

Search for Supersymmetry at the LHC

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- Introduction to Supersymmetry
- Challenges in data analysis
- Tools for SUSY searches
- Detailed example for a SUSY search
- Outlook



The Standard Model





Dark Matter



observed

10

M33 rotation curve

expected from

uminous dis

R (kpc)

We don't know much about the largest part of the universe!



Corrections to the Higgs Mass



Up to 1-loop corrections

Most dangerous !



Corrections to the Higgs Mass



Where do we expect Λ ?



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Where we are...



Open questions of the Standard Model



- Challenges in data analysis
- Tools for SUSY searches
- Detailed example for a SUSY search
- Outlook



Supersymmetry



Each SM particle gets assigned a SUSY partner particle with spin differing by $^{1\!\!/_2}$ SUSY transformation:

- ↓ Q |fermion> = |boson>

Name convention:

- ← Fermion ← → S-fermion
- → Boson \leftarrow → bos(on)-ino



SUSY Particles



Overview of Particles

		-	a		
Names	Spin	P_R	Gauge Eigenstates	Mass Eigenstates	
Higgs bosons	0	+1	$H^0_u \; H^0_d \; H^+_u \; H^d$	$h^0 \hspace{0.1 cm} H^0 \hspace{0.1 cm} A^0 \hspace{0.1 cm} H^{\pm}$	5 physical Higgs
			$\widetilde{u}_L \widetilde{u}_R \widetilde{d}_L \widetilde{d}_R$	(same)	Dosons
squarks	0	-1	$\widetilde{s}_L \widetilde{s}_R \widetilde{c}_L \widetilde{c}_R$	(same)	
			$\widetilde{t}_L \widetilde{t}_R \widetilde{b}_L \widetilde{b}_R$	$\widetilde{t}_1 \widetilde{t}_2 \widetilde{b}_1 \widetilde{b}_2$	
			$\widetilde{e}_L \widetilde{e}_R \widetilde{ u}_e$	(same)	
sleptons	0	-1	$\widetilde{\mu}_L \widetilde{\mu}_R \widetilde{ u}_\mu$	(same)	
			$\widetilde{ au}_L \ \widetilde{ au}_R \ \widetilde{ u}_ au$	$\widetilde{ au}_1 \ \widetilde{ au}_2 \ \widetilde{ au}_{ au}$	
neutralinos	1/2	-1	$\widetilde{B}^0 \ \widetilde{W}^0 \ \widetilde{H}^0_u \ \widetilde{H}^0_d$	$\widetilde{N}_1 \ \widetilde{N}_2 \ \widetilde{N}_3 \ \widetilde{N}_4$	$\widetilde{\chi}_1^{\ 0}, \widetilde{\chi}_2^{\ 0}, \widetilde{\chi}_3^{\ 0}, \widetilde{\chi}_4^{\ 0}$
charginos	1/2	-1	\widetilde{W}^{\pm} \widetilde{H}^+_u \widetilde{H}^d	\widetilde{C}_1^\pm \widetilde{C}_2^\pm	$\widetilde{\chi}_1^{+/-} \widetilde{\chi}_2^{+/-}$
gluino	1/2	-1	\widetilde{g}	(same)	
goldstino (gravitino)	$\frac{1/2}{(3/2)}$	-1	\widetilde{G}	(same)	

SUSY Mass Eigenstates



Mass eigenstates are calculated from gauge eigenstates, e.g. $f_1 = f_L \cos(M_f) + f_R \sin(M_f)$, where the mass matrix M_f depends on:

- M_1 , M_2 , tan β , μ : SUSY masses and breaking
- → m_Z , m_W , sin² θ_W : EWSB mixing: B,W → Z, γ

Neutralino mixing: (\widetilde{B} , \widetilde{W} , \widetilde{H}_{d} , \widetilde{H}_{u}) $\rightarrow \widetilde{\chi}^{0}_{1,2,3,4}$

