EBERHARD KARLS
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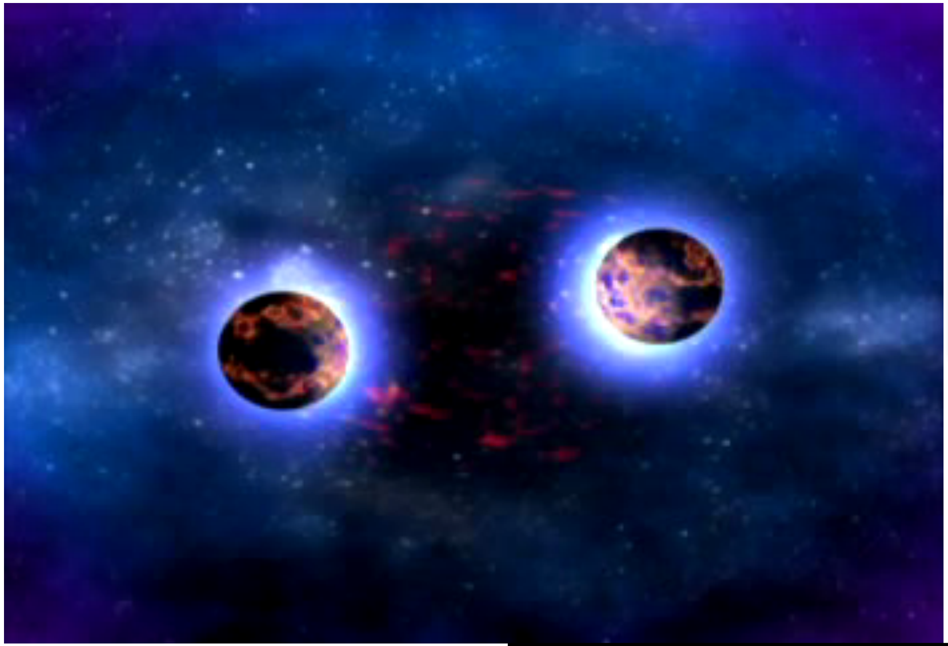
Gravitational Wave Sources

Kostas Kokkotas

Coalescing BH/NS Binaries

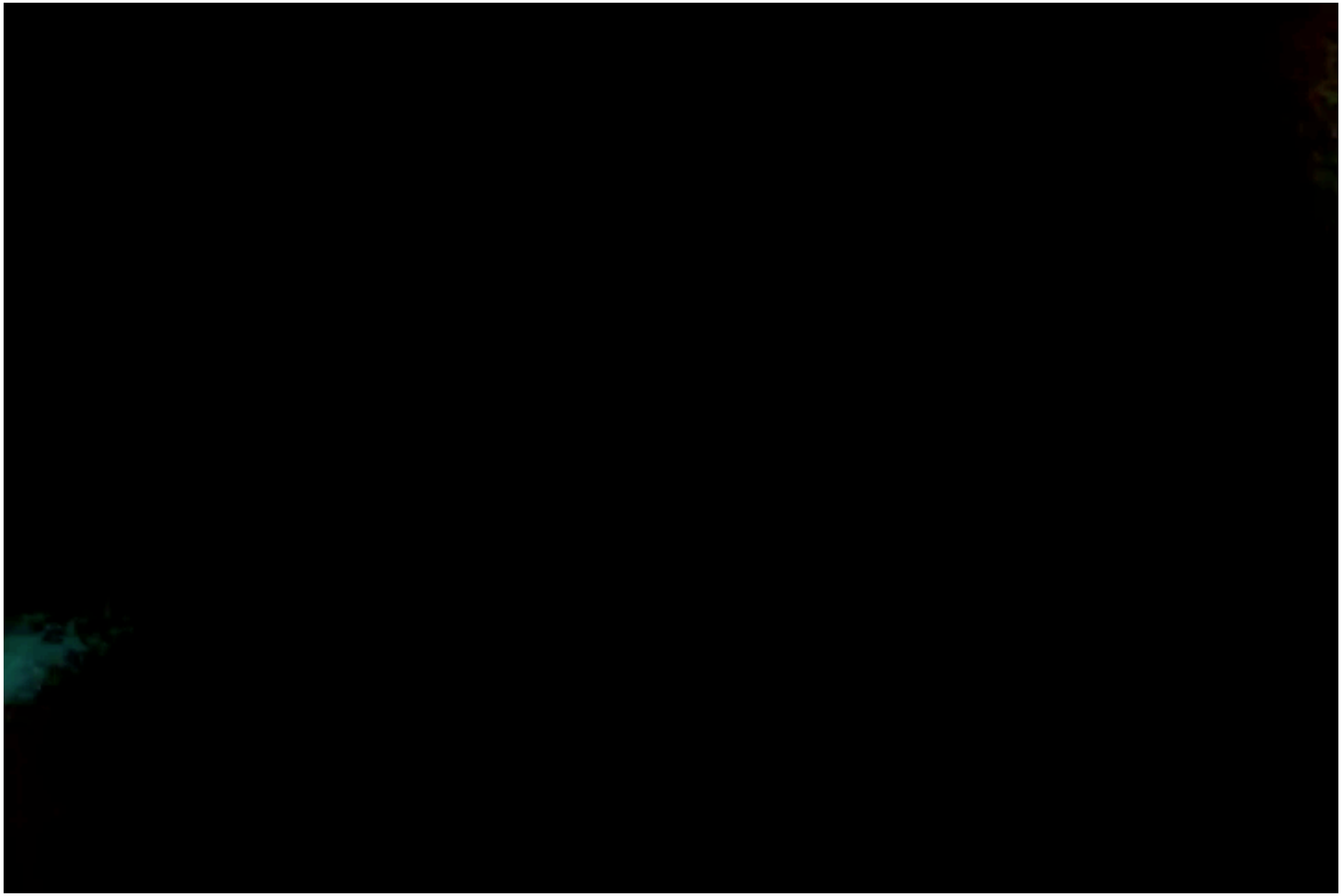


Binaries are the primary sources for GW detectors both ground based and space

Merging Super-massive BHs can be observed from the edges of the Universe

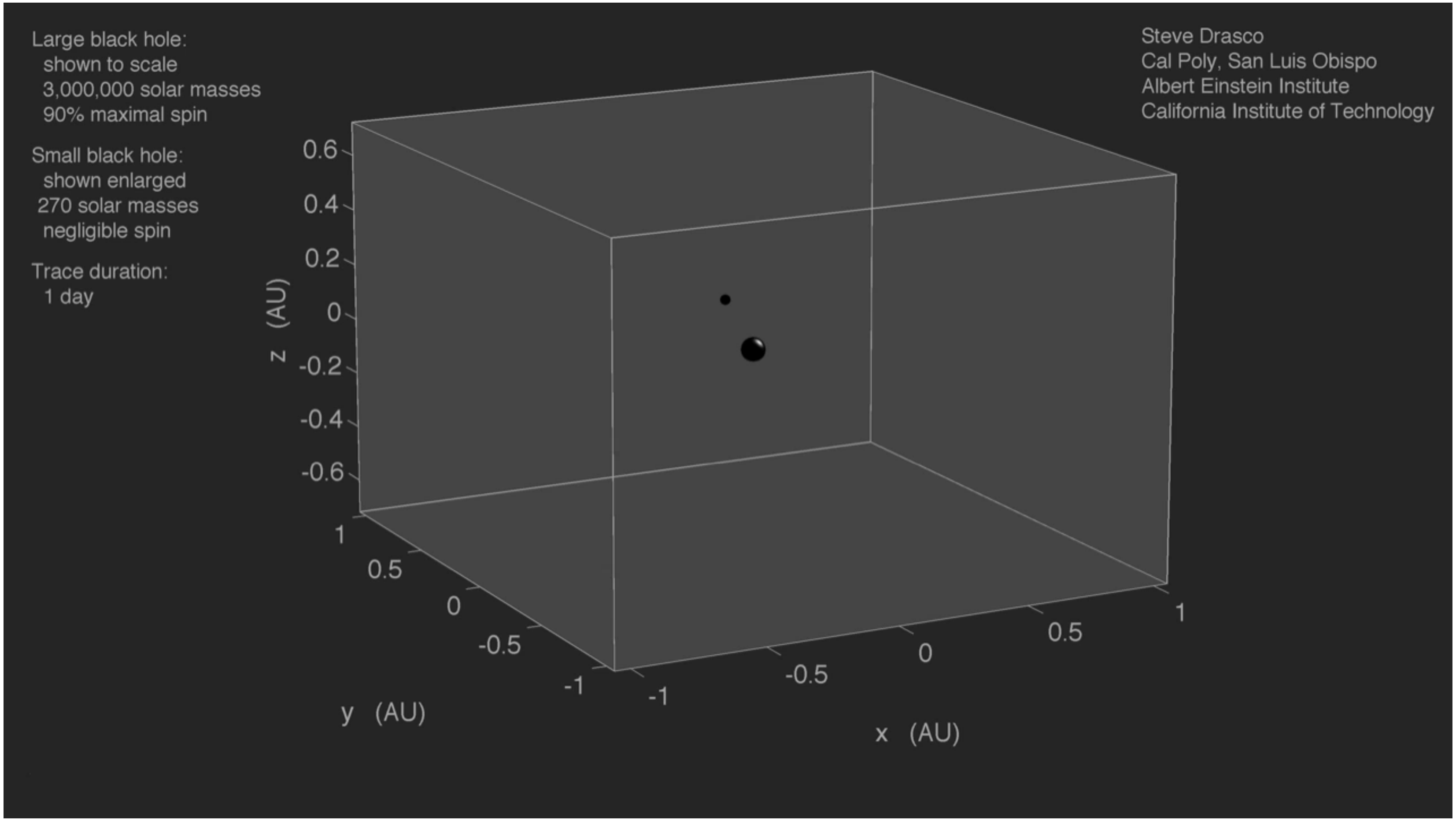
Events/year	Virgo/LIGO 2007	Adv. Detectors 2015+	ET 2025
NS/NS	~0.02	~40	millions
BH/NS	~0.006	~10	10^4
BH/BH	~0.01	~20	millions
Total	~0.04	$\gtrsim$70	

Merging Galaxies



Merging Galaxies

Black Hole “Melody”



Coalescence of Compact Binaries

- During the frequency change from 100-200Hz GWs carry away **$5\times10^{-3} M_{\odot} c^2$.**
- In LIGOs band
  - **NS/NS** (~**16000** cycles)
  - **NS/BH**(~**3500** cycles)
  - **BH/BH**(~**600** cycles)
- The GW amplitude is:
- **Larger total mass**

improves detection probability.

Events/year	**Virgo/LIGO 2007**	**Adv. Detect. 2015+**	**ET 2025**
NS/NS	**~0.02**	**~40**	**millions**
BH/NS	**~0.006**	**~10**	**10^4**
BH/BH	**~0.01**	**~20**	**millions**
Total	**~0.04**	**$\gtrsim$70**	

- ✓ **Phase effects are important**, if the signal and the template get out of phase their cross correlation will be reduced.
- ✓ **High accuracy templates** are needed for accurate detection.

$$h \sim \frac{M_{chirp}^{5/3} f^{2/3}}{r}$$

$$h \approx 7.5\times10^{-23}\left(\frac{M}{2.8M_{\odot}}\right)^{2/3}\left(\frac{\mu}{0.7M_{\odot}}\right)\left(\frac{f}{100Hz}\right)^{2/3}\left(\frac{100Mpc}{r}\right)$$

An interesting observation

- The observed frequency change will be:

$$\dot{f} \sim f^{11/3} M_{chirp}^{5/3}$$

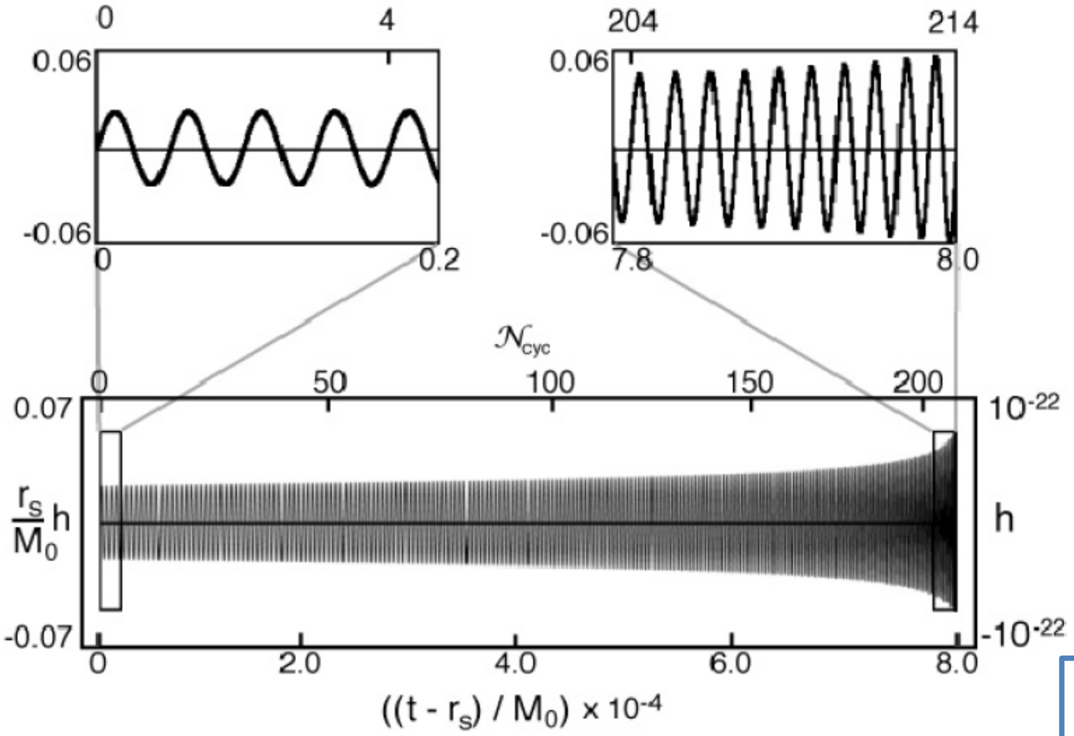
$$M_{chirp}^{5/3} = \mu M^{2/3}$$

- The corresponding amplitude will be :

$$h \sim \frac{M_{chirp}^{5/3} f^{2/3}}{r} = \frac{\dot{f}}{f^{3} r}$$

- Since both frequency and its rate of change are measurable quantities, we can immediately compute the chirp mass.
- The third relation provides us with a direct estimate of the distance of the source
- Post-Newtonian relations can provide the individual masses

The inspiral signal



The chirp signal

$$h_+ \propto \frac{\mathcal{M}^{5/3}}{r} f^{2/3} \cos(2\pi f t)$$

$$h_\times \propto \frac{\mathcal{M}^{5/3}}{r} f^{2/3} \sin(2\pi f t)$$

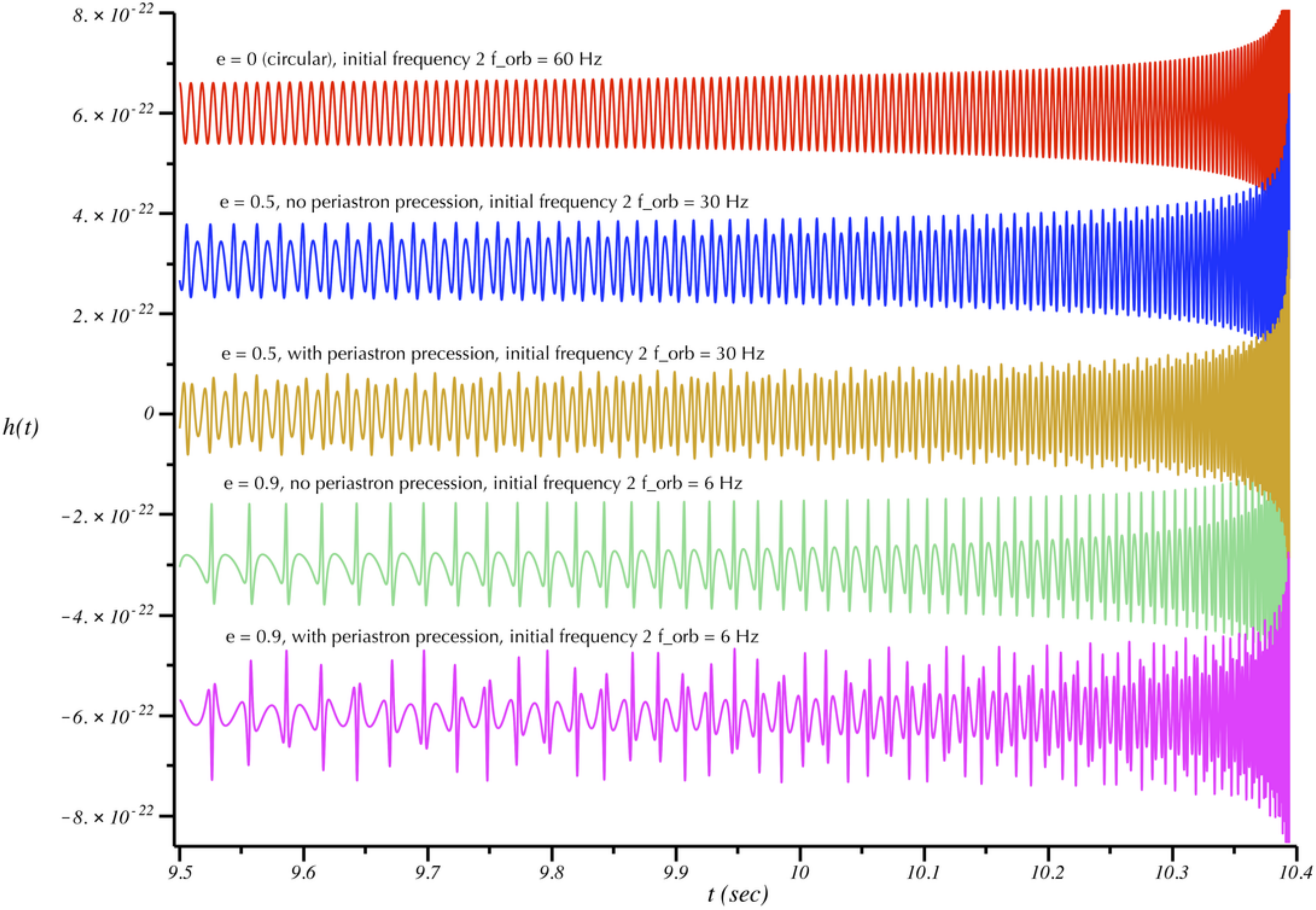
$$f = K_0 \, \mathcal{M}^{-5/8} (t_{\rm coal} - t)^{-3/8}$$

