Simulation and analysis of source related effects for KATRIN

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Column density
Tritium purity
Magnetic field strength
Temperature
Contents

- KATRIN
- Windowless Gaseous Tritium Source
- Simulation
  - Density profile
  - Temperature profile
  - Source Spectrum Calculation
- Analysis
  - Ensemble method
    - Column density
    - Temperature
  - Profile likelihood
    - Tritium purity
- Summary and Outlook
Direct determination of neutrino mass: $\beta$-decay

Tritium $\beta$-decay, $E_0 = 18.6$ keV

$$m_v^2 = \sum_i \left| U_{ei} \right|^2 \cdot m_i^2$$

$$\frac{dN_i}{dE} \propto (E_0 - E) \sqrt{(E_0 - E)^2 - m_i^2c^4}$$

Model independent approach: kinematics & energy conservation
The KATRIN experiment

Calibration and monitoring system

WGTS cryostat

Transport section

Spectrometers and detector section

Strong $\beta$-source

$>10^{11}$ e$^-$/s

Remove molecules and ions

Transmit e$^-$ with $E>qU$

$\Delta E=0.93$ eV

Count e$^-$

$\sim 1$ e$^-$/s
Windowless Gaseous Tritium Source WGTS

Column density
Tritium purity
Magnetic field strength
Temperature

DPS1-R
Module 2
3.6 T
85 K
120 K

Module 1
3.6 T
30 K
120 K

WGTS-tube
3.6 T
30 K (Standard mode)
120 K (Krypton mode)

Solenoid
T$_2$/Kr injection chamber
Beamtube

16 m
10 m

DPS1-F
Module 1
5.6 T
30 K
120 K

Module 2
5.6 T
85 K
120 K

R = Rear system
F = Forward system

Markus Hötzel – Simulation and analysis of source related effects for KATRIN
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Motivation: Systematic effects of the source

- KATRIN sensitivity: \( m_\nu < 200 \text{meV/c}^2 \) (90% C.L.)

- 3 years measurement time: \( \sigma_{\text{stat}} = 0.018 \text{eV}^2/\text{c}^4 \)

- Systematic effects: \( \sigma_{\text{syst}} \leq 0.017 \text{eV}^2/\text{c}^4 \)

- “4 out of 5 systematic effects are related with the WGTS”
  - Monitoring of the column density
  - Energy losses due to elastic/inelastic scattering
  - Magnetic field variations in the WGTS
  - Description of the final state distribution
Simulation
1-D density calculations

- Hydrodynamical regime
- Transitional regime
- Free molecular flow
- Unified calculation based on Boltzmann equation
2-D / Pseudo-3-D density profile

- Azimuthal temperature gradient causes 2-D deviations

2-phase LNe, $T_0 = 30$ K
Column density of the WGTS

Deviations on the $10^{-5}$ level

$\rightarrow$ Negligible for KATRIN
Temperature profile (Measurement)

- „Demonstrator“ tests, 2011
- Original components, cryosystem
- Test of beam tube cooling
- No tritium circulation

Temperature stability

\[ \Delta T / T = 1 \cdot 10^{-4} \]
Temperature profile (Measurement)

- „Demonstrator“ tests, 2011
- Original components, cryosystem
- Test of beam tube cooling
- No tritium circulation

- Temperature gradient ~ 1 K
- Increased thermal radiation
- Thermal conduction identified

→ Solved at assembly of the WGTS
Source Spectrum Calculation

- Combine various models of source parameters
  - Gas dynamics
  - Magnetic field
  - Energy spectrum of $T_2$
  - Doppler effect
  - Scattering of electrons in the source

- Use "voxelized" description of the WGTS

- Calculate expected count rates at detector
Analysis
Variance of ML estimator: MC method

- Simulate a „measured spectrum“ of KATRIN
- Use slightly different source parameters as “theoretical spectrum”
- Fit and store best fit value $\hat{m}_v^2$
- Repeat e.g. 4000 times
- Read off systematic shift $\Delta m_v^2$ and statistical uncertainty $\sigma_{\text{stat}}$
Systematic influence of the column density

\[ \Delta m^2_{\nu} (\text{eV}^2) \times 10^{-3} \]

\[ \Delta \rho_d / \rho_d \]

\[ \Delta p_{\text{in}} / p_{\text{in}} \]

unaccounted shift

\[ -7.5 \times 10^{-3} \text{eV}^2 \]
Systematic influence of the source temperature

\[ \Delta \frac{T}{T} \times 10^{-3} \]

\[ \text{systematic shift } \Delta m^2 \text{ (eV}^2\text{)} \]

\[ + 7.5 \cdot 10^{-3} \text{ eV}^2 \]
Influence of the temperature profile

- Increased temperature gradient $>3K$ due to additional thermal radiation

- Neglect temperature profile:

$$\Delta m^2_\nu = (1.0 \pm 2.3) \cdot 10^{-4} \text{ eV}^2$$
### Requirements & Achievements