(M_1	0	$-m_Z c_\beta s_W$	$m_Z s_\beta s_W$	
	0	M_2	$m_Z c_\beta c_W$	$-m_Z s_\beta c_W$	
	$-m_Z c_\beta s_W$	$m_Z c_eta c_W$	0	$-\mu$	
	$m_Z s_\beta s_W$	$-m_Z s_\beta c_W$	$-\mu$	0	/

SUSY Mass Eigenstates



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If the theory presenteds from a GUTs in the twees M_1 and M_2 : $M_2 \sim 2M_1$ 11



- → Dark matter → lightest SUSY particle → viable Dark Matter candidate
- Gravity \rightarrow can be included in SUSY
- ✤ Unification of forces at 10¹⁶ GeV:



Neutrino masses: expected in SUSY

Corrections to the Higgs Mass



Corrections to the Higgs mass by scalar partner particles



 $\begin{array}{ll} \mbox{Complete cancellation only if masses are exactly the same, if not:} \\ \delta M_h{}^2 \sim log(|M_B{}^2-M_F{}^2|) \\ \mbox{To solve the hierarchy problem, expect at least a few SUSY masses below 1 TeV} \\ \mbox{Isabell Melzer-Pellmann} & GK Workshop Bad-Liebenzell 22.-24.9.2014} \end{array}$

Corrections to the Higgs Mass



More SUSY particles protecting the Higgs mass:



Cornerstones of naturalness:

- Higgsino directly bound by Higgs mass parameter μ
- Top squark mass should not be heavier than 1 TeV (1-loop)
- Gluino can be a bit heavier (2-loops)

SUSY Breaking



Perfect symmetry: SUSY particles have same mass as SM particles **BUT:** experimentally not found \rightarrow broken symmetry can lead to higher masses Allow all possible mass terms and couplings which don't violate gauge couplings:

squark masses (e.g. m _Q ² : 3x3 matrix, 6 real + 3 phases)	$\mathcal{L}_{\text{soft}} = -\left[\tilde{Q}_{i}^{\dagger}\mathbf{m}_{\mathbf{Q}_{ij}}^{2}\tilde{Q}_{j} + \tilde{d}_{\mathbf{R}i}^{\dagger}\mathbf{m}_{\mathbf{D}_{ij}}^{2}\tilde{d}_{\mathbf{R}j} + \tilde{u}_{\mathbf{R}i}^{\dagger}\mathbf{m}_{\mathbf{U}_{ij}}^{2}\tilde{u}_{\mathbf{R}j} + \tilde{u}_{\mathbf{R}i}^{\dagger}\mathbf{m}_{\mathbf{U}_{ij}}^{2}\tilde{u}_{\mathbf{R}j} + \tilde{u}_{\mathbf{R}i}^{\dagger}\mathbf{m}_{\mathbf{U}_{ij}}^{2}\tilde{u}_{\mathbf{R}j} + \tilde{u}_{\mathbf{R}i}^{\dagger}\mathbf{m}_{\mathbf{U}_{ij}}^{2}\tilde{u}_{\mathbf{R}j}\right]$
slepton / Higgs m	$\frac{+\mathcal{L}_{i} \mathbf{m}_{Lij} \mathcal{L}_{j} + \mathbf{e}_{Ri} \mathbf{m}_{Eij} \mathbf{e}_{Rj} + m_{H_{u}} \mathbf{n}_{u} \mathbf{n}_{u} \mathbf{n}_{u} \mathbf{n}_{H_{d}} \mathbf{n}_{u} \mathbf{n}_{u}}{\mathbf{asses}}$
	$-\frac{1}{2}\left[M_1\bar{\lambda}_0\lambda_0+M_2\bar{\lambda}_A\lambda_A+M_3\bar{\tilde{g}}_B\tilde{g}_B\right]$
gaugino masses (M1,M2,M3) M': CP violation	$-\frac{i}{2} \left[M_1' \bar{\lambda}_0 \gamma_5 \lambda_0 + M_2' \bar{\lambda}_A \gamma_5 \lambda_A + M_3' \bar{\tilde{g}}_B \gamma_5 \tilde{g}_B \right]$
	+ $\left[(\mathbf{a}_{\mathbf{u}})_{ij} \epsilon_{ab} \bar{Q}_i^a H_u^b \tilde{u}_{Rj}^{\dagger} + (\mathbf{a}_{\mathbf{d}})_{ij} Q_i^a H_{da} d_{Rj}^{\dagger} + (\mathbf{a}_{\mathbf{e}})_{ij} L_i^a H_{da} \tilde{e}_{Rj}^{\dagger} + \text{h.c.} \right]$
qqH, IIH couplings	+ $\left[(\mathbf{c}_{\mathbf{u}})_{ij} \epsilon_{ab} \tilde{Q}^a_i H^{*b}_d \tilde{u}^{\dagger}_{\mathbf{R}j} + (\mathbf{c}_{\mathbf{d}})_{ij} \tilde{Q}^a_i H^{*}_{ua} \tilde{d}^{\dagger}_{\mathbf{R}j} + (\mathbf{c}_{\mathbf{e}})_{ij} \tilde{L}^a_i H^{*}_{ua} \tilde{e}^{\dagger}_{\mathbf{R}j} + \text{h.c.} \right]$
Higgs masses	$+ \begin{bmatrix} bH_u^a H_{da} + h.c. \end{bmatrix}, \qquad (8.1)$