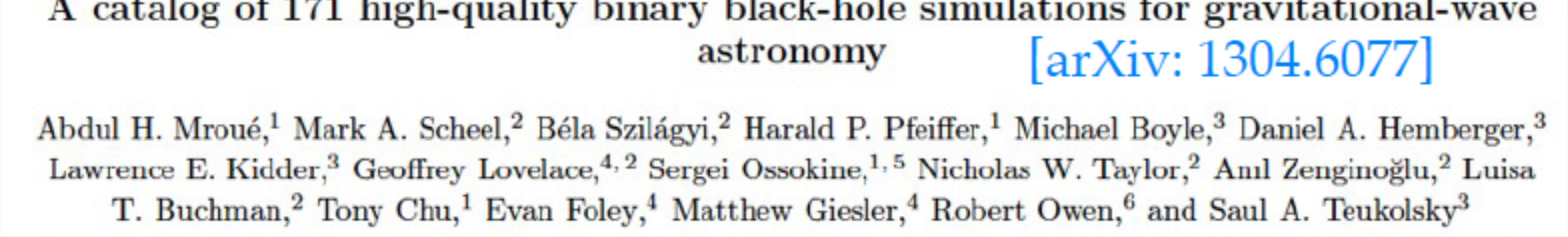
A more precise formula

Orbital phase at the 3.5PN approximation

$$
\begin{aligned}
\phi(t) = -\frac{1}{\nu}\Bigg\{ & \tau^{5/8} + \left(\frac{3715}{8064} + \frac{55}{96}\nu\right)\tau^{3/8} - \frac{3}{4}\pi\tau^{1/4} \\
+ & \left(\frac{9275495}{14450688} + \frac{284875}{258048}\nu + \frac{1855}{2048}\nu^2\right)\tau^{1/8} + \left(-\frac{38645}{172032} - \frac{15}{2048}\nu\right)\pi \ln\left(\frac{\tau}{\tau_0}\right) \\
& + \left(\frac{831032450749357}{57682522275840} - \frac{53}{40}\pi^2 - \frac{107}{56}C + \frac{107}{448}\ln\left(\frac{\tau}{256}\right)\right. \\
& + \left[-\frac{123292747421}{4161798144} + \frac{2255}{2048}\pi^2 + \frac{385}{48}\lambda - \frac{55}{16}\theta\right]\nu + \frac{154565}{1835008}\nu^2 \\
& \left. - \frac{1179625}{1769472}\nu^3\right)\tau^{-1/8} + \left(\frac{188516689}{173408256} + \frac{140495}{114688}\nu - \frac{122659}{516096}\nu^2\right)\pi\tau^{-1/4}\Bigg\}
\end{aligned}
$$

- T. Damour, G. Schaefer, L. Blanchet, C.M. Will
- B.R.Iyer, E. Poisson, P. Jaranowski,…


8. × 10^-22
6. × 10^-22
4. × 10^-22
2. × 10^-22
0
−2. × 10^-22
−4. × 10^-22
−6. × 10^-22
−8. × 10^-22
h(t)
9.5
9.6
9.7
9.8
9.9
10
10.1
10.2
10.3
10.4
t (sec)
e = 0 (circular), initial frequency 2 f_orb = 60 Hz
e = 0.5, no periastron precession, initial frequency 2 f_orb = 30 Hz
e = 0.5, with periastron precession, initial frequency 2 f_orb = 30 Hz
e = 0.9, no periastron precession, initial frequency 2 f_orb = 6 Hz
e = 0.9, with periastron precession, initial frequency 2 f_orb = 6 Hz



A catalog of 171 high-quality binary black-hole simulations for gravitational-wave astronomy

[arXiv: 1304.6077]

Abdul H. Mroué,[1] Mark A. Scheel,[2] Béla Szilágyi,[2] Harald P. Pfeiffer,[1] Michael Boyle,[3] Daniel A. Hemberger,[3] Lawrence E. Kidder,[3] Geoffrey Lovelace,[4, 2] Sergei Ossokine,[1, 5] Nicholas W. Taylor,[2] Anıl Zenginoğlu,[2] Luisa T. Buchman,[2] Tony Chu,[1] Evan Foley,[4] Matthew Giesler,[4] Robert Owen,[6] and Saul A. Teukolsky[3]

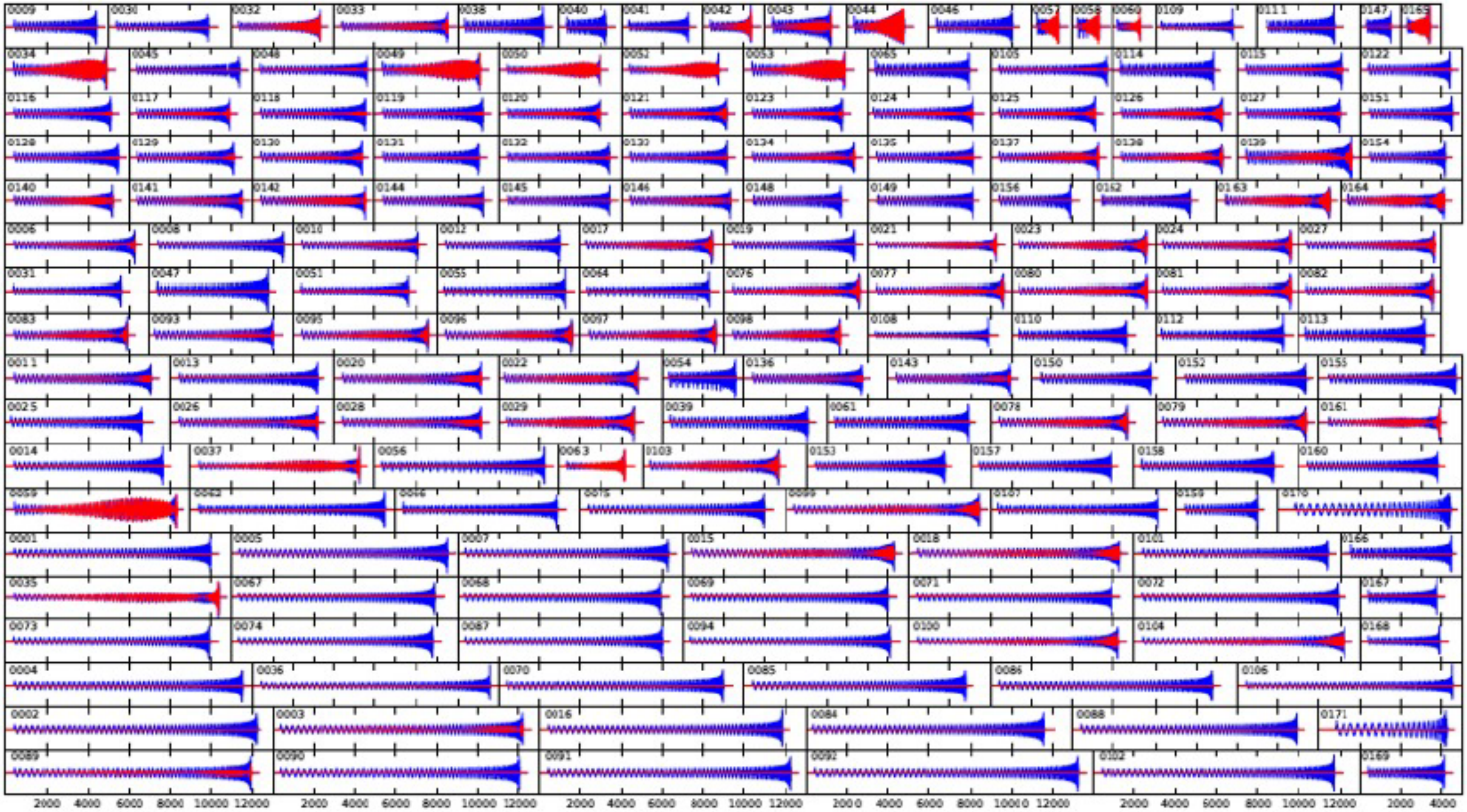


FIG. 3: Waveforms from all simulations in the catalog. Shown here are h_+ (blue) and $h_\times$ (red) in a sky direction parallel to the initial orbital plane of each simulation. All plots have the same horizontal scale, with each tick representing a time interval of $2000M$, where M is the total mass.

The Data Analysis Challenge

$$d(t) = s(t) + n(t)$$

$$\langle a | b \rangle = \int_{f_1}^{f_2} \frac{\tilde{a}(f)\tilde{b}^*(f)}{S_n(f)} df$$

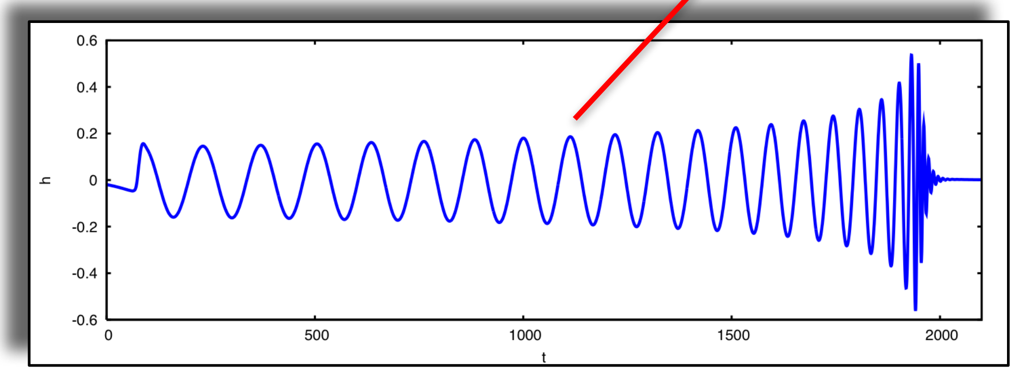
Overlap

Data = Signal + Noise

$$M(h \mid d) = \max_{\vec{\xi},\vec{\beta}} \frac{\langle h | d \rangle}{\sqrt{\langle h | h \rangle \langle d | d \rangle}}$$

Match

Template

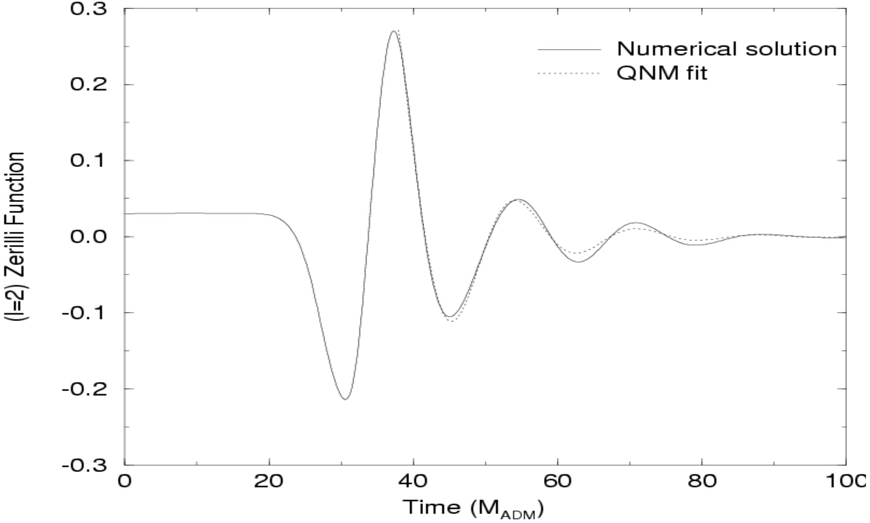


Matches > 0.98 are needed

Without the gravitational wave templates from numerical relativity, detection and characterization of sources will be extremely difficult.

Black-Hole “Ringing”

- The newly formed BH is ringing till settles down to the stationary Kerr state (QNMs).
- The **amplitude** of the ringdown waves and their energy depends on the distortion of the BH.
- Typical frequencies: 10^{-4}-10^{4} Hz
- Binary system merging
- Supermassive BHs absorbing smaller BHs or stars
- The ringing due to the excitation by the fallback material (after the collapse) might last for secs
- The **energy** emitted in GWs by the falling material is: $\Delta E \gtrsim 0.01\mu c^2(\mu/M)$.



$$\omega \approx \frac{1}{M}(0.37+0.19a) \approx 12kHz\left(\frac{M_\odot}{M}\right)$$

$$\tau \approx M(1.48+2.09a) \approx 0.05ms\left(\frac{M}{M_\odot}\right)$$

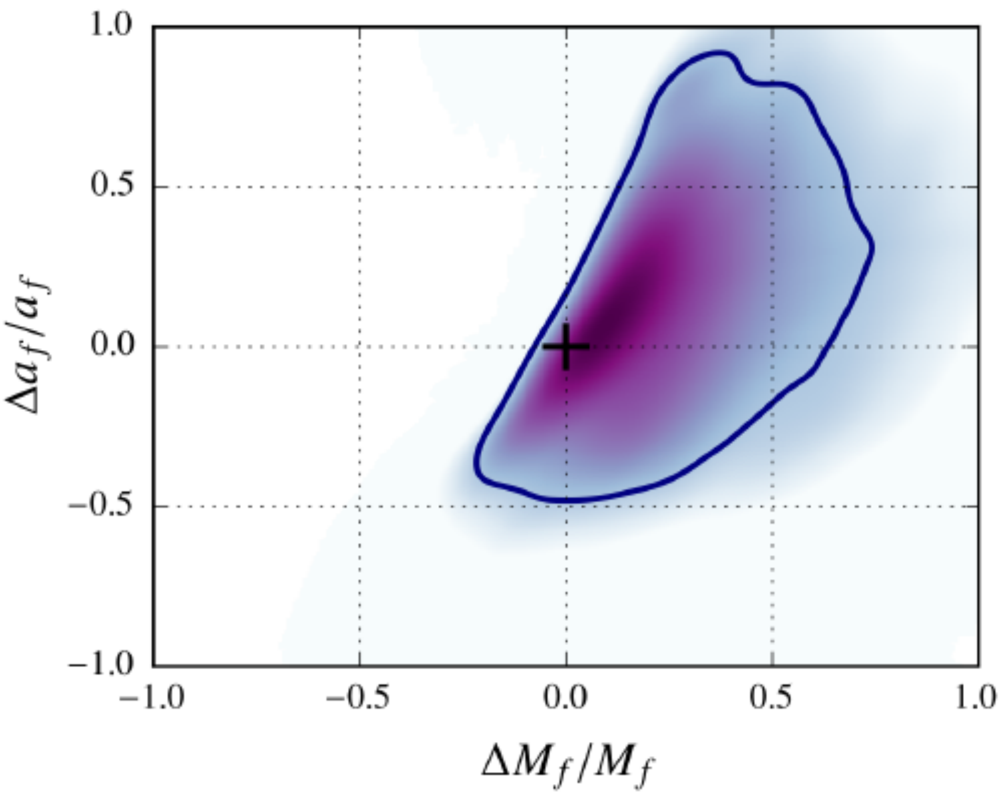
$$h_{\mathrm{eff}} \approx 2\times10^{-21}\left(\frac{\varepsilon}{0.01}\right)\left(\frac{d}{10\mathrm{Mpc}}\right)^{-1}\left(\frac{\mu}{M_\odot}\right)$$

GWs from BHs: a unique probe of their existence

Head-on Collision

of Equal-Mass BHs with Transverse Spins

Black-Hole “Ringing”



GW150914

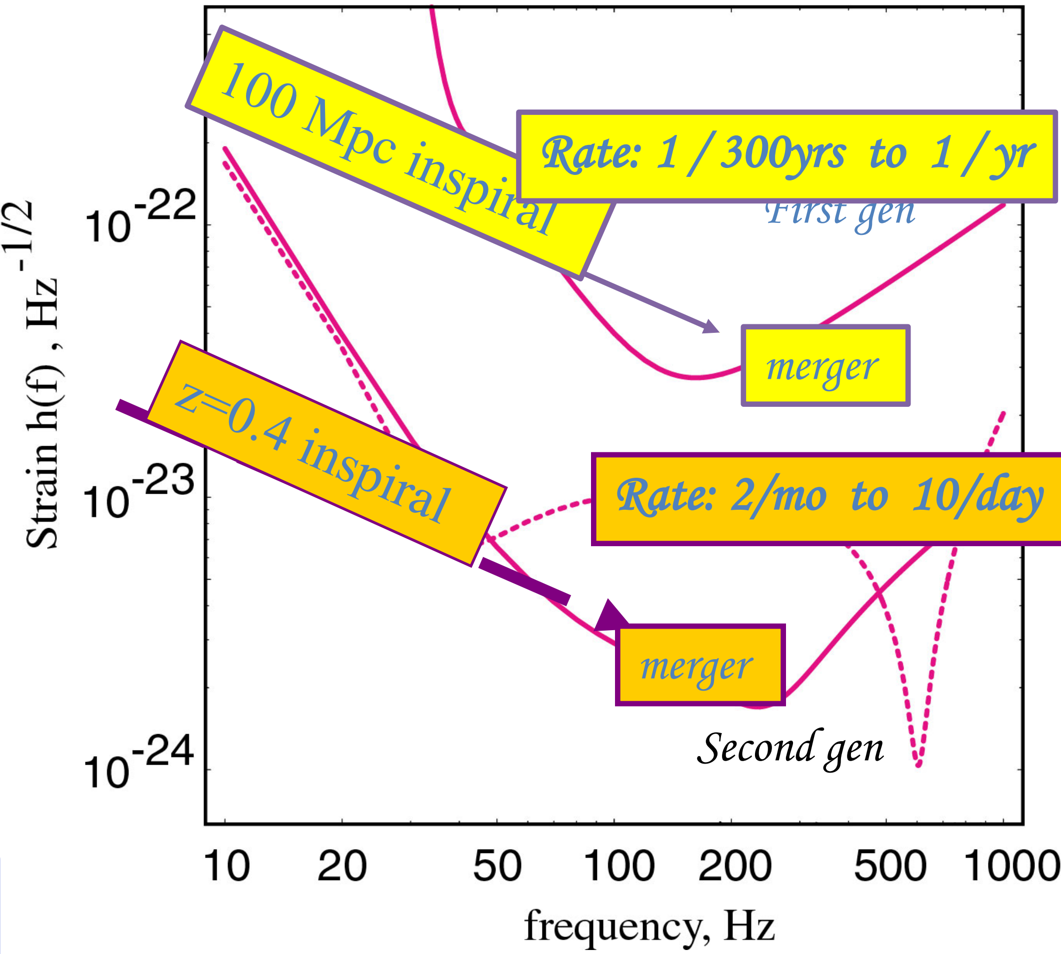
$$\omega \approx \frac{1}{M}(0.37+0.19a) \approx 12kHz\left(\frac{M_\odot}{M}\right)$$

$$\tau \approx M(1.48+2.09a) \approx 0.05ms\left(\frac{M}{M_\odot}\right)$$

GWs from BHs: a unique probe of their existence

Possible First Source:
Binary Black Hole Coalescence

- $10M_\odot + 10\,M_\odot$ BH/BH binary
- Event rates based on population synthesis
- mostly globular cluster binaries.
- Totally quiet!!



