CPT and Lorentz violations

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Outline

- CPT and Lorentz symmetries.
- Convensions and calculations
- CPT and Lorentz violations
- Discussions and generalisations

- The pillars of modern particle physics are Lorentz, CPT and Gauge symmetries.
- Lorentz transformation is given by:

$$x'^{\alpha} = \Lambda^{\alpha}{}_{\beta} x^{\beta} \tag{1}$$

where $x^{\alpha} = (ct, x, y, z)$ and

$$\Lambda^{\alpha}{}_{\beta} = \begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

with, $\beta = \frac{v}{c}$, $\gamma = \frac{1}{\sqrt{1-\beta^2}}$.

- Though Special Relativity is replaced by Generel relativity but all physical fields have to obey the laws of local Lorentz invariance.

- An important property of local quantum field theory in minkowski space time is CPT invariance, where,
  C: Charge conjugation,
  P: Parity,
  T: Time reversal.

- The CPT theorem states that every local relativistic quantum field theory is symmetric under the combined operation of C, P and T.

- So any violation of Lorentz and CPT symmetries must be very small and they may give a hint of new physics at vastly different scales.

- There are many ways CPT and Lorentz violation can occur. We shall just focus on a Particular case that is CPT and Lorentz violation due to topologically non-trivial spacetime.

Convention

- We are considering chiral gauge theories which is anomaly free. For eg. Chiral Yang-Mills with gauge group $G = SO(10)$.
- In our calculation we consider the chiral gauge theory is defined over a four-dimensional Euclidean flat manifold ($M = \mathbb{R}^3 \times \mathbb{S}^1$) with noncompact coordinates $x^1, x^2, x^3 \in \mathbb{R}$ and compact $x^0 \in \mathbb{S}^1$.
- For the calculation purpose metric is taken to be euclidean flat metric $g_{ab} = \text{diag}(1, 1, 1, 1)$.
- Another assumption is about the gauge fields as follows

$$A_m(x) = A_m(x^1, x^2, x^3), \quad \text{and } A_0(x) = 0. \tag{3}$$

- The four dimensional fermionic action for chiral fermions is given by

$$S[\psi_L^\dagger, \psi_L, A] = \int d^4x \mathcal{L}[\psi_L^\dagger, \psi_L, A] \tag{4}$$

- Now we shall look at the effective action of the gauge fields.

- In vacuum the virtual creation and annihilation of fermion-antifermion pairs interact with the classical gauge fields. The effective action $\Gamma[A]$ is a functional which takes these interactions into account.

Effective action

In feynman's path integral formalism the functional $\Gamma[A]$ is obtained by integrating out the fermionic degrees of freedom

$$\exp(\Gamma[A]) = \int \mathcal{D}\psi_L^\dagger(x)\mathcal{D}\psi_L(x) \exp\left(i \int_M d^4x \; \mathcal{L}[\psi_L^\dagger, \psi_L, A]\right), \tag{5}$$

which is formally equal to the determinant of the operator $[\sigma_-^a(\partial_a + A_a)]$. This is "formally" because this operator has an unbounded spectrum so that the determinant is infinite. So to regularize this expression the suitable regularization method is used.

Effective action

- The left-handed fermionic fields in the action can be expanded into Fourier modes

$$\psi(x) = \sum_{n=-\infty}^{\infty} e^{\frac{2\pi i n x^0}{L}} \xi_n(x^1, x^2, x^3), \tag{6}$$

and

$$\psi^\dagger(x) = \sum_{n=-\infty}^{\infty} e^{-\frac{2\pi i n x^0}{L}} \xi_n^\dagger(x^1, x^2, x^3). \tag{7}$$

- The corresponding effective action can be factorized as

$$\exp\left[-\Gamma_W(A)\right] \propto \prod_{n=-\infty}^{\infty} \int \mathcal{D}\chi_n^\dagger \mathcal{D}\chi_n \quad \exp[-I_n(\chi_n^\dagger, \chi_n, A)], \tag{8}$$

- where,

$$I_n(\chi_n^\dagger, \chi_n, A) = \int_M d^3x \; \chi_n^\dagger \left[\sigma^i(\partial_i + A_i) + \frac{2\pi i n}{L}\right] \chi_n \tag{9}$$

We focused on $n = 0$ sector

The perturbative calculation gives rise to the one-loop result

$$\Gamma_W(A)] \subseteq \int_M d^3x \int_0^L dx^0 \; \frac{s_0(1+a)\pi}{L} \; \omega_{CS}[A] \tag{10}$$

where

$$\omega_{CS}[A] \simeq \epsilon^{ijk}(A_i \partial_j A_k - \frac{2}{3} A_i A_j A_k) \tag{11}$$

is the Chern-Simons density

- The CPT transformation of the anti-Hermitian gauge field is given by

$$A_\mu(x) \to A_\mu^T(-x). \tag{12}$$

- For the hermitian electromagnetic vector potential $A_\mu(x)$ CPT transformation is given by $A_\mu(x) \to -A_\mu(-x)$.

- Clearly the term $\omega_{CS}[A]$ are not invariant under CPT transformation. They are CPT odd terms.

- In the above term not every Lorentz index is contracted with a four-vector. So this term is obviously not Lorentz invariant.

Discussion

- The extent of the CPT non-invariance is inversely proportional to the length of the compactified coordinate.
- the mass scale of the CPT-violating term for the photon field is of the order of

$$\alpha L^{-1} \sim 10^{-35} ev(\frac{\alpha}{1/137})(\frac{1.5 \times 1^{10} lyr}{L}) \tag{13}$$

- The CPT and Lorentz violation may be important in the very early universe.

- The CPT and Lorentz violation holds for the spacetime manifolds M having at least one compact spatial dimension can be factored out. For example $(R \times S^1 \times S^1 \times S^1)$.

- In lower dimension, the two-dimensional chiral U(1) theory over torus, where the chiral determinant is known exactly, the CPT violation also have been calculated. For example the 3, 4, 5 model (three chiral fermions with charges $q_{R1} = 3$, $q_{R2} = 4$, $q_{L3} = 5$)

- We are looking for more generalised results, where the gauge fields depends upon the compactified coordinate(s) also.

References

1. G. Luders, Ann. Phys. (N. Y.) 2 (1957) (1).

2. J. Sakurai, Invariance Principles and Elementary Particles, Princeton Univ. Press, Princeton, 1964.

3. R. Streater, A. Wightman, PCT, Spin and Statistics, and All That, Ben- jamin, New York, 1964.

4. Nucl. Phys B 578, 277 (2000)

5. Phys. Rev. D 63, 097701 (2001)

THANK YOU!