SUSY Breaking



We know SUSY is a broken symmetry – but how? Different theories about the hidden sector on the market:



Most known are:

SUGRA:

→ Mediating interactions are gravitational → LSP is usually χ_1^0 GMSB:

- Mediating interactions are ordinary electroweak and QCD gauge interactions → LSP is usually the gravitino
- AMSB, Gaugino-mediation:
 - SUSY breaking happens on different brane in a higher-dimensional theory

Soft Supersymmetry Breaking: Give different masses to SM particles and their superpartners but preserve the structure of couplings of the theory

Gravitino in SUSY



- ♦ When standard symmetries are broken spontaneously → a massless boson appears for every broken generator
- If the symmetry is local, this boson is absorbed into the longitudinal components of the gauge boson, which becomes massive
- The same is true in SUSY → here, a massless fermion appears, called the Goldstino
- In the case of local supersymmetry, this Goldstino is absorbed into the Gravitino
- ◆ Coupling of the Goldstino (gravitino) to matter → proportional to $1/\sqrt{m_{\tilde{G}}M_{Pl}}$
- Mass of the gravitino in GMSB:

$$m_{\tilde{G}} \sim \frac{F}{M_{Pl}} \simeq 10^{-6} - 10^{-9} \text{GeV}$$

SUSY Parameter Space



 $\frac{1}{2}M_Z^2 = \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2 \beta}{\tan^2 \beta} - \mu^2 \approx -m_{H_u}^2 - \mu^2$

MSSM \rightarrow 105 free parameters (masses, couplings, phases) **Key parameters** are:

- μ = SUSY version of the SM Higgs mass
- $tan\beta$ = Ratio of vacuum expectation values of H_u/H_d
- $m_h = Mass of h^0$ $m_h^2 \le M_Z^2 + \Delta m_{rad}^2 (A_t, tan\beta, \mu, m_{\tilde{t} 1,2}, m_t, v^{**})$

•
$$m_A = Mass of A^0$$

- $m_{H^+} = Mass of H^{+/-}$
- m_{Hu²}, m_{Hd²} from SUSY breaking
- M₀² = Squark 3x3 mass term

$$-=m_0^2$$
 at GUT scale*

- M_{12} = Slepton 3x3 mass term
- M_1 = Bino mass term
- M_2 = Wino mass term

•
$$M_3$$
 = gluino mass term

- $A_{u,d,e}$ ~Yukawa-like 3x3 matrix = A_0 at GUT scale*
- = m_{1/2} at GUT scale*

 - In Planck-scale mediated SUSY breaking models like mSUGRA *

**
$$v = \sqrt{(v_u^2 + v_d^2)}$$

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SUSY Models



MSSM \rightarrow 105 free parameters (masses, couplings, phases)

pMSSM \rightarrow 19 free parameters (first two sfermion generations degenerate, and with negligible Yukawa couplings)



Full vs. simplified Model



Past: interpretation in CMSSM

Present: try to make it more easier for theorists to compare their model to our result \rightarrow use simplified model!