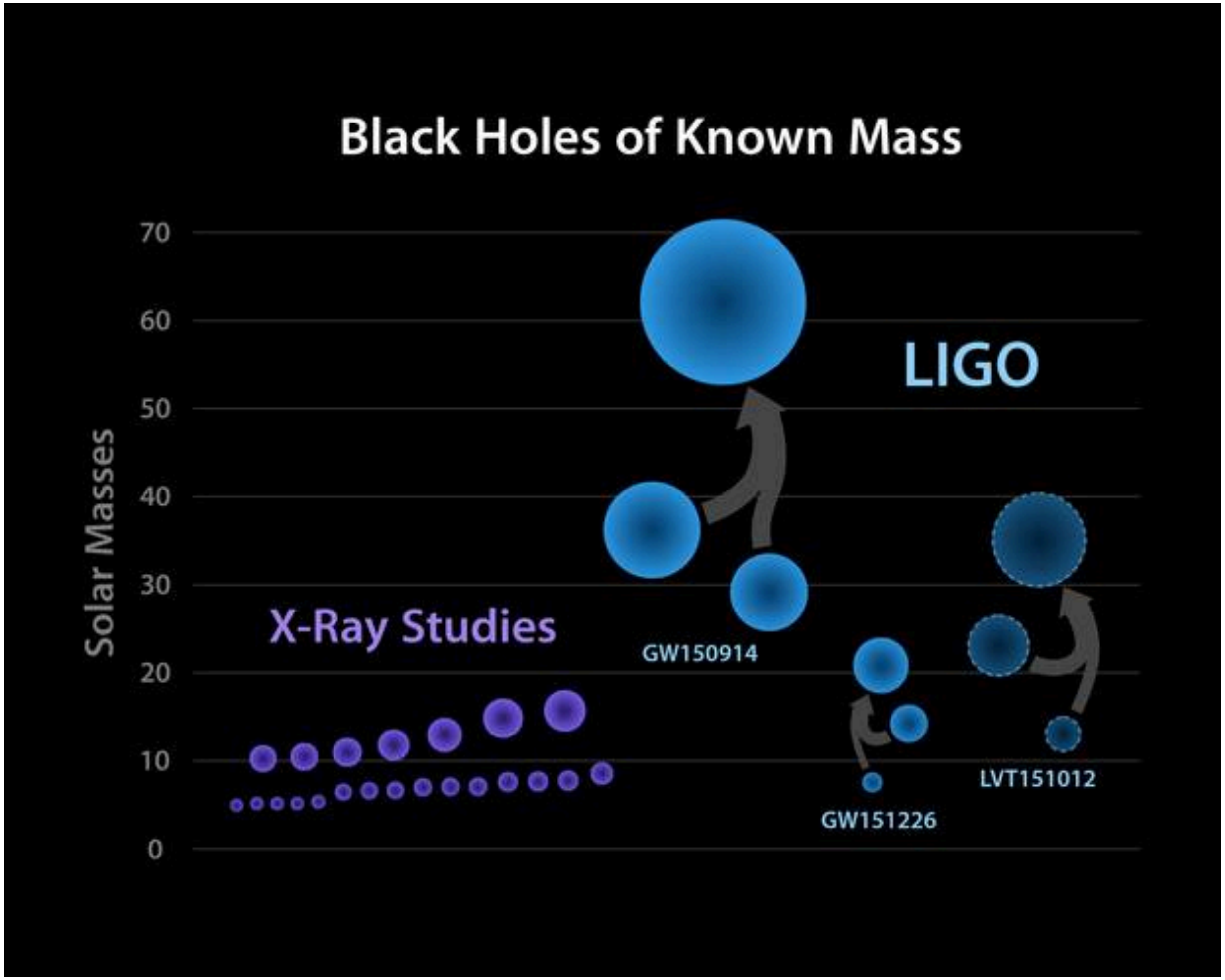
Remember: $\dot{f} \sim f^{11/3} \mathcal{M}_{chirp}^{5/3}$

A new challenge for Astrophysics
Size of Stellar Mass BHs in the Universe

GW150914
SNR =24
Net spin of BBH~0
Final spin of BH ~ 70% max

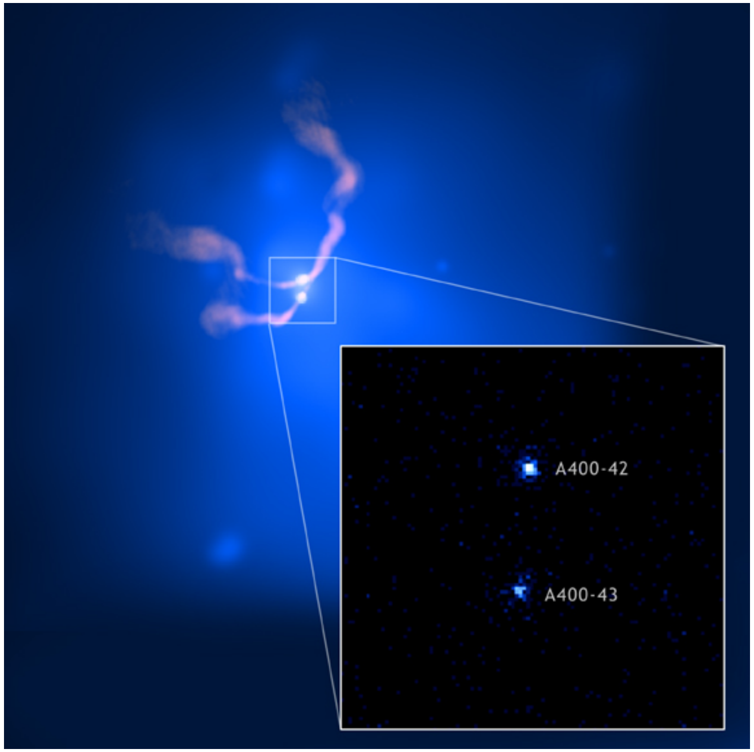
GW151226
SNR =13
Net spin of BBH~29% max
Final spin of BH ~ 75% max

LVT151012
SNR =10
Net spin of BBH~0
Final spin of BH ~ 70% max



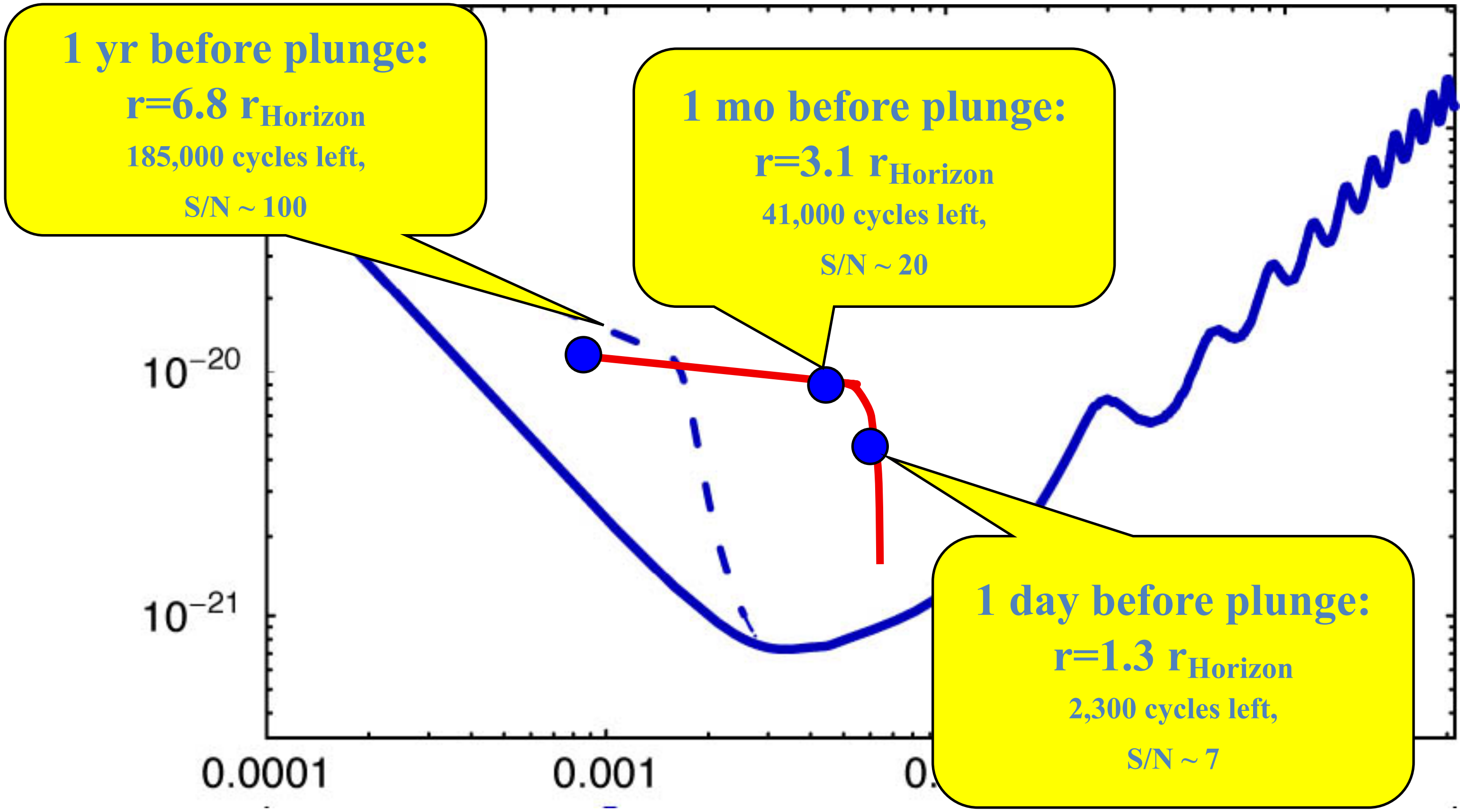
Massive Black Holes in Galaxies

- Detected masses from 10^6 -$10^9 M_\odot$. Smaller masses possible.
- Galaxy mergers should produce BH mergers.
  - Rate uncertain: 1/yr for 10^6 $M_\odot$ at $z=1$?
- Protogalaxy mergers may be richer. ***Phinney***: possibly 10^3/yr for 10^5 $M_\odot$ at $z = 7$.
- **Stellar BHs fall into massive BHs more often, but weaker radiation.**



- ***LISA can test GR in the dynamical, strong field regime…if we know the merger waveforms***
- When $m_1 \neq m_2$, GW emission is asymmetric → recoil kick

EXAMPLE: Circular, Equatorial orbit; $10\,M_\odot + 10^6\,M_\odot$; fast spin



1 yr before plunge:
$r=6.8\,r_{\mathrm{Horizon}}$
185,000 cycles left,
S/N ~ 100

1 mo before plunge:
$r=3.1\,r_{\mathrm{Horizon}}$
41,000 cycles left,
S/N ~ 20

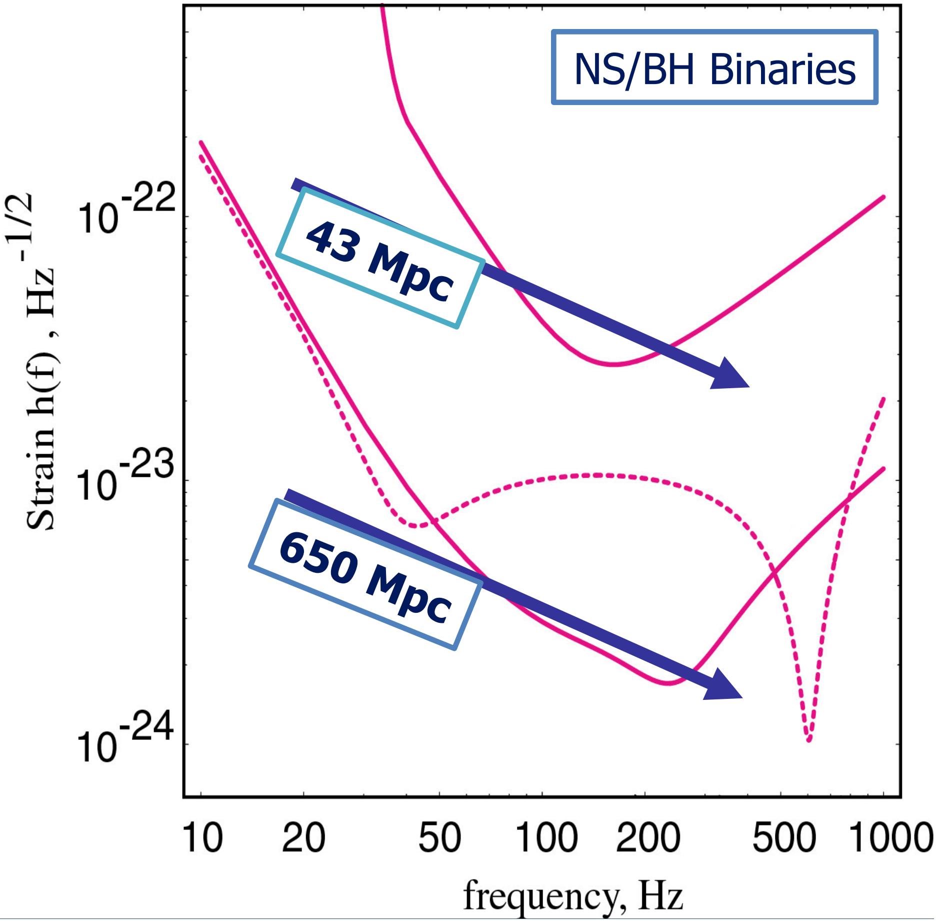
1 day before plunge:
$r=1.3\,r_{\mathrm{Horizon}}$
2,300 cycles left,
S/N ~ 7

Problems:

- Seeing them under SMBHs
- Separating them from each other

NS-BH inspiral and NS Tidal Disruption

- **NS-BH Event rates**
  - Based on *Population Synthesis*
- **Initial interferometers**
  - Range: 43 Mpc
  - 1/1000 yrs to 1per yr
- **Advanced interferometers**
  - Range: 650 Mpc
  - 2 per yr to several per day



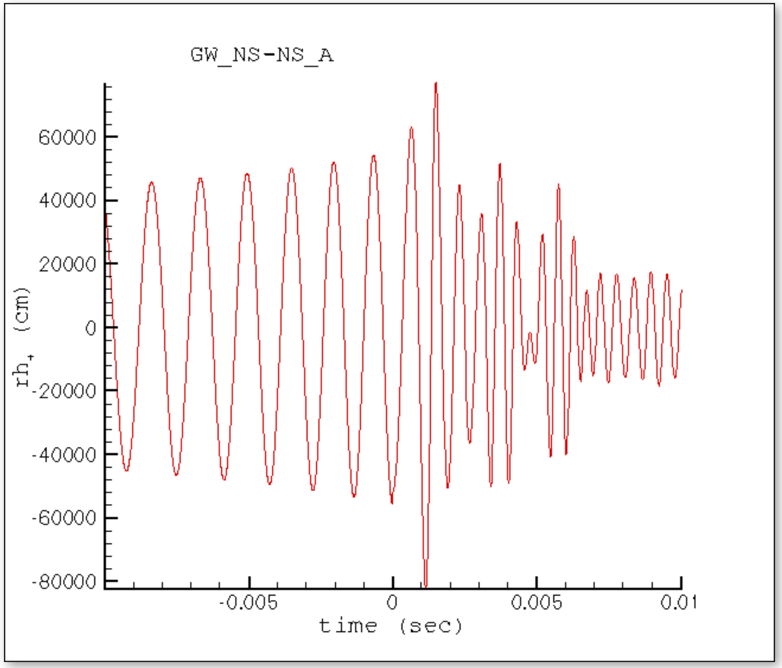
Merging phase: NS/NS & BH/NS

Tidal disruption of a NS by a BH

- **GWs could carry information about the EOS of NS eg. estimation of NS radius (15% error).**
- **The disruption waves lie in the band 300-1000Hz**
- **A few events per year at 140Mpc (Adv LIGO)**

Merging of NS-NS

- **Imprint of the NS radii just before merging ($f \lesssim 1$kHz)**
- **During the merging we could get important information about the EOS ($f \gtrsim 1$kHz)**