<table>
<thead>
<tr>
<th>source of syst. uncertainty</th>
<th>requirements</th>
<th>( \Delta m^2_{\nu} ) (10^{-3} eV^2)</th>
<th>achievements</th>
</tr>
</thead>
<tbody>
<tr>
<td>variations of column density</td>
<td>( \frac{\Delta \rho d}{\rho d} &lt; 2 \cdot 10^{-3} )</td>
<td>&lt; 1.5</td>
<td></td>
</tr>
<tr>
<td>injection pressure</td>
<td>( \frac{\Delta \rho_{\text{in}}}{\rho_{\text{in}}} &lt; 2 \cdot 10^{-3} )</td>
<td></td>
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<tr>
<td>exit pressure</td>
<td>( \frac{\Delta \rho_{\text{ex}}}{\rho_{\text{ex}}} &lt; 0.06 )</td>
<td></td>
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<tr>
<td>temperature</td>
<td>( \frac{\Delta T}{T} &lt; 2 \cdot 10^{-3} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>tritium purity</td>
<td>( \frac{\Delta \varepsilon_T}{\varepsilon_T} &lt; 2 \cdot 10^{-3} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>WGTS magnetic field</td>
<td>( \frac{\Delta B_S}{B_S} &lt; 2 \cdot 10^{-3} )</td>
<td>&lt; 2</td>
<td>1.3 ( \cdot 10^{-4} )</td>
</tr>
<tr>
<td>WGTS potential</td>
<td>( \Delta U &lt; 10 \text{ mV} )</td>
<td>&lt; 1.2</td>
<td></td>
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</tbody>
</table>

Experimental achievements reported in [arXiv:1205.5421](https://arxiv.org/abs/1205.5421)
Profile likelihood & Systematics

- Include systematics into analysis


At KATRIN

- Parameter of interest: \( m_{\nu}^2 \)

- Nuisance parameters: \( \vec{\theta} = \{ \rho d, \varepsilon_T, \ldots \} \)

- Profile likelihood: 
  \[
  \lambda(m_{\nu,0}^2 \mid \bar{X}) = \frac{\sup \{L(m_{\nu,0}^2, \bar{\theta} \mid \bar{X}); \bar{\theta}\}}{\sup \{L(m_{\nu}^2, \bar{\theta} \mid \bar{X}); m_{\nu}^2, \bar{\theta}\}}
  \]
Profile likelihood

\[- \log L(\bar{X} \mid m_v^2, \bar{\theta}) = - \sum_i p(X_i \mid m_v^2, \bar{\theta})\]
Profile likelihood with constraints (pull method)

\[-\log L(\bar{X} \mid m_v^2, \bar{\theta}) = -\sum_i p(X_i \mid m_v^2, \bar{\theta}) - \frac{(|\epsilon_T - \bar{\epsilon}_T|^2)}{2\sigma^2}\]

Constraint, e.g., external measurement of tritium purity $\bar{\epsilon}_T$.
Summary

- „4 out of 5 systematic uncertainties are related with the WGTS“
- Simulation with detailed source model
  - Density profile
  - Temperature profile
  - Spectrum Calculation
- Analysis
  - MC methods
  - Profile likelihood to include systematics
- Results
  - Requirements on source parameters validated
  - Experimental achievements e.g. „Demonstrator“ measurements

Outlook

- Full 3-D gasdynamics simulation of pumping chambers
- Use provided analysis routines to investigate further systematic effects
2-D density calculations

Injection region

Pumping chamber
Bulk velocity in the WGTS

![Diagram showing bulk velocity $u_z$ as a function of radial distance $r$ to the beam axis for different $z$ values (0.5 m, 2.0 m, 4.0 m, 4.5 m).]
Systematic influence of the tritium purity

![Graph showing systematic shift vs. unaccounted shift]

- Systematic shift $\Delta m_y^2$ (eV$^2$)
- Unaccounted shift $\Delta \varepsilon_T / \varepsilon_T$