CMSSM

Future: ???

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SUSY Production (Squarks and Gluinos)



Colored production has highest cross section at the LHC (if colored particles are not too heavy)



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SUSY Production (3rd Generation and Chargino-Neutralino)







In the MSSM, **proton could decay** through SUSY particle exchange with new couplings $W = \lambda' LQD + \lambda LLE + \lambda'' UDD$:



Solution: *R*-parity conservation, with *R* defined as:

 $R = (-1)^{2s+3B+L}$ with s: Spin, B: baryon number, L: lepton number

R=1: SM particle

R=-1: SUSY particle

If *R*-parity is conserved (RPC):

- → Single SUSY particle cannot decay into just SM particles
- → Lightest SUSY particle (LSP) absolutely stable
- → LSP candidates are: lightest neutralino, gravitino

Other option: **small violation of B or L** (aka **RPV** models)



Summary: Introduction to Supersymmetry

SUSY: a beautiful and straight-forward extension of the SM...

- Only possible extension of the Poincare group
- Solves most SM questions (Includes gravity, Dark Matter, unification of the forces
- Predicts a light Higgs
- Perturbative → predictive

- Predicts many new scalar particles
- We don't know their masses
- Hard to find (at least up to now...)
- Adds new quantum number to prevent p decay, but otherwise not theoretically motivated...

- Escaped 30 years of searches!
- SUSY breaking not understood (soft breaking in hidden sector?)
- We are flowed by 105 new parameters...

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Where we are...



- Open questions of the Standard Model
- Introduction to Supersymmetry

-----> Challenges in data analysis <-----

- Tools for SUSY searches
- Detailed example for a SUSY search
- Outlook





How to find direct evidence of (weak-scale) SUSY at LHC?

- SUSY cross-section is weak (pb-fb) and SM background is huge
- SUSY mass spectrum is a priori unknown (well, we can use naturalness as guide...)
- SUSY signatures can be numerous and (more or less) striking
- Long decay chains difficult to reconstruct

Experimental challenges = systematics = search sensitivity

- Changing LHC conditions (especially pile-up) and experiments
- Trigger can kill the signal ...
- Object reconstruction in hadronic environment
- Detector understanding (timing, ...) crucial for non-standard SUSY
- Data/Monte-Carlo (dis-)agreement in hadronic environment























Event Selection needed!

Problem: Signals much smaller than background!

Big challenge to find the rare exciting events!!

Introduction to Supersymmetry

Challenges in data analysis

Tools for SUSY searches

Detailed example for a SUSY search

Open questions of the Standard Model

Outlook

Where we are...

Need to find variables that **distinguish BSM from SM** signatures!

- Missing transverse energy (expected for only weakly interacting neutral particles leaving the detector)
 - Calculated from all energies in calorimeter or all Particle Flow objects: E_T^{miss} (or MET)
 - Calculated from all jets: H_T^{miss} (or MHT)
- $\Delta \Phi$: angle between E_T^{miss} and leading jet(s)
- → M_{T2}, Razor: two heavy particles in two hemispheres decaying to LSP
- **Transverse mass** calculated from lepton p_T and E_T^{miss} : M_T
- Sum of transverse energies of all jets above certain p_T threshold:

$$H_T = \sum_{jets} p_T^{jet}$$

Effective mass of all objects: M_{eff}

$$M_{eff} = \sum_{jets} p_T^{jet} + p_T^{lepton} + E_T^{miss}$$
(typically peaks at 1.8(M²_{SUSY} - M²_{LSP})/M_{SUSY}

Crucial for all SUSY searches (and experimentally very challenging)

- Real MET: Presence of a neutral weakly interacting particle in the event (i.e. v)
- Fake MET: Mismeasurement + detector malfunctions, poorly instrumented regions

→ Agreement data – Monte Carlo key for SUSY searches

High MET might be caused by **high** p_T **mismeasured jets** $\rightarrow \Delta \Phi$: angle between E_T^{miss} and leading jet(s)

 $\Delta \Phi$ (jet, MET)_{min}>0.2-0.5 (0-lepton events)

QCD sample (0.5<p_T<1.1 TeV)

Reverting this cut provides a very nice QCD enriched sample for background studies...