The NEXT SOURCE: Neutron Star Binaries

The 2nd most promising and exciting source:
NS-NS binaries

Rezzolla etal

Neutron Star Binary Inspiral

NS-NS coalescence event rates

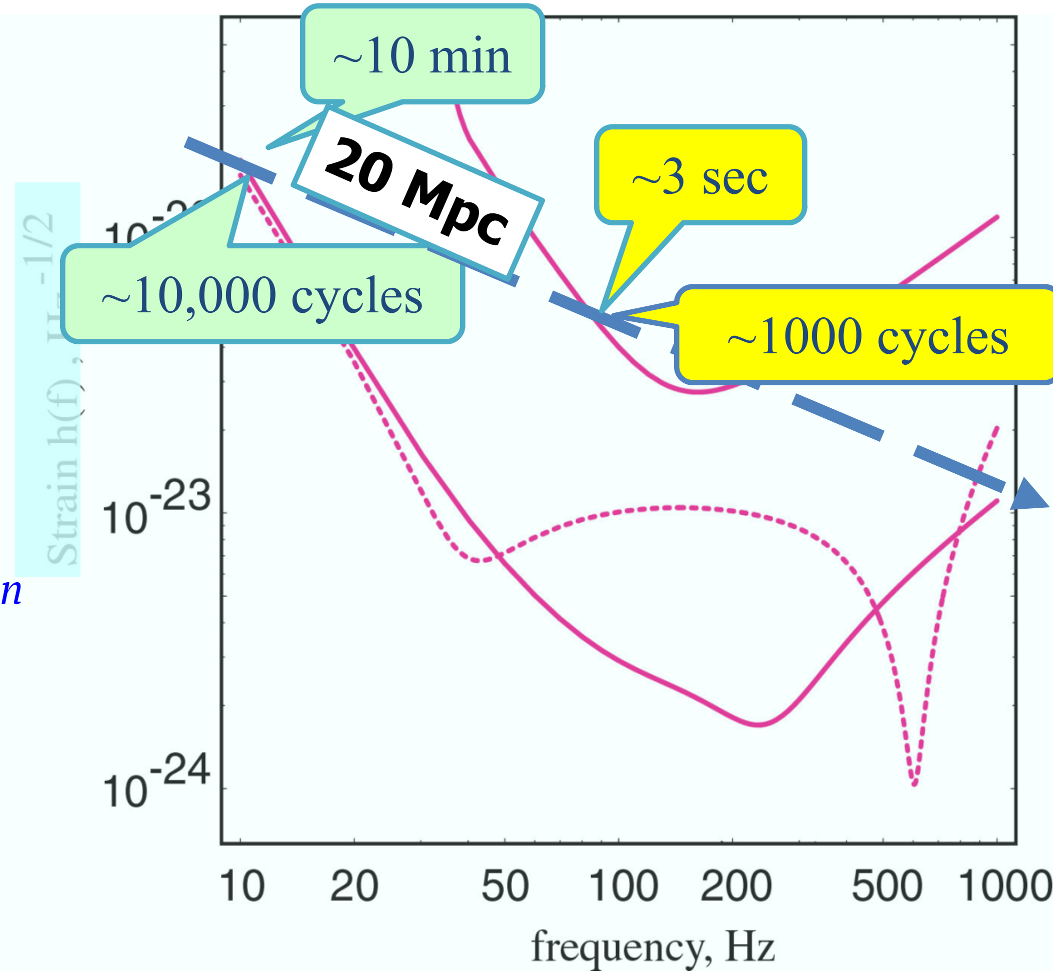
- Initial interferometers

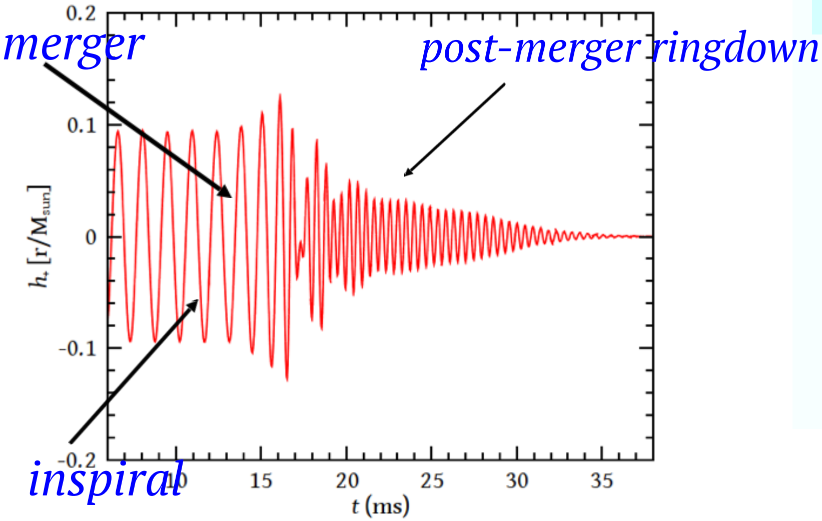
  Range: 20 Mpc

  1 per 40 yrs to 1 per 2 yrs
- Advanced interferometers

  Range: 300Mpc

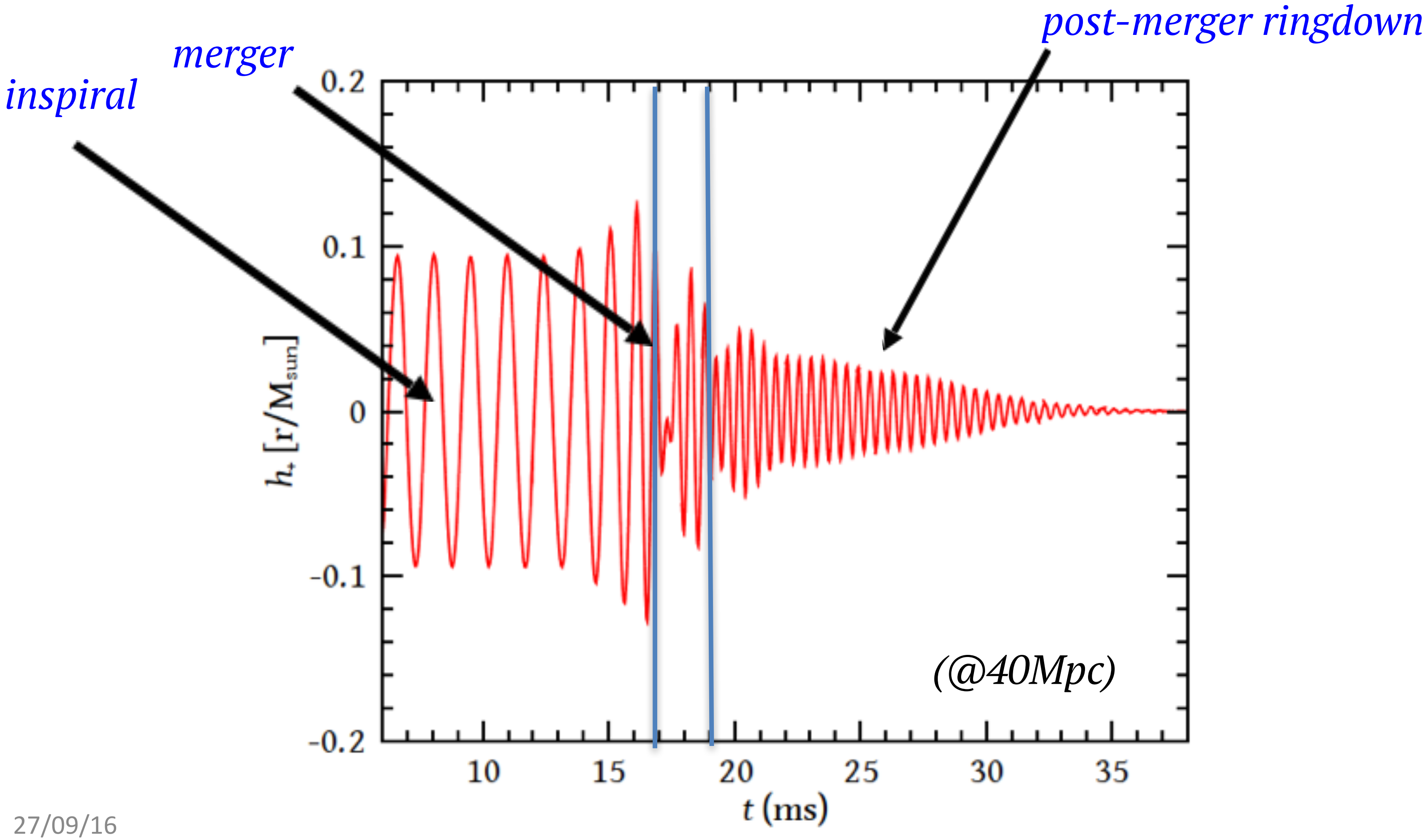
  few per yr to several per day





Binary Neutron Star Mergers
the standard scenario

The GW signal can be divided into **three distinct phases**



Binary Neutron Star Mergers

the standard scenario

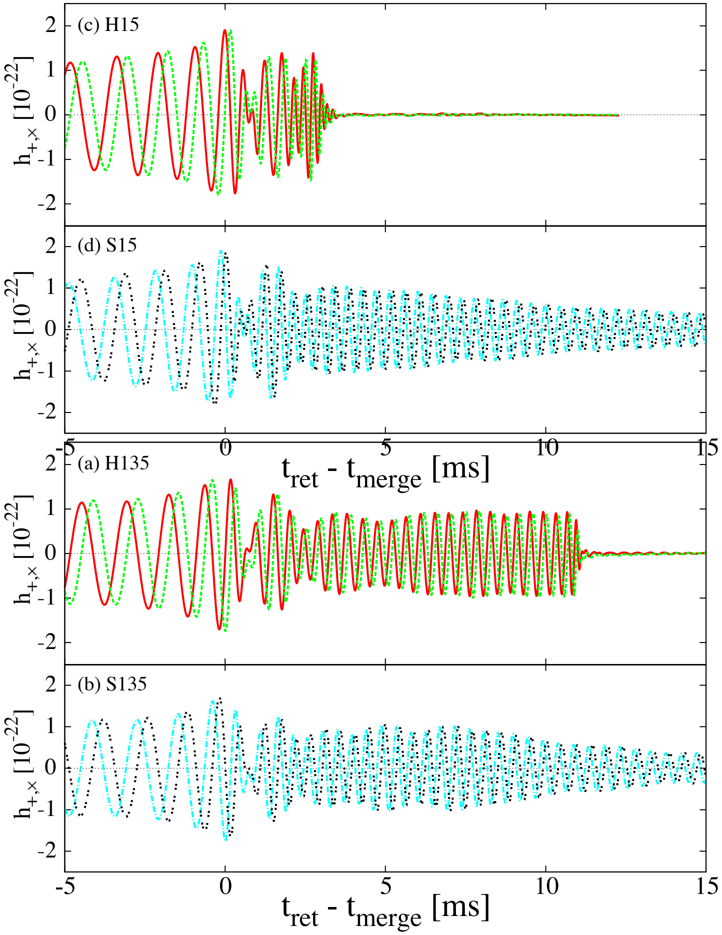
I. After the merging the final body most probably will be a supramassive NS (2.5-3 $M_\odot$)

II. The body will be **initially** differentially rotating

III. The “averaged” magnetic field will amplified due to magnetic field instabilities (up to 2-4 orders of magnitude)

IV. The strong magnetic field and the emission of GWs will drain rotational energy

V. This phase will last only a few tenths of msecs and can potentially provide information for the EOS



Kiuchi, Sekiguchi, Kyutoku, Shibata2012

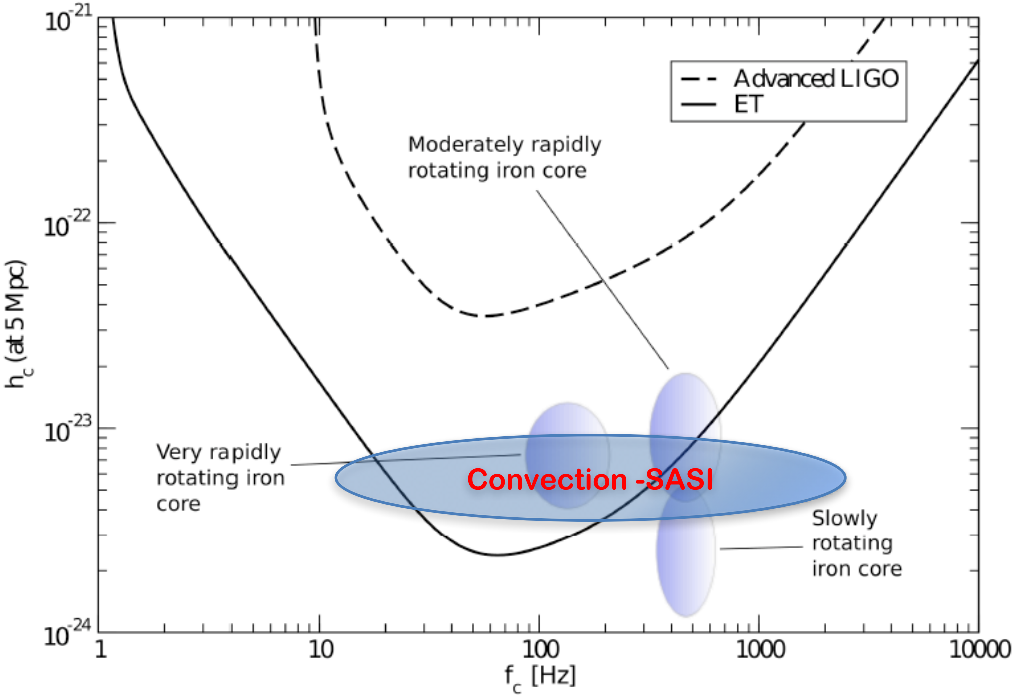
The High Frequency Window

- **Supernova Core Collapse**
  - The violent dynamics associated with a supernova core collapse is expected to lead to GW emission through a number of channels
- **Rotating Deformed Neutron Stars**
  - Asymmetries, generated either by strains in the star's crust or by the magnetic field, are expected to slowly leak rotational energy away from spinning neutron stars.
- **Oscillations and Instabilities of NS**
  - Neutron stars have rich oscillation spectra which, if detected, could allow us to probe the internal composition "GW Asteroseismology"
- **Magnetars**
  - Magnetar flares emit huge amounts of EM radiation, if a small percentage is emitted in GW they can be a promising source.

Core Collapse

Leads to GW emission through a number of channels related to:

- ✓ **The dynamics of the PNS and its immediate environment**
- ✓ **The convective zone behind the shock front**

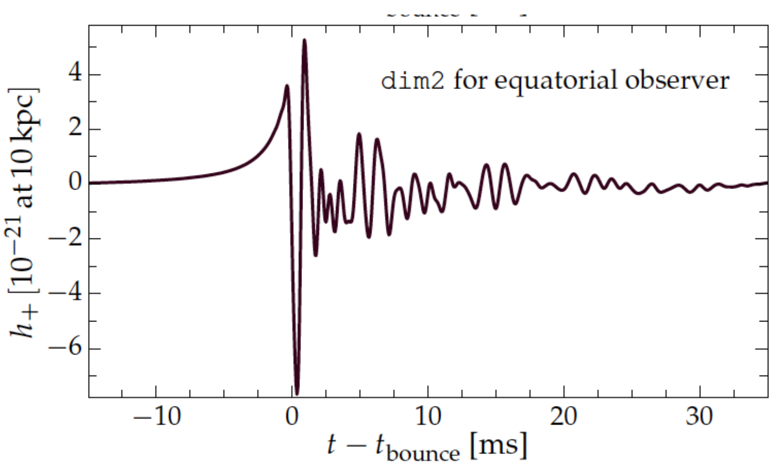


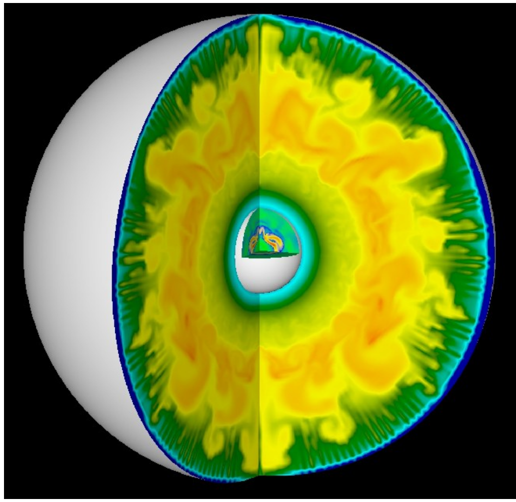
1. **Slowly rotating iron cores : bounce and initial ringdown (700-900Hz)**
2. **Faster rotation amplifies the bounce signal (400-800 Hz)**
3. **Very rapid rotation leads to bounce at subnuclear densities (100-200 Hz)**
4. **Prompt convection shortly after core bounce due to negative lepton gradients (50-1000Hz)**
5. **Neutrino-driven convection and SASI (Standing Accretion Shock Instability) (100-800 Hz)**

- ✓ **A major uncertainty connected with supernova models is the initial state, in particular the angular momentum distribution in the iron core.**
- ✓ **Current expectations from stellar evolution calculations imply a slowly rotating core as a canonical case**

Reviews: Ott 2009, Andersson etal 2011, Gossan etal 2016

Core Collapse





- **GW burst from core collapse**

$$h \simeq 10^{-20} - 10^{-21}\frac{10kpc}{d},\ f \approx 600Hz$$

- **GW from convective boiling…**

$$h \simeq 10^{-21}\frac{10kpc}{d},\ f \approx 700Hz$$

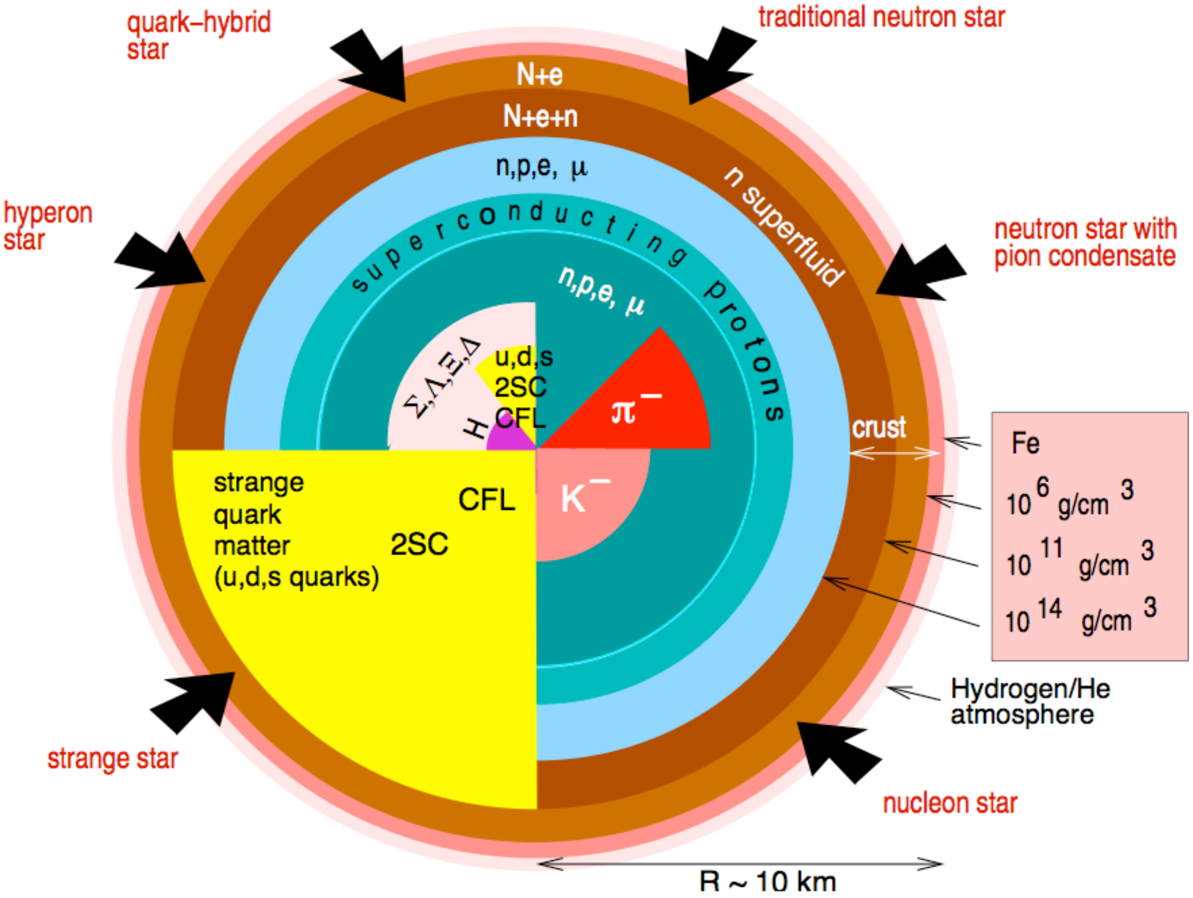
GW from the coupling of an unstable l=1 g-mode in PNS to higher harmonics…

$$h \simeq 10^{-21} - 4\times 10^{-20}\frac{10kpc}{d},$$
$$f \approx 800-1000Hz$$

