \bar{u}_{e, S} \gamma_\mu (1 - \gamma^5) \frac{i(\not{p} + m_j)}{p_0^2 - \not{p}^2 - m_j^2 + i\epsilon} (1 + \gamma^5) \gamma_\nu u_{e, D}. \]
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\[ \cdot \sum_j \mathcal{M}^\mu \mathcal{M}^{\nu*} |U_{ej}|^2 \int \frac{d^4p}{(2\pi)^4} e^{-ip_0(t_2-t_1) + i\vec{p}(\vec{x}_2 - \vec{x}_1)} \]

\[ \cdot \bar{u}_e, S \gamma_\mu (1 - \gamma^5) \frac{i(p + m_j)}{p_0^2 - \vec{p}^2 - m_j^2 + i\epsilon} (1 + \gamma^5) \gamma_\nu u_{e,D}. \]

Evaluation:

- \( dt_1 \, dt_2 \)-integrals \( \rightarrow \) energy-conserving \( \delta \) functions \( \rightarrow \) \( p_0 \)-integral trivial
- \( d^3x_1 \, d^3x_2 \)-integrals are Gaussian
- \( d^3p \)-integral: Use Grimus-Stockinger theorem
The Grimus-Stockinger theorem

Let $\psi(\vec{p})$ be a three times continuously differentiable function on $\mathbb{R}^3$, such that $\psi$ itself and all its first and second derivatives decrease at least like $1/|\vec{p}|^2$ for $|\vec{p}| \to \infty$. Then, for any real number $A > 0$,

$$\int d^3 p \frac{\psi(\vec{p}) e^{i\vec{p}\vec{L}}}{A - \vec{p}^2 + i\epsilon} \xrightarrow{|\vec{L}| \to \infty} \frac{2\pi^2}{L} \psi(\sqrt{A}L) e^{i\sqrt{A}L} + O(L^{-\frac{3}{2}}).$$

$\Rightarrow$ Quantification of requirement of on-shellness for large $L = |\vec{L}|$.

From the amplitude to the transition rate

Amplitude:

\[ iA = \frac{-i}{2L} \mathcal{N} \delta(E_S - E_D) \exp \left[ -\frac{E_S^2 - m_j^2}{2\sigma_p^2} \right] \sum_j M^\mu M^{\nu*} |U_{ej}|^2 e^{i\sqrt{E_S^2 - m_j^2}L} \]

\[ \sigma_p^{-2} = (m_{H\omega_H,S} + m_{He\omega_{He,S}})^{-1} + (m_{H\omega_H,D} + m_{He\omega_{He,D}})^{-1} \]
From the amplitude to the transition rate

Amplitude:

\[ iA = -\frac{i}{2L} N \delta(E_S - E_D) \exp \left[ -\frac{E_S^2 - m_j^2}{2\sigma_p^2} \right] \sum_j M^\mu M^{\nu *}|U_{ej}|^2 e^{i\sqrt{E_S^2 - m_j^2}L} \]

\[ \cdot \bar{u}_{e,S} \gamma_\mu \frac{1-\gamma^5}{2}(\rho_j + m_j)^{1+\gamma^5} \gamma_{\nu} u_{e,D}, \]

\[ \sigma_p^{-2} = (m_{H\omega_{H,S}} + m_{He\omega_{He,S}})^{-1} + (m_{H\omega_{H,D}} + m_{He\omega_{He,D}})^{-1} \]

Transition rate: Integrate $|A|^2$ over densities of initial and final states

\[ \Gamma \propto \int_{0}^{\infty} dE_{H,S} \; dE_{He,S} \; dE_{He,D} \; dE_{H,D} \]

\[ \cdot \delta(E_S - E_D) \rho_{H,S}(E_{H,S}) \rho_{He,D}(E_{He,D}) \rho_{He,S}(E_{He,S}) \rho_{H,D}(E_{H,D}) \]

\[ \cdot \sum_{j,k} |U_{ej}|^2 |U_{ek}|^2 \exp \left[ -\frac{2E_S^2 - m_j^2 - m_k^2}{2\sigma_p^2} \right] e^{i\left(\sqrt{E_S^2 - m_j^2} - \sqrt{E_S^2 - m_k^2}\right)L} \]

Analogue of Lamb-Mössbauer factor
(Recoil-free fraction)

Oscillation phase
The Lamb-Mössbauer factor

The Lamb-Mössbauer factor is the relative probability of recoil-free emission and absorption, compared to the total emission and absorption probability.

\[ \exp \left[ -\frac{2}{E_i} - \frac{m_j^2}{2\sigma^2} - \frac{m_k^2}{2\sigma^2} \right] = \exp \left[ -\left( \frac{p_{\text{min}}^{jk}}{2\sigma^2} \right)^2 \right] \exp \left[ -\left| \Delta m^2_{jk} \right| \frac{2\pi}{\sigma^2} \right] \]

where \( (p_{\text{min}}^{jk})^2 = E_i^2 - \max(m_j^2, m_k^2) \).

\[ 4\pi\sigma_x E_i / \sigma_p \lesssim L_{\text{osc}}^{jk}, \]

which is easily fulfilled in realistic situations.
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⇒ Emission and absorption of lighter mass eigenstates is suppressed compared to that of heavy mass eigenstates
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Convenient reformulation:

\[
\exp \left[ - \frac{2E_S^2 - m_j^2 - m_k^2}{2\sigma_p^2} \right] = \exp \left[ - \frac{(p_{jk}^{\text{min}})^2}{\sigma_p^2} \right] \exp \left[ - \frac{|\Delta m_{jk}^2|}{2\sigma_p^2} \right]
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where \((p_{jk}^{\text{min}})^2 = E_S^2 - \max(m_j^2, m_k^2)\).
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⇒ Localization condition

\[4\pi \sigma_x E / \sigma_p \lesssim L^{\text{osc}}_{jk},\]

(with \(\sigma_x = 1/2\sigma_p\)) is satisfied if \(L^{\text{osc}}_{jk} \gtrsim 2\pi \sigma_x\), which is easily fulfilled in realistic situations.
Line broadening

Energy levels of $^3$H and $^3$He in the source and detector are smeared e.g. due to spin-spin interactions, crystal impurities, lattice defects, etc.

R. S. Raghavan, hep-ph/0601079
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Good approximation:

$$\rho_{A,B}(E_{A,B}) = \frac{\gamma_{A,B}/2\pi}{(E_{A,B} - E_{A,B,0})^2 + \gamma_{A,B}^2/4}$$
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Result for two neutrino flavours:

$$\Gamma \propto \frac{(\gamma_S + \gamma_D)/2\pi}{(E_{S,0} - E_{D,0})^2 + (\gamma_S + \gamma_D)^2} \cdot \left\{ 1 - 2s^2c^2 \left[ 1 - \frac{1}{2}(e^{-L/L_{coh}^S} + e^{-L/L_{coh}^D}) \cos \left( \pi \frac{L}{L_{osc}} \right) \right] \right\}$$

$$L_{coh}^{S,D} = 4\bar{E}^2/\Delta m^2\gamma_{S,D}$$
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In realistic cases: $L^{coh}_{S,D} \gg L_{osc} \Rightarrow$ Decoherence is not an issue.
Outline

1. The Mössbauer neutrino experiment
2. Oscillations of Mössbauer neutrinos: Qualitative arguments
3. Mössbauer neutrinos in QFT
4. Conclusions
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- **Wave packet treatment**: 

  - Coherence and localization conditions are irrelevant for realistic experiments.
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    - Only properties of the source and the detector are put in by hand.
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    - Nonzero line width leads to coherence condition.
    - Both conditions are easily fulfilled in realistic experiments.
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Thank you!