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In W \rightarrow Iv decay, transverse mass M_T has an **endpoint at the true W mass**:

$$\begin{split} m_W^2 &= m_l^2 + m_\nu^2 + 2(E_T^l E_T^\nu \cosh \Delta \eta - \mathbf{p}_T^l \cdot \mathbf{p}_T^\nu) \ge \\ m_T^2 &= m_l^2 + m_\nu^2 + 2(E_T^l E_T^\nu - \mathbf{p}_T^l \cdot \mathbf{p}_T^\nu) \end{split}$$

In **RPC SUSY: two decay chains** with an unobserved child (c1 and c2) at each end. The "stransverse" mass M_{T2} : extension of M_T for the SUSY case of two unobserved particles:

$$M_{T2}(m_c) = \min_{p_T^{c(1)} + p_T^{c(2)} = p_T^{miss}} \left[\max\left(m_T^{(1)}, m_T^{(2)}\right) \right]$$

If m_c were known, the endpoint of M_{T2} could be used to calculate the parent mass M_p Simpler, but with similar efficiency: M_{CT}

$$m_{\rm CT}^2(v_1, v_2) = [E_{\rm T}(v_1) + E_{\rm T}(v_2)]^2 - [\mathbf{p}_{\rm T}(v_1) - \mathbf{p}_{\rm T}(v_2)]^2$$

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Interesting Variables: M_{T2}

• Simplest case: no extra jets (ISR/FSR); $m_c=0$:

$$(M_{T2})^2 = 2p_T^{vis(1)}p_T^{vis(2)}(1 + cos\phi_{12})$$

- For signal with symmetric systems, $p_T^{vis(1)} = p_T^{vis(2)}$:
 - → M_{T2} ~ MET
- For background:

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- Well-measured back-to-back dijets: $M_{T_2} \sim 0$
- Mis-measured events: $M_{T2} < E_T^{miss}$
- Multi-jet events \rightarrow divided into 2 pseudo-jets

CMS, vs = 7 TeV, L = 4.73 fb

QCD W+iet Z+iets

M_{ro} Analysis

200

400

10 Events

10^t

10⁴

10³

600

M_{T2} [GeV]

Main Backgrounds

CMS

 $\widetilde{\chi}_{1}^{0}$

a

р

ã

ĝ

ã

 \widetilde{X}_{2}^{0}

р

Background differ depending on the final state:

Z(→vv)+ jets

(W,t)+jets; $W \rightarrow \tau v$

q

 \widetilde{X}_{1}^{0}

D

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Where we are...

- Open questions of the Standard Model
- Introduction to Supersymmetry
- Challenges in data analysis
- Tools for SUSY searches

Detailed example for a SUSY search

Outlook

Example: Inclusive All-Hadronic Search Event Display

Inclusive All-Hadronic Search: Introduction

 $\tilde{\chi}_1^0$

Signature: Many jets and large missing transverse energy

- Least model-dependent analysis
- Large backgrounds:
 - Z+jets with $Z \rightarrow vv$ (irreducible)
 - W+jets and ttbar with W \rightarrow Iv and lost lepton or $\tau \rightarrow$ hadrons + v
 - QCD multijet events with large missing transverse momentum due to:
 - Leptonic decays of heavy flavor hadrons inside jets
 - Jet energy mismeasurement
 - Instrumental noise
 - Non-functioning detector components