GWs from QNM oscillations

$$h \simeq 10^{-21} - 3\times 10^{-20}\frac{10kpc}{d},$$
$$f \approx 1-5kHz$$

Zooming into a Neutron Star

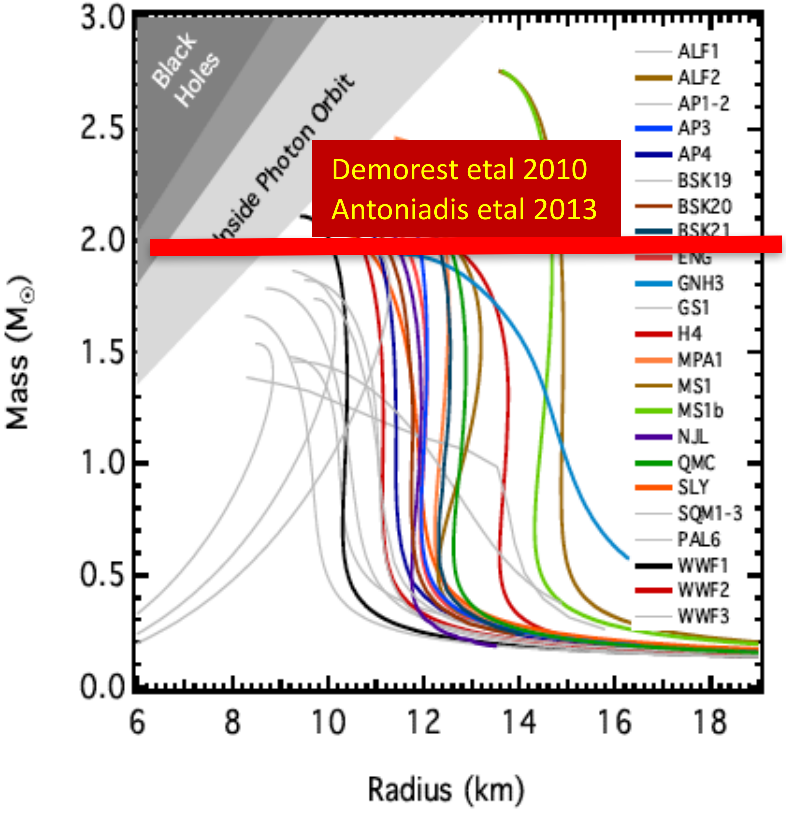


- ➢ The holy grail of NS astrophysics… is the determination of the equation of state (EOS) of matter at supra-nuclear densities.
- ➢ The most direct way of constraining the EOS is to measure simultaneously the neutron star mass and radius.

Neutron Stars: Mass vs Radius

Static Models

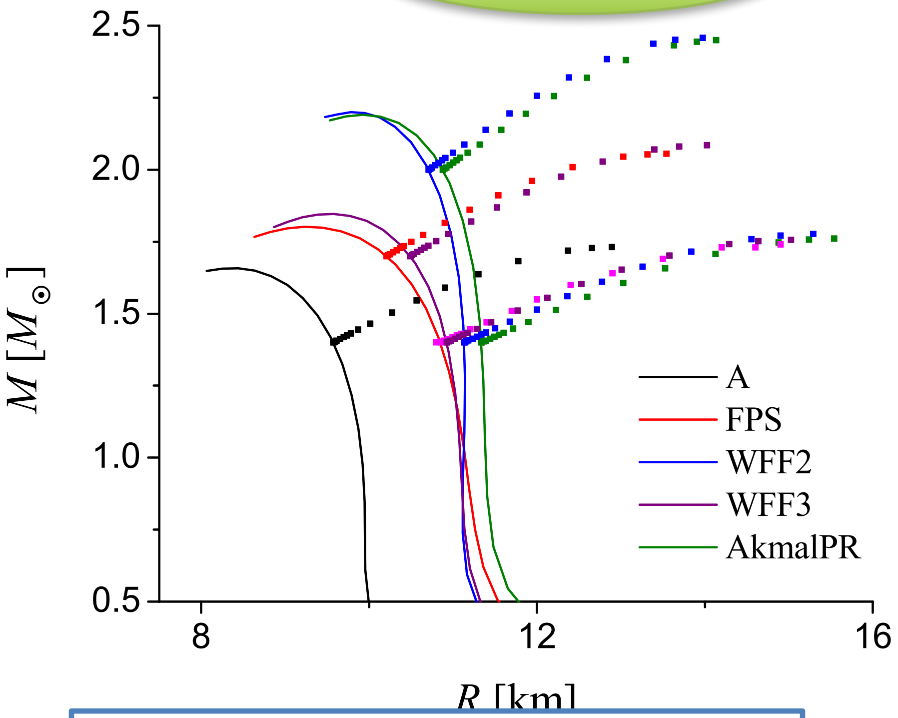
Rotating Models



Demorest etal 2010
Antoniadis etal 2013

Özel & Freire (2016)

supramassive



$$M_{max} \simeq (1.1962 + 0.0108) M_{TOV}$$

Breu-Rezzolla 2015

Constraints on Neutron Star Radius
GW observations

Main methods in EM spectrum:

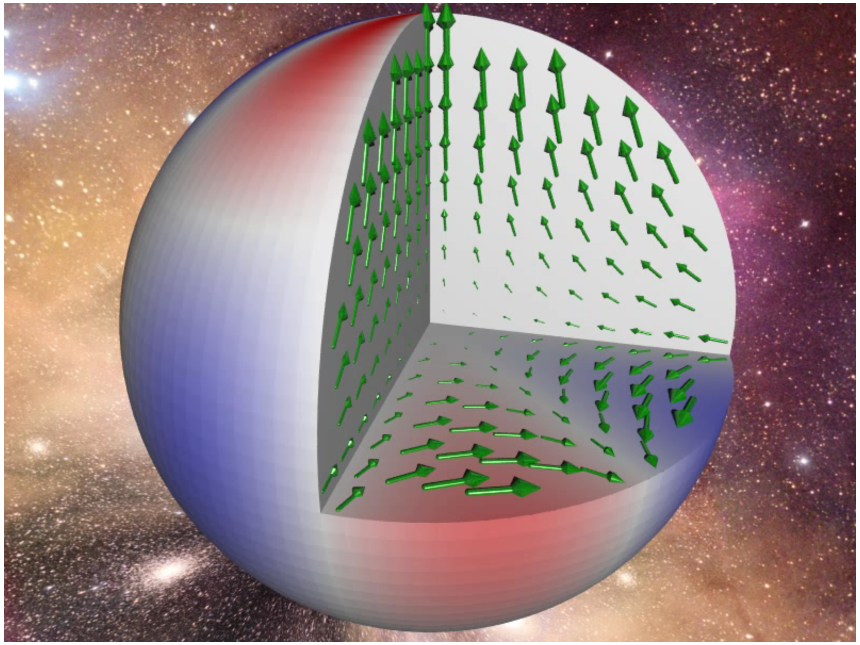
- **Thermonuclear X-ray bursts (photospheric radius expansion)**
- **Burst oscillations (rotationally modulated waveform)**
- **Fits of thermal spectra to cooling neutron stars**
- **khZ QPOs in accretion disks around neutron stars**
- **Pericenter precession in relativistic binaries (double pulsar J0737)**

Constraints on Neutron Star Radius
GW observations

Main methods in GW spectrum:

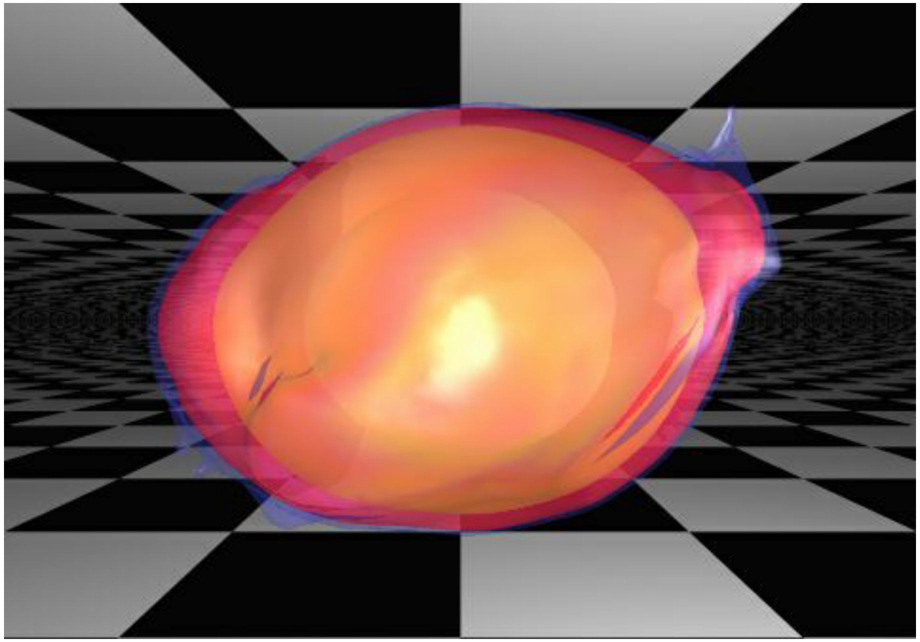
- **Tidal effects on waveform during inspiral phase of NS-NS mergers**
- **Tidal disruption in BH-NS mergers**
- **Oscillations in post-merger phase of NS-NS mergers & collapse**

Gravitational Wave Asteroseismology



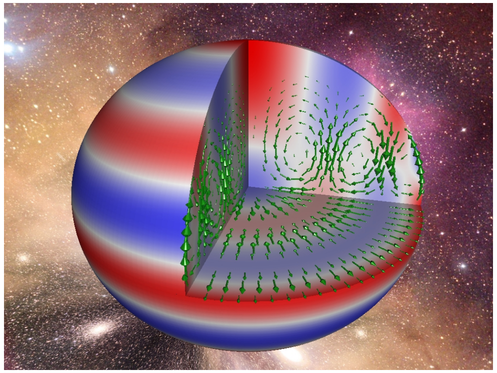
Neutron Stars oscillate wildly during the very first seconds of their life

We can potentially estimate their masses, radii, equations of state by analysing the seismic data via the emitted gravitational waves



Rotation is responsible for a number of instabilities which emit copious amounts of GWs

Neutron Star "ringing"



p-modes: main restoring force is the pressure (f-mode) *(>1.5 kHz)*

$$\sigma \approx \sqrt{\frac{M}{R^3}}$$

Inertial modes: (r-modes) main restoring force is the Coriolis force

$$\sigma \approx \Omega$$

w-modes: pure space-time modes (only in GR) *(>5kHz)*

$$\sigma \approx \frac{1}{R}\left(\frac{M}{R}\right)$$

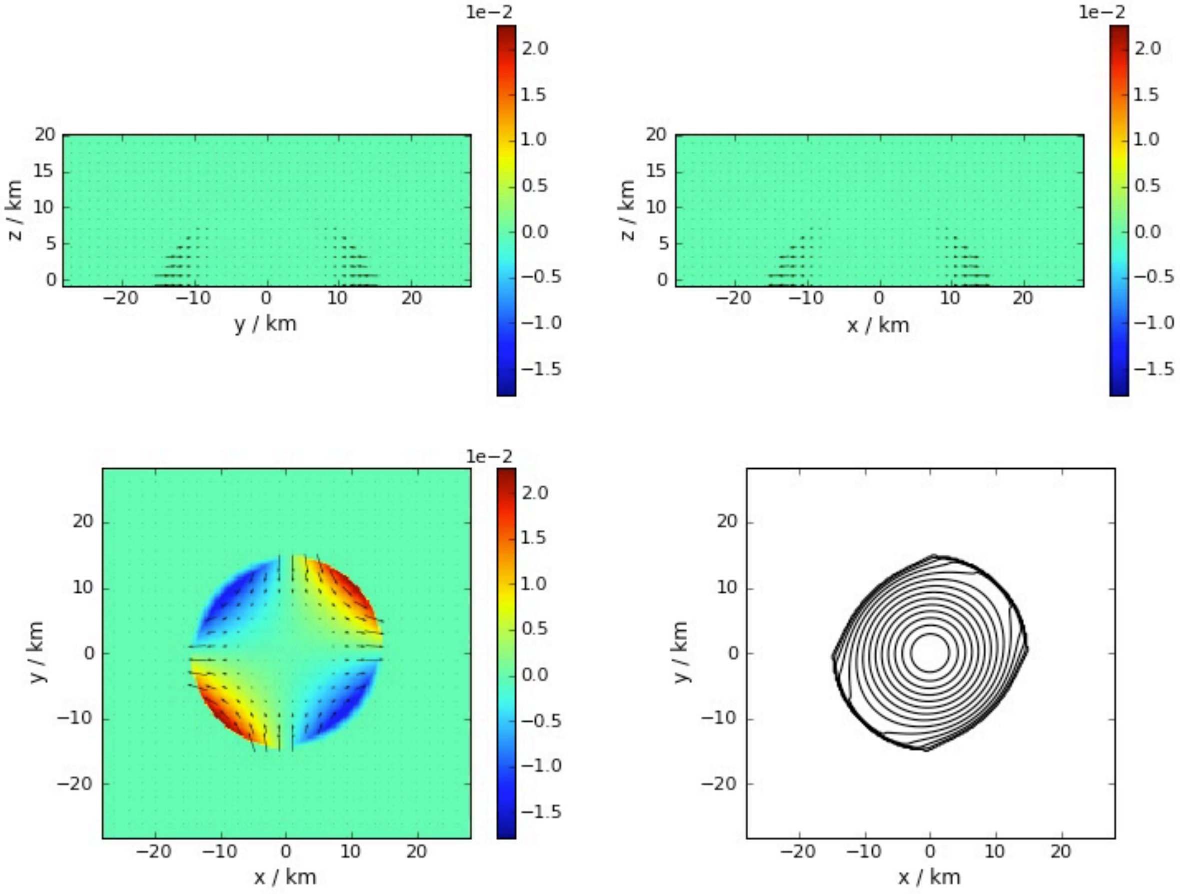
Torsional modes (t-modes) *(>20 Hz)* shear deformations. Restoring force, the weak Coulomb force of the crystal ions.

$$\sigma \approx \frac{v_S}{R}$$

… and many more

Animation of the l=m=2 f-mode

Kastaun, Willburger, Kokkotas (2010)



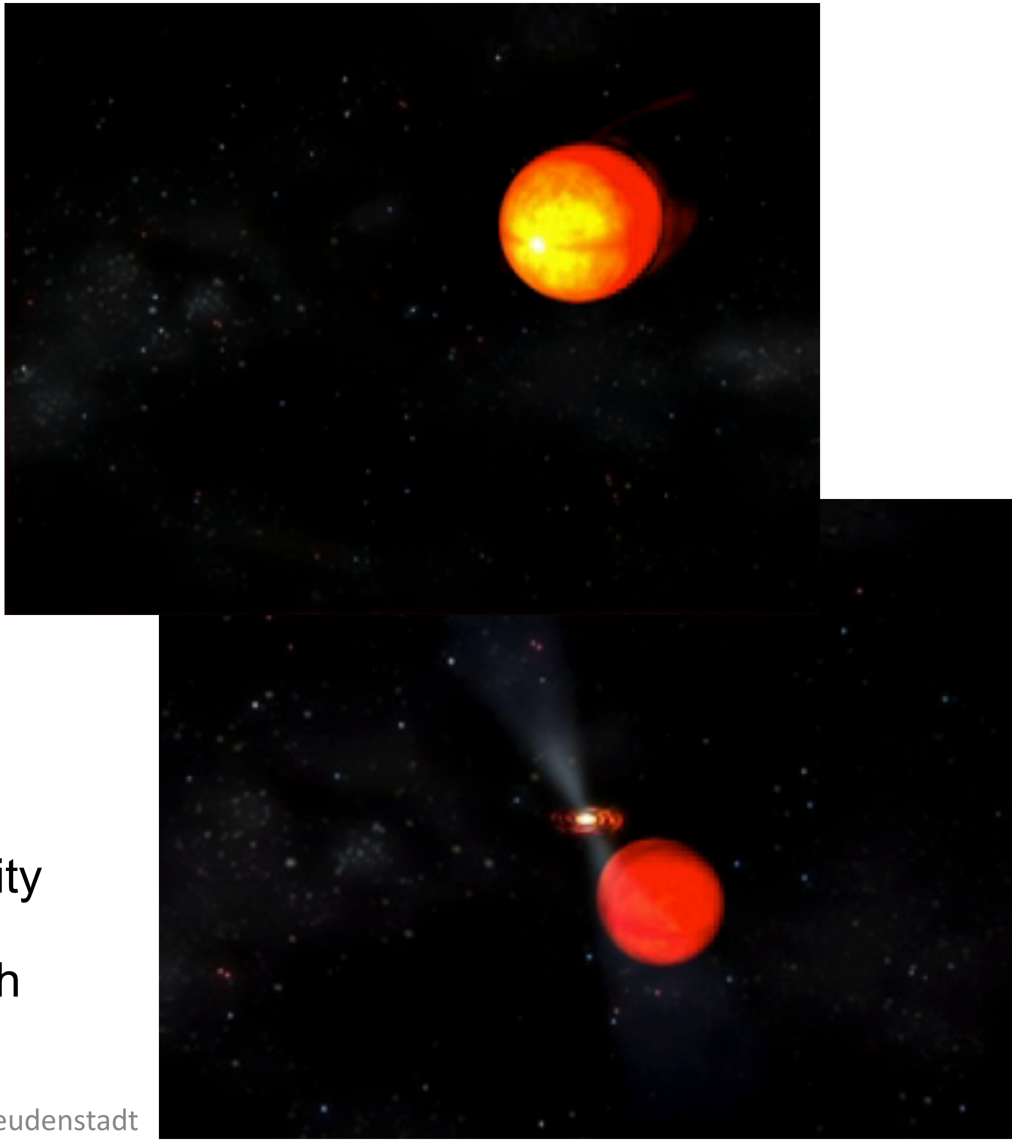
- ✓ **Quasi-Radial &Axisymmetric:** damped due to shock formation
- ✓ **Non-axisymmetric:** damped due to wave breaking on the surface

Unstable NS in Binary systems

Key idea: Emission of GW balances accretion torque.

Observational evidence : clustering of spin-frequencies in LMXB (250-700 Hz)

Narrow banding



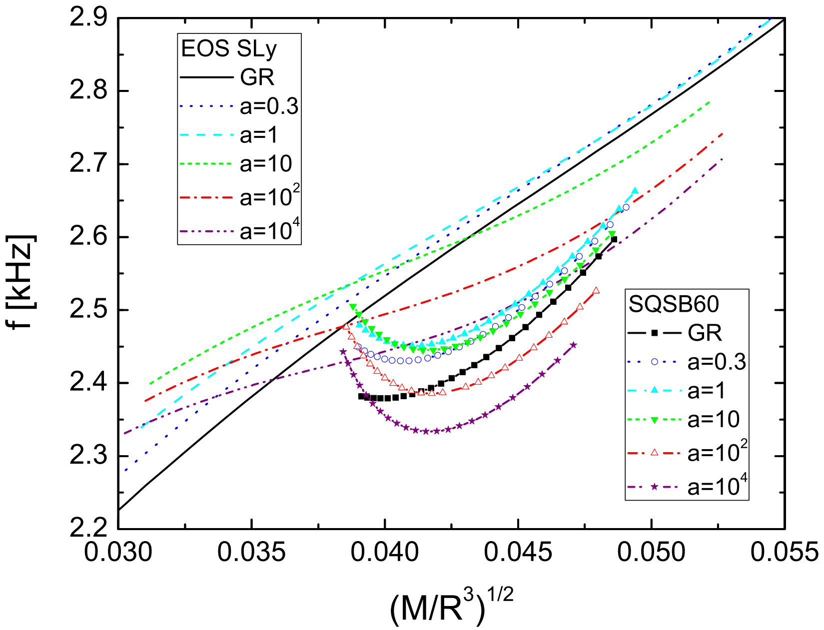
Possible GW mechanisms:

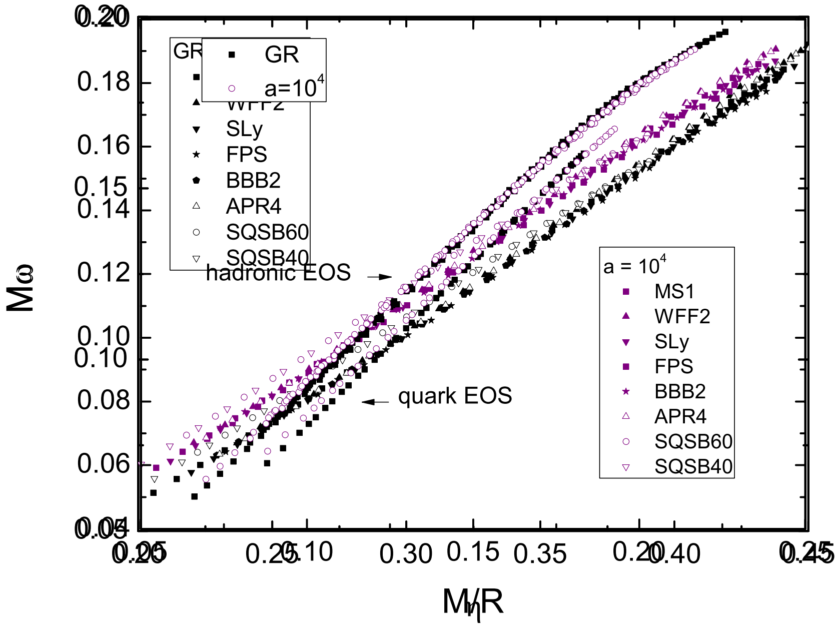
– accretion induced asymmetry

– unstable r-modes: strong bulk viscosity may shift instability window to lower temperatures; accreting stars can reach quasi-equilibrium state

Asteroseismology in GR and *R² gravity*

- **EOS independent** with suitable choice of normalization
- **Alternative normalizations show nicer relations**

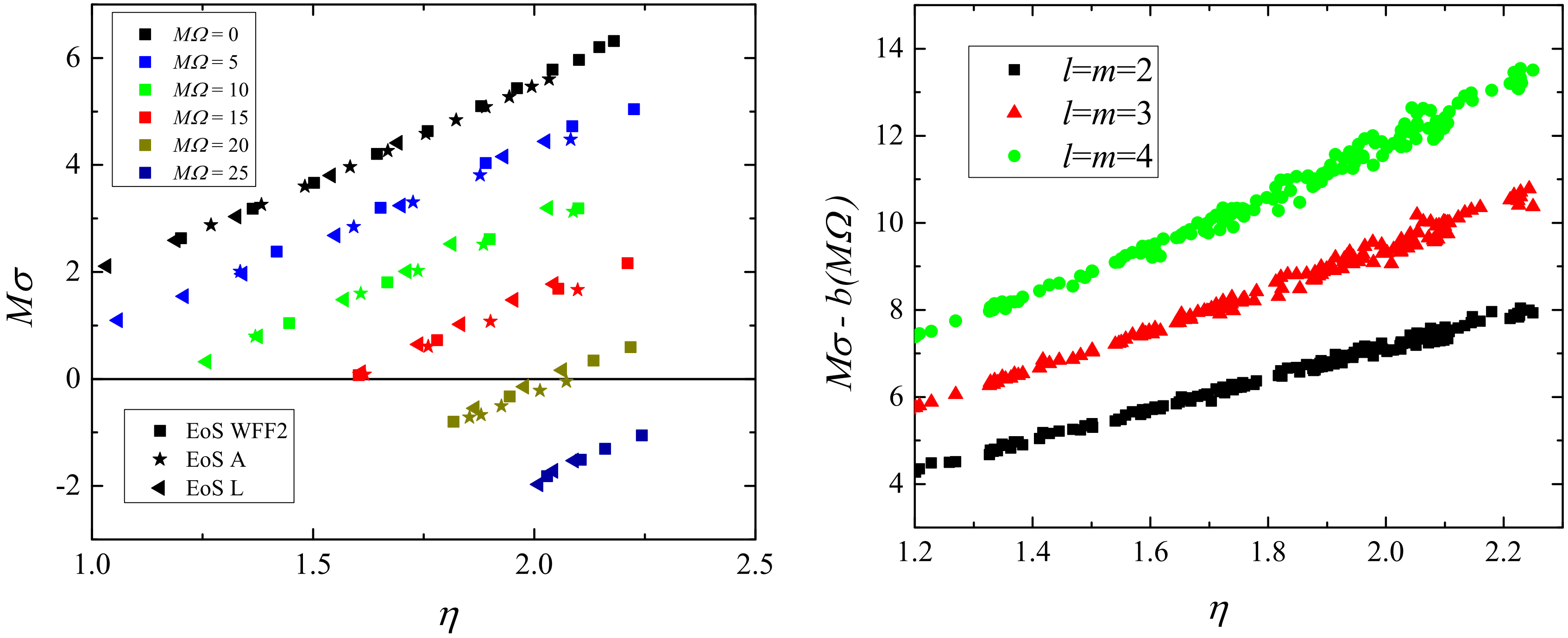




$$\eta = \sqrt{M^3 / I}$$

Asteroseismology: f-modes

$$M\sigma_i^{unst} = \left[(0.56 - 0.94\ell) + (0.08 - 0.19\ell)M\Omega + 1.2(\ell + 1)\eta\right]$$



The l = 2 f-mode oscillation frequencies as functions of the parameter η

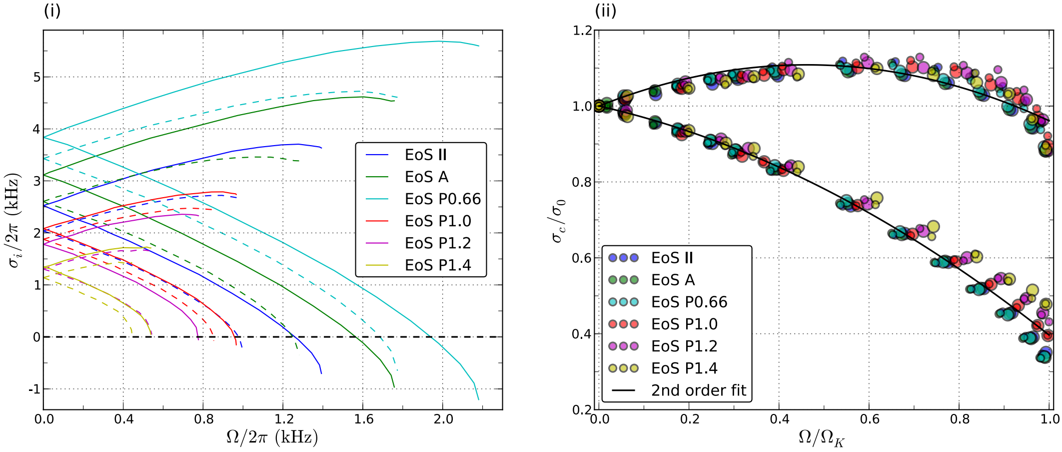
$$\eta = \sqrt{M^3 / I}$$

Doneva-Kokkotas 2015

Asteroseismology: f-modes

We can produce empirical relation relating the parameters of the *rotating neutron stars* to the observed frequencies.

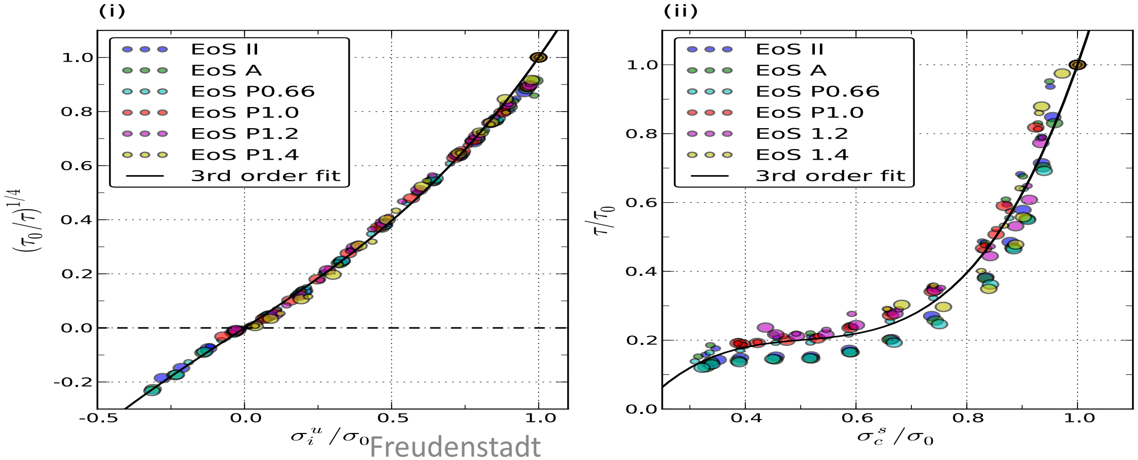
Frequency



Cowling Approximation

Gaertig-Kokkotas 2008, 2010, 2011
Doneva, Gaertig, Kokkotas, Krüger (2013)

Damping/ Growth time

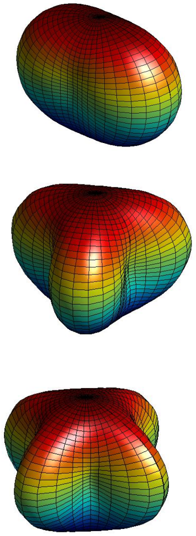


Asteroseismology: f-modes

Stable Branch

$$\frac{\omega_c^s}{\omega_0} = 1 - 0.235\left(\frac{\Omega}{\Omega_K}\right) - 0.358\left(\frac{\Omega}{\Omega_K}\right)^2$$

Unstable Branch



$$\frac{\omega^u_{c\ l=2}}{\omega_0} = 1 + 0.402\left(\frac{\Omega}{\Omega_K}\right) - 0.406\left(\frac{\Omega}{\Omega_K}\right)^2$$

$$\frac{\omega^u_{c\ l=3}}{\omega_0} = 1 + 0.373\left(\frac{\Omega}{\Omega_K}\right) - 0.485\left(\frac{\Omega}{\Omega_K}\right)^2$$

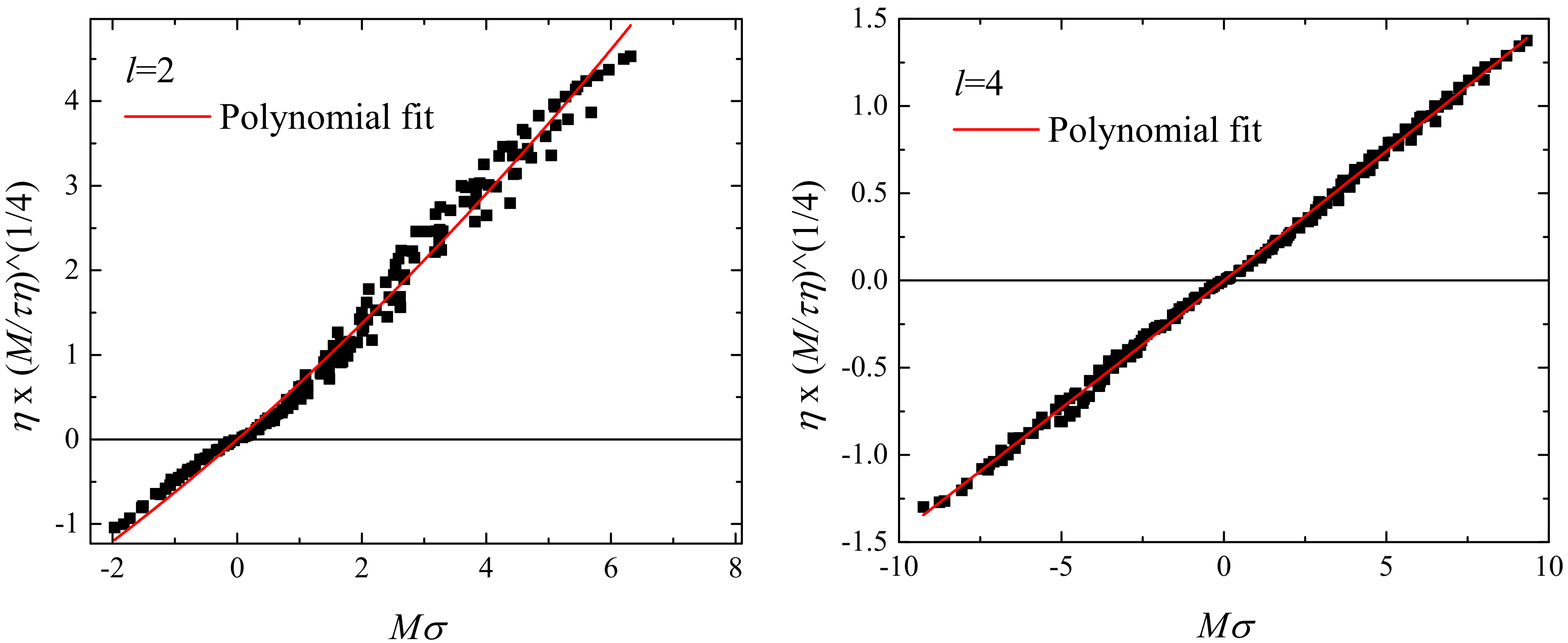
$$\frac{\omega^u_{c\ l=4}}{\omega_0} = 1 + 0.360\left(\frac{\Omega}{\Omega_K}\right) - 0.543\left(\frac{\Omega}{\Omega_K}\right)^2$$

Unstable Branch

$$\frac{\tau_0}{\tau} = \mathrm{sgn}(\omega_i^u)\left(0.900\left(\frac{\omega_i^u}{\omega_0}\right) - 0.057\left(\frac{\omega_i^u}{\omega_0}\right)^2 + 0.157\left(\frac{\omega_i^u}{\omega_0}\right)^3\right)^{2l}$$

Doneva, Gaertig, KK, Krüger (2013))

Asteroseismology: f-modes



The **normalized damping time** $\eta\left(\frac{M}{\tau\eta^{2}}\right)^{(1/2\ell)}$ where $\eta=\sqrt{M^{3}/I}$

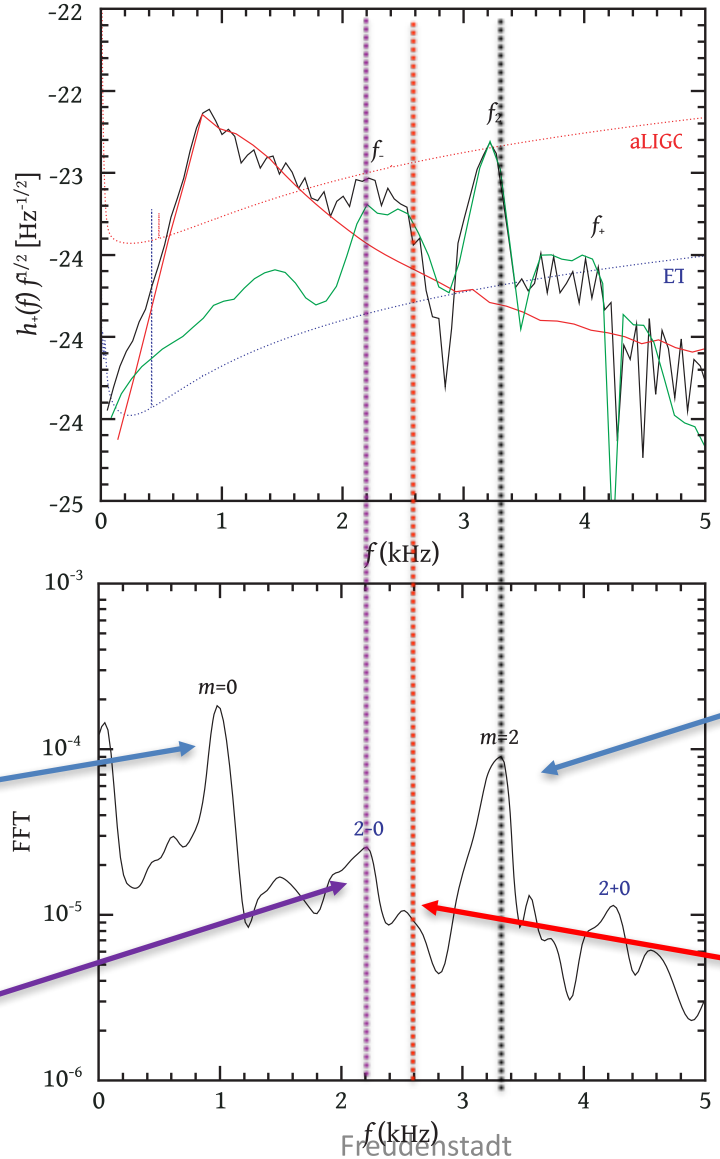
as a function of the normalized oscillation frequency ***Mσ*** for l = m = 2 & l = m = 4 f-modes.

Doneva-KK 2015

Binary Neutron Star Mergers

Post-merger Oscillations & GWs

GRAVITATIONAL WAVES



Stergioulas et al. (2011)
Bauswein, Stergioulas (2015)
…
Similar results from all groups

NEUTRON STAR OSCILLATIONS

l=m=2
linear f-mode (f_{peak})

l=m=0 linear quasi-radial mode

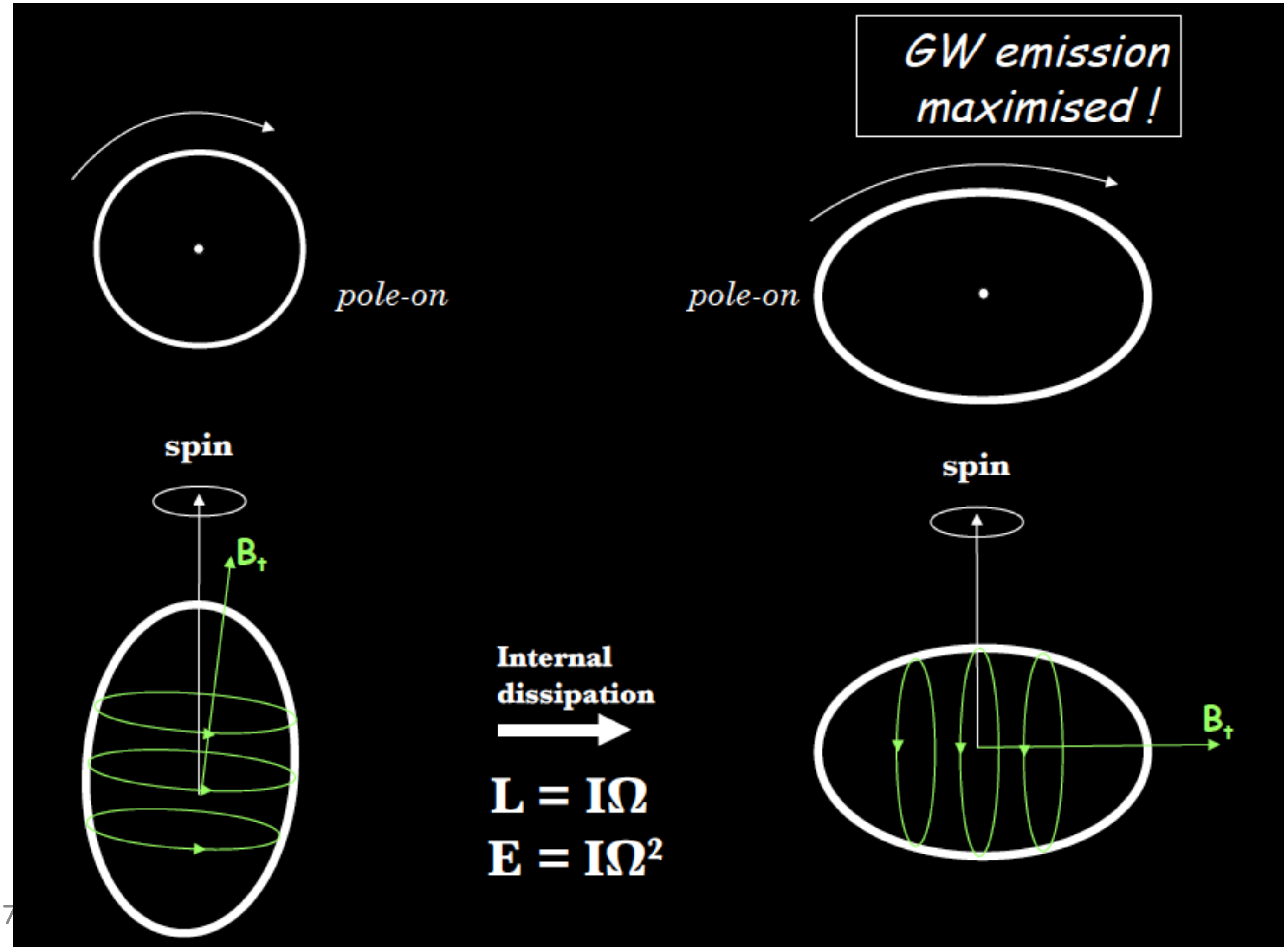
Quasi-linear combination frequency (f_{2-0})

nonlinear spiral frequency

Binary Neutron Star Mergers

Deformed Magnetar Model

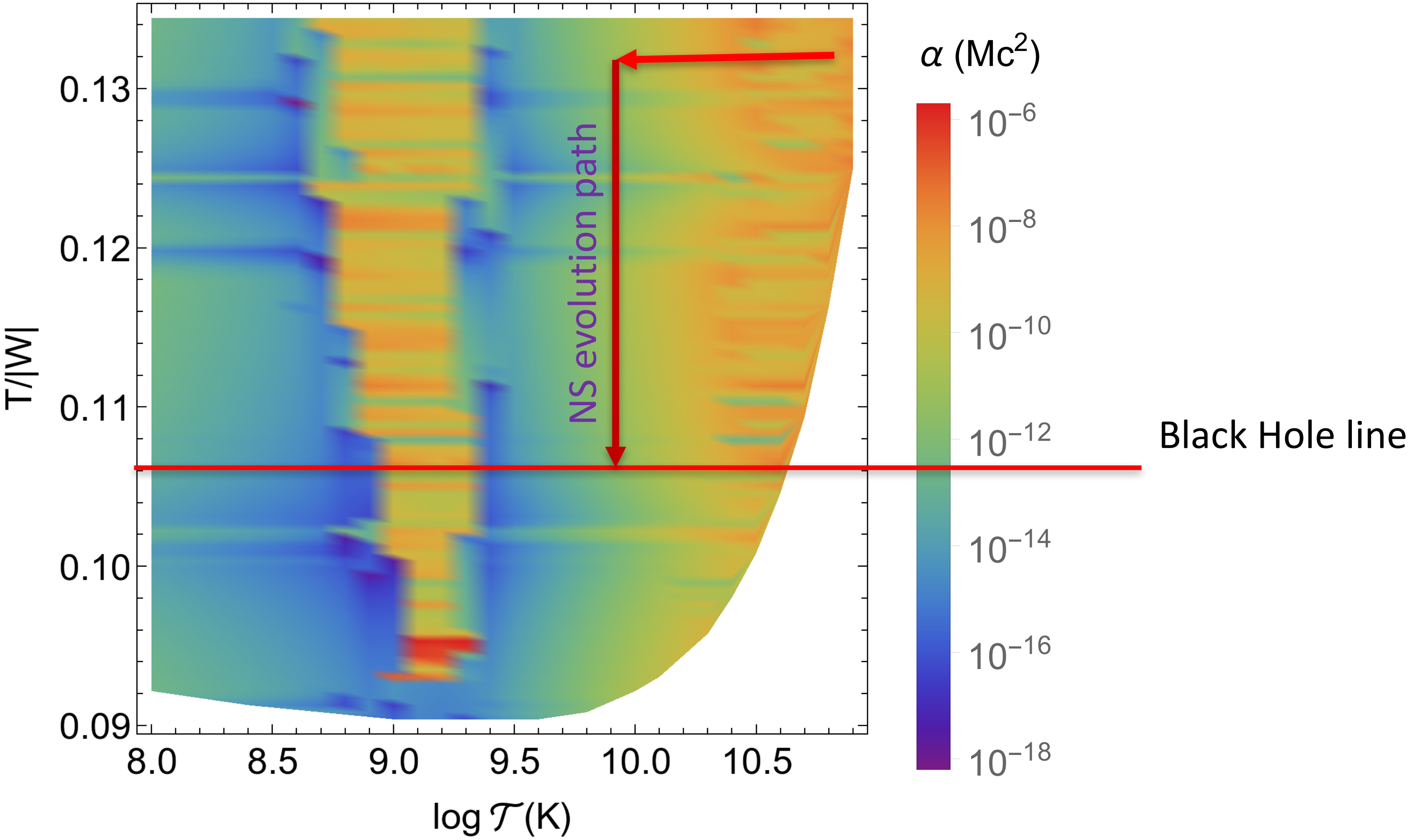
Viscous dissipation drives the magnetic symmetry axis orthogonal to the spin axis, thus maximizing the strength of the emitted GW radiation.



Dall'Osso-Giacomazzo-Perna-Stella 2015

Saturation of the Instability

Parametric Resonance

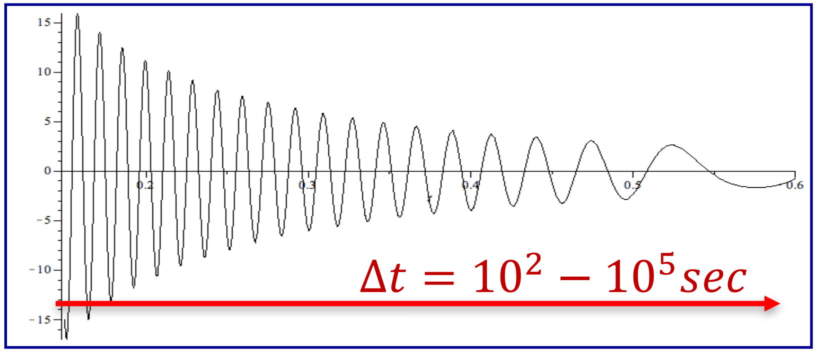


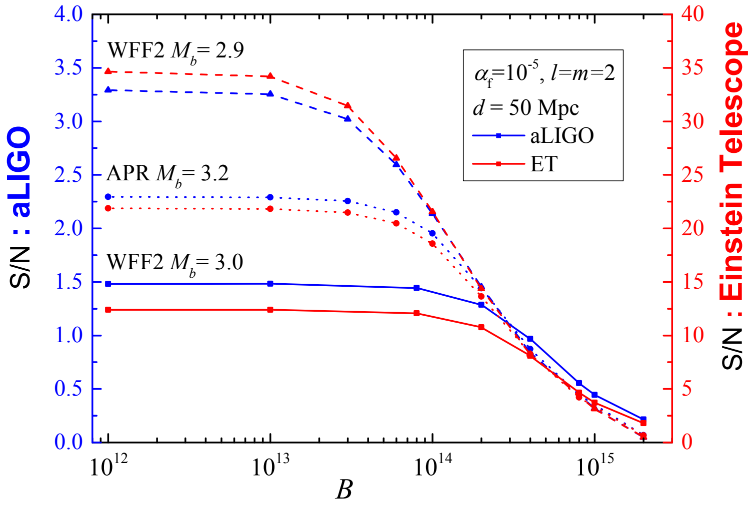
NOTE: these are purely Newtonian estimations

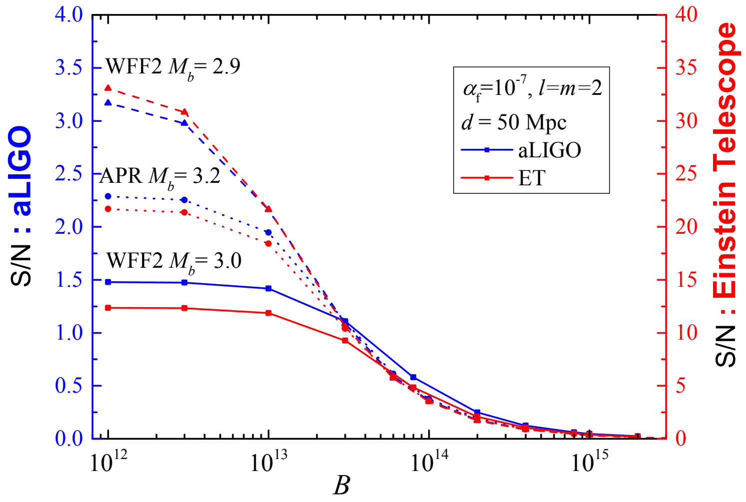
Pnigouras, Kokkotas 2015,16

Post-Merger NS:

F-mode instability vs Magnetic field







Competition between the B-field and the secular instability

GW frequencies:

WW2a: 920-1000 Hz
APR: 370–810 Hz
WFF2b: 600–780 Hz

Doneva-Kokkotas-Pnigouras 2015

Magnetars

- **Young, slowly spinning (P~10s) systems (about 21)**

- **Exhibit regular γ-ray flares**
  - Believed to be powered by magnetic field
  - Either trigger or are preceded by starquakes
  - Some linked to glitches or **anti-glitches**

- **Three giant flares observed with peak luminosities ~10^{47} erg/s**
  - March 5, 1979 : SGR 0526-66
  - August 27, 1998 : SGR 1900+14
  - December 27, 2004: SGR 1806-20

- **Giant flares**
  - QPOs – 10's -100's of Hz
  - Magnetic field reconstruction
  - **Possible f-mode excitation**

Magnetars: SGR 1806-20

Only Crust Oscillations
- Sotani, Kokkotaa, Stergioulas 2007,2008
- Samuelsson, Andersson 2007
- Sotani, Colaiuda, Kokkotas 2008
- Steiner, Watts 2009
- …
- Sotani etal 2012-16

Without Crust
- Levin 2007
- Sotani, Kokkotas, Stergioulas 2008
- Colaiuda, Beyer, Kokkotas 2009
- Cerda-Duran, Stergioulas, Font 2009

Fluid + Crust
- Van Hoven, Levin 2011, 2012
- Cerda-Duran, Stergioulas, Font 2011
- Colaiuda, Kokkotas 2011
- Gabler etal 2012
- Gabler etal 2013 …

Superfluidity
- Passamonti, Lander 2012
- Sotani etal 2013
- Gabler etal 2013

Mixed axial-polar
- Colaiuda, Kokkotas 2012
- Lee, Yoshida 2015

Non-axisymmetric
- Sotani, Kokkotas 2012

EoS : APR (NV)

Mass: $M=1.4M_\odot$

Radius: 11.57 km

B-field : 2×10^{15} G

Crust : 0.099 R

Colaiuda, Kokkotas (2011-12)

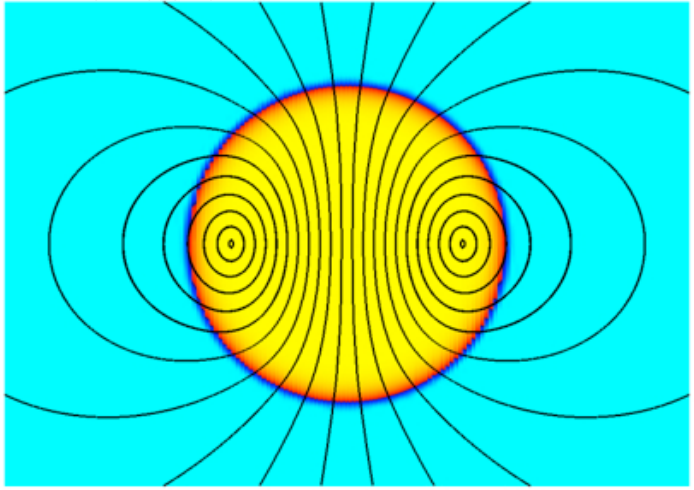
2005-6 Israel etal Watts & Strohmayer	**2010-12** Colaiuda-KK	**2011** **Hambaryan, Neuhaüser, KK**	**2013-15**
	16	16	
18	18		
	22	22	
26	26		
29.5	30		
	39	37	
	44		
	53		
	56		56
	60	59 &61	
	66		
	74		
	80		
	82		
92	**92**		92

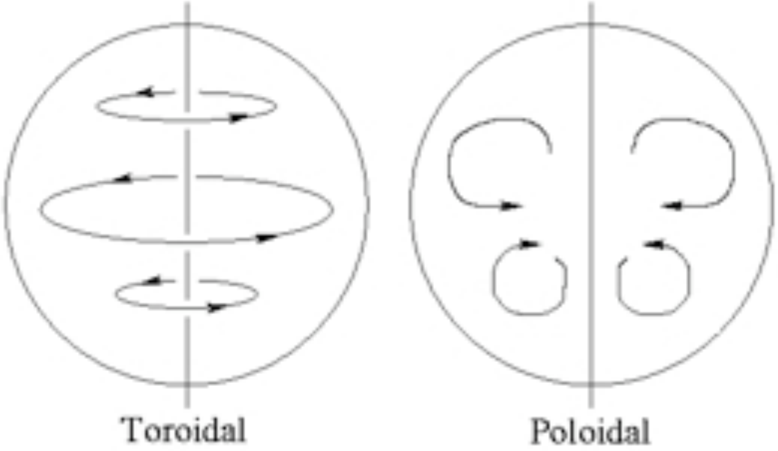
Magnetars & Grav. Waves

- **3D - GRMHD simulations of known and arbitrary initial magnetic field configurations**

- **Magnetic field instabilities relevant for flare generation** instability mechanisms, relevant timescales, phenomenology (GR)

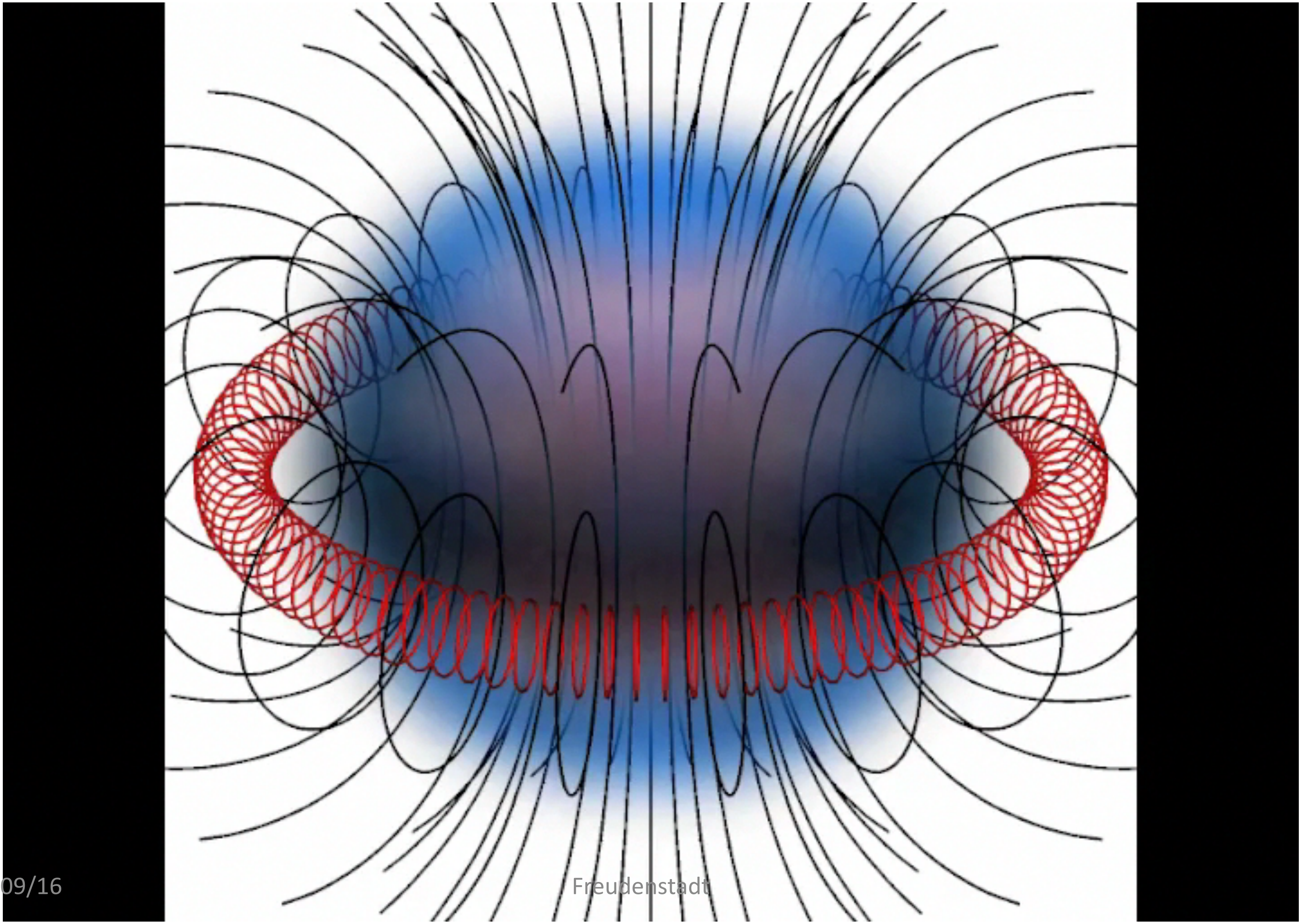
- **Understanding stable magnetic field configurations** mixed poloidal - toroidal configurations, relevant strengths of components, multipolar structure...





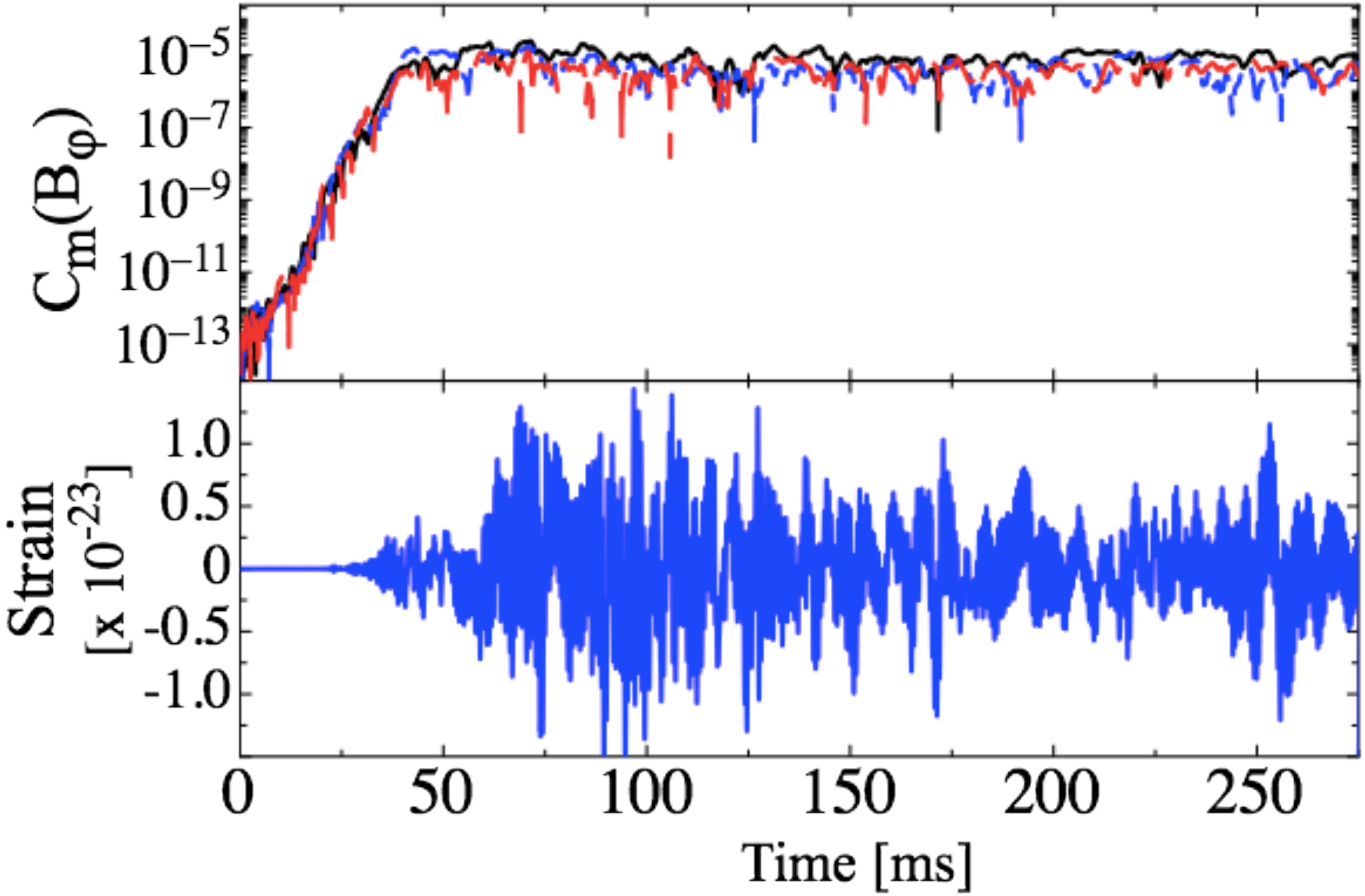
Simulation of Magnetic Field Instability

Lasky, Zink, Kokkotas, Glampedakis ApJL (2011)

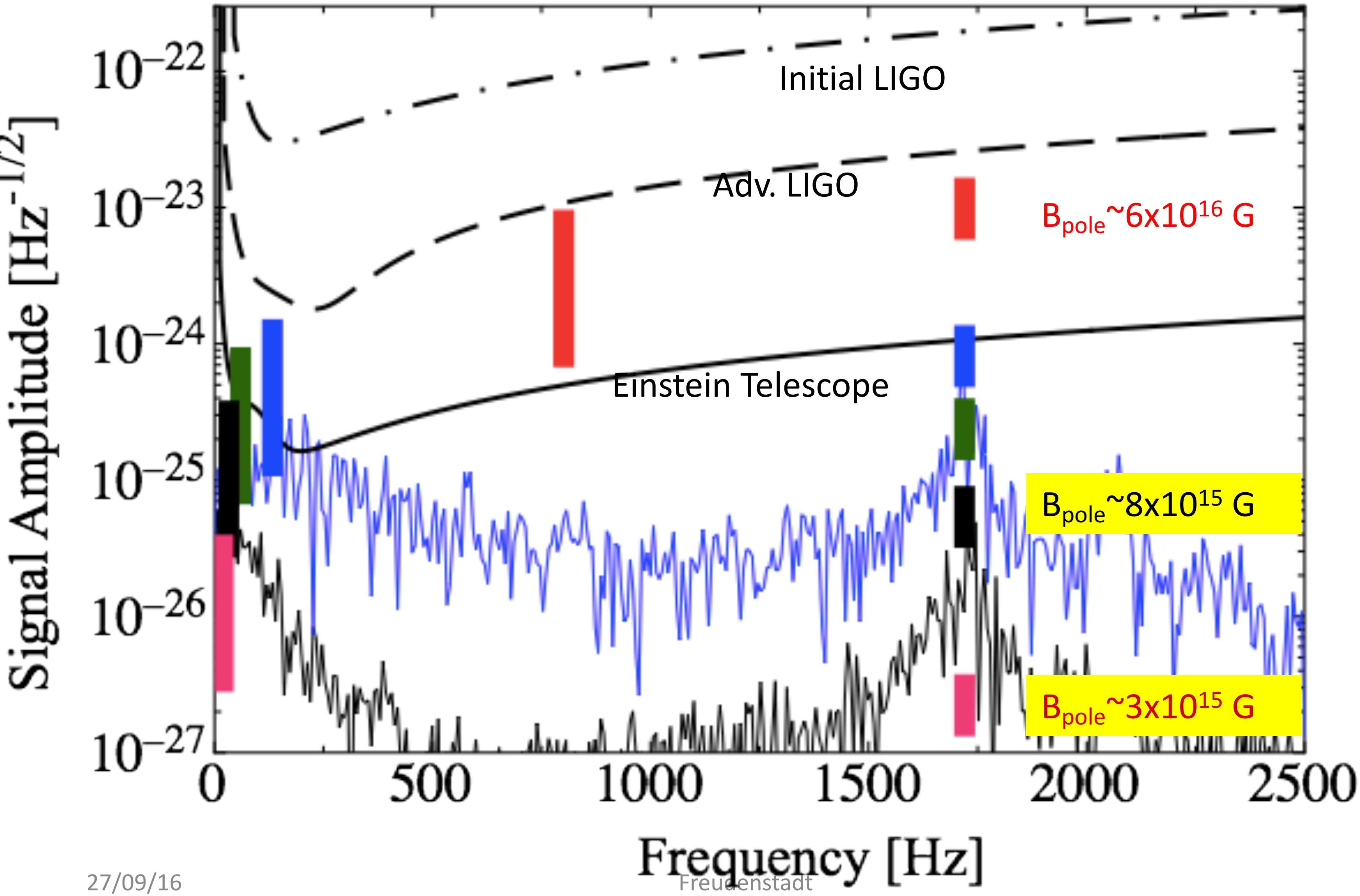


Gravitational Waves from Magnetars

Zink, Lasky, KK (2011)



Detectability II



Stochastic Background of Grav. Waves

- Energy density:

$$\boxed{\rho_{GW} = \frac{c^2}{32\pi G} < \dot{h}_{ab}\,\dot{h}^{ab} >}$$

- Characterized by log-frequency spectrum:

$$\boxed{\Omega_{GW}(f) = \frac{1}{\rho_c}\,\frac{d\rho_{GW}(f)}{d\ln f}}$$

- Related to the strain spectrum:

$$\boxed{S(f) = \frac{3H_0^2}{10\pi^2}\,\frac{\Omega_{GW}(f)}{f^3}}$$

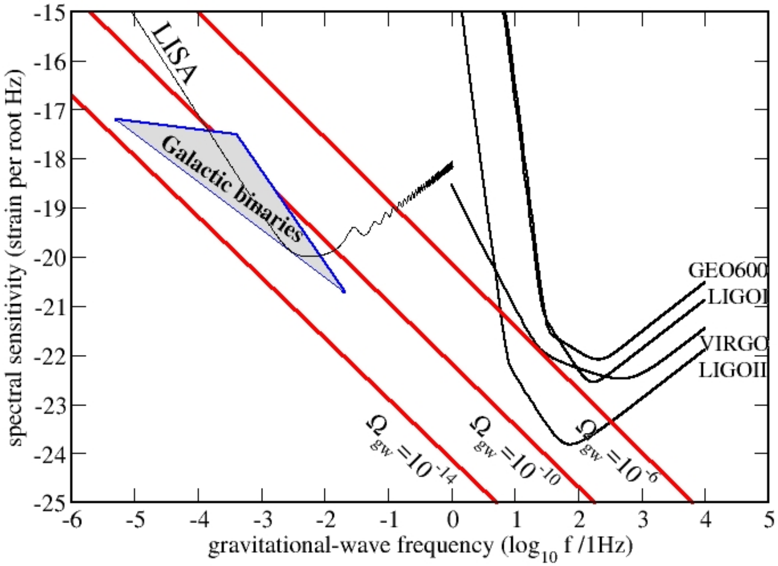
- Strain scale:

$$\boxed{h(f) = 6.3 \times 10^{-22}\,\sqrt{\Omega_{GW}(f)}\,\left(\frac{100\ \mathrm{Hz}}{f}\right)^{3/2}\ \mathrm{Hz}^{-1/2}}$$

GWs from the Big Bang

Stochastic background reflecting fundamental physics in the early universe;

- Phase transitions
- Inflation
- Topological defects
- String-inspired cosmology
- Higher dimensions



After the Big Bang, photons decoupled after 10^5 years, neutrinos after 1s, **GWs before10^{-24} s!**

$$\rho_c = \frac{3H_0^2}{8\pi G}$$

Detection: Requires cross-correlation of detectors. Best window, free of "local" GW sources, is around 0.1-1 Hz. Need LISA follow-on mission?

Strength expressed as fraction of closure energy density;

$$\Omega_{gw} = \frac{f}{\rho_c}\frac{d\rho_{gw}}{df}$$

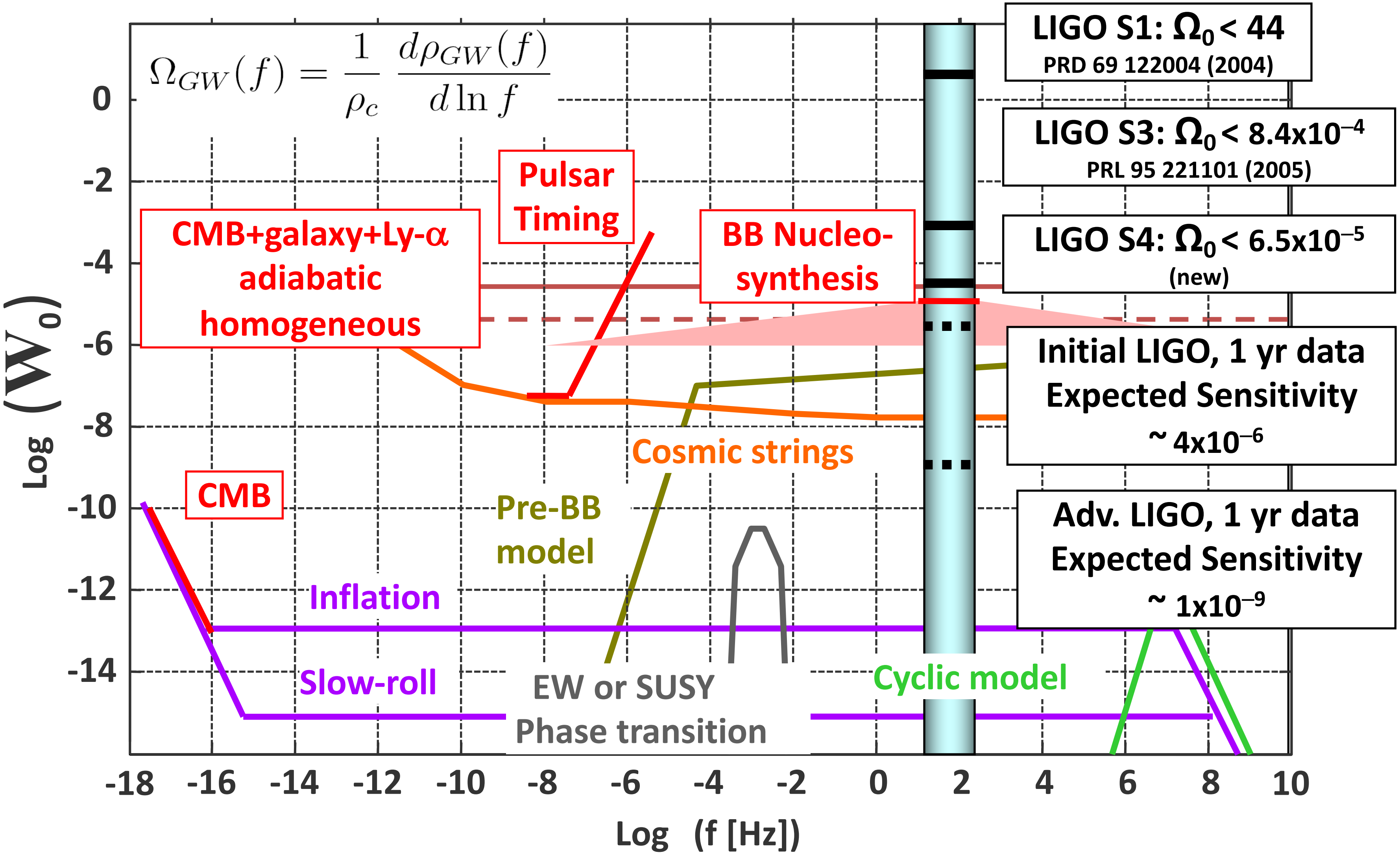
$$10^{-14} \approx \Omega_{gw} < 10^{-5}$$

simple inflation nucleosynthesis

$$h_c \approx 10^{-18}\left(\frac{1Hz}{f}\right)\sqrt{h_0^2\Omega_{gw}(f)}$$

One of the most fundamental observations possible!

Stochastic search limits



$$\Omega_{GW}(f) = \frac{1}{\rho_c}\frac{d\rho_{GW}(f)}{d\ln f}$$

LIGO S1: $\Omega_0 < 44$
PRD 69 122004 (2004)

LIGO S3: $\Omega_0 < 8.4\text{x}10^{-4}$
PRL 95 221101 (2005)

LIGO S4: $\Omega_0 < 6.5\text{x}10^{-5}$
(new)

Initial LIGO, 1 yr data
Expected Sensitivity
~ $4\text{x}10^{-6}$

Adv. LIGO, 1 yr data
Expected Sensitivity
~ $1\text{x}10^{-9}$