Inclusive All-Hadronic Search: Event Selection

Baseline selection

- At least 3 jets with $p_T^{jet} > 50$ GeV and $|\eta| < 2.5$
- ✤ H_T > 350 GeV
- $H_T^{miss} > 200 \text{ GeV}$
- ↓ $|\Delta \Phi (J_{1,2}, H_T^{miss})| > 0.5$ and $|\Delta \Phi (J_3, H_T^{miss})| > 0.3$ to veto events where H_T^{miss} is aligned in transverse plane with one of the 3 leading jets
- Veto on isolated muons and electrons

Inclusive All-Hadronic Search: Background Estimation for $Z \rightarrow vv$

Background estimation with γ **+jets :**

Strategy:

- Declare photon invisible to emulate neutrinos
- Then re-calculate H_T^{miss} for this event
- Correct for the photon reconstruction efficiency and neutrino branching ratio
- Then scale the result with the production cross section ratio R_{Z/y}

SUSY signals could bias the prediction!

- \rightarrow Cross check with Z \rightarrow µµ+jets:
- Drawback: Low statistics in signal region, but comparable result in baseline selection

Inclusive All-Hadronic Search: W and Top Background Estimation

Lost Lepton Background Estimation

• Muon control sample with $M_T < 100 \text{ GeV}$ with $M_T = \sqrt{(2p_T^{\mu} E_T^{\text{miss}} (1-\cos \phi))}$ used to model:

- Non-isolated (but identified) leptons
- Non-identified leptons (ratio id/nonid taken from Monte Carlo)

τ Background Estimation

- Determined with a muon control sample
- Substitute μ with τ jet using response template to model the fraction of visible momentum
- Recalculate all quantities like H_T, H_T^{miss}

Inclusive All-Hadronic Search: QCD Background Estimation

SUS-13-012 JHEP 06 (2014) 055 arXiv:1402.4770

Most difficult background, derived here by '**Rebalance & Smear**' method:

- Rebalance all jets to overall p_T balance (=kind of `generator level jet', robust against seed jet mismeasurements and non-QCD processes)
- Smear p_T of each seed jet by a factor derived from jet resolution distribution (from simulation, and corrected for data/MC differences)

Smearing of the jets results in artificially created E_T^{miss} used to estimate the real E_T^{miss} distribution

Inclusive All-Hadronic Search: QCD Background Estimation

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Inclusive All-Hadronic Search: Results

Result measured in bins of H_T, H_T^{miss} and Njets Different search regions sensitive to different signals

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Recap

Today we discussed:

- Introduction to Supersymmetry
- Challenges in data analysis
- Tools for SUSY searches
- Detailed example for a SUSY search

Next:

- Overview over other SUSY searches
- Outlook

Event selection done – but still some backgrounds left Reconstruction of physics objects [e.g. muons]

- Suppose SUSY Model XYZ implies that we should be looking for a signature of one muon, plus 3 jets to do:
 - Use a combination of Monte Carlo simulation of all known processes [e.g. W+3 jets with W→µv] that give this signature plus data events with 1µ+3jets
 - But what about another background: Z+3 jets, for which we lose one lepton from the $Z \rightarrow \mu\mu$ decay?!

Problem: we can only get a feeling for the size of the effect from Monte Carlo and detector simulation, MC+simulation will never get the answer completely right

- One needs to find a way of calculating this efficiency from the only source that speaks the absolute truth: the **data**!
- → Need to apply "data-driven" methods / techniques

Obtaining (in)efficiencies from data

Tag-and-probe method, e.g. with $Z \rightarrow \mu\mu$ events:

- Make a selection based on one muon that "tags" the type of event (e.g. passes tight cuts; or passes the trigger)
- Then demand that second muon does the same

Basic Variables

- Number (N) of ÷
 - ↓ Jets

- Model dependence, e.g. mean N_{jet} can vary from 0 to 4 for 1-lepton events in mSUGRA (CMS LM points)
- Leptons
- Transverse momentum (p_T) of
 - ↓ Jets
 - Leptons
- Model dependence (softer or harder spectra possible)
- Angle ϕ : no (large) ϕ dependence expected good crosscheck
- Pseudorapidity $\eta = -\ln(\tan \theta/2)$
- Relative isolation within a cone ΔR defined as:

