Adam Falkowski
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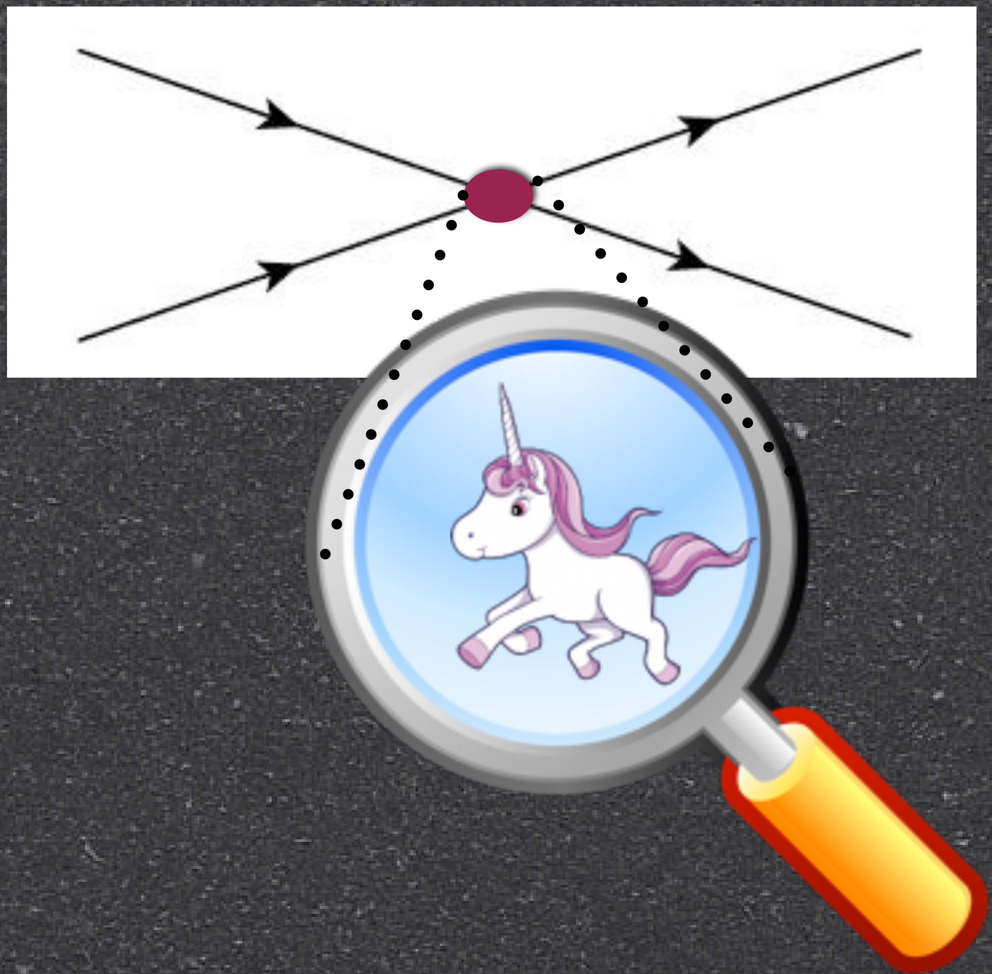
Freudenstadt,
29-30 September 2015

Lectures on Effective Field Theory Approach to Physics Beyond the Standard Model



Plan

Part I

- Short introduction and motivations
- (Illustrated) philosophy of effective field theory

Part II

- Effective Lagrangian for physics beyond the SM
- From D=6 operators to collider observables

Part III

- Constraints on EFT from LHC Higgs physics

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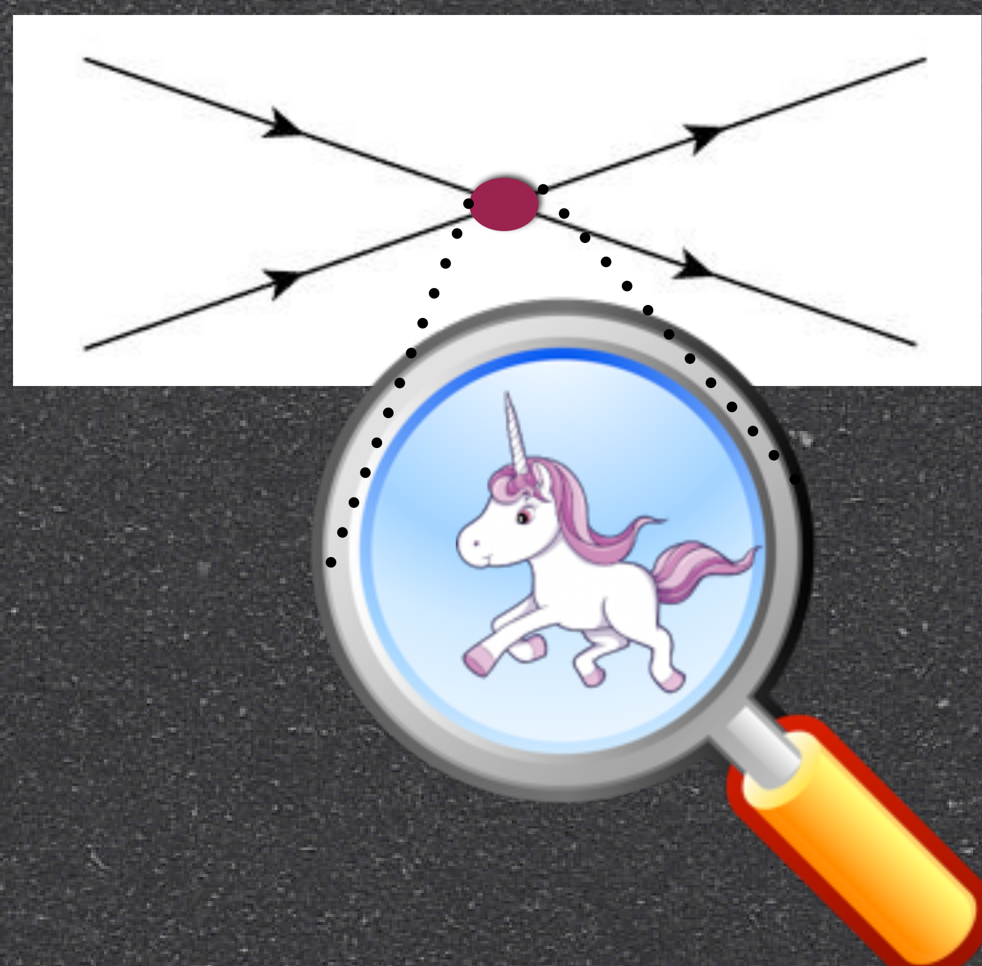

part I

Plan

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Part III

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Introduction
and
Motivations

Life after Higgs discovery

- Discovery of 125 GeV Higgs boson is last piece of puzzle that falls into place
- No more free parameters in SM
- Overwhelming evidence that particle interactions obey linearly realized SU(3)xSU(2)xU(1) local symmetry
- All data consistent with electroweak symmetry breaking SU(2)xU(1)→U(1) proceeding via a single doublet Higgs field
- No new particles from beyond the SM with masses below 0.5 - 2 TeV

What about new physics?

- We know physics beyond SM exists (neutrino masses, dark matter, inflation, baryon asymmetry)
- There are also some theoretical hints for new physics (strong CP problem, flavor hierarchies, gauge coupling unifications, naturalness problem)
- But there isn't one model or a class of models that is strongly preferred
- How to keep open mind on many possible forms of new physics that may show up in experiment?

Several approaches to new physics searches

Model specific

pick one well-defined, "motivated",
often UV complete model

E.g. 2HDM, MSSM, NMSSM, NNMSSM, ...,
composite Higgs, minimal walking technicolor

Simplified models

pick simple well-defined model
that captures some aspects of phenomenology
of large class of specific models

E.g. singlet scalar, gluino+neutralino, heavy
top quark, vector triplet,

Model independent

parametrize low-energy effects
large class of models as higher-dimensional
contact interaction of light particles

Effective field theory

Effective Field Theory Framework

- An effective field theory (EFT) is a QFT for low energy degrees of freedom, where heavy particles that cannot be directly produced in experiment have been integrated out
- Effects of heavy particles are encoded into contact interactions of low energy particles
- EFT Lagrangian can be defined as consistent expansion in inverse mass scale of the heavy particles
- Under certain assumptions, EFT framework allows one to describe effects of new physics beyond the SM in a model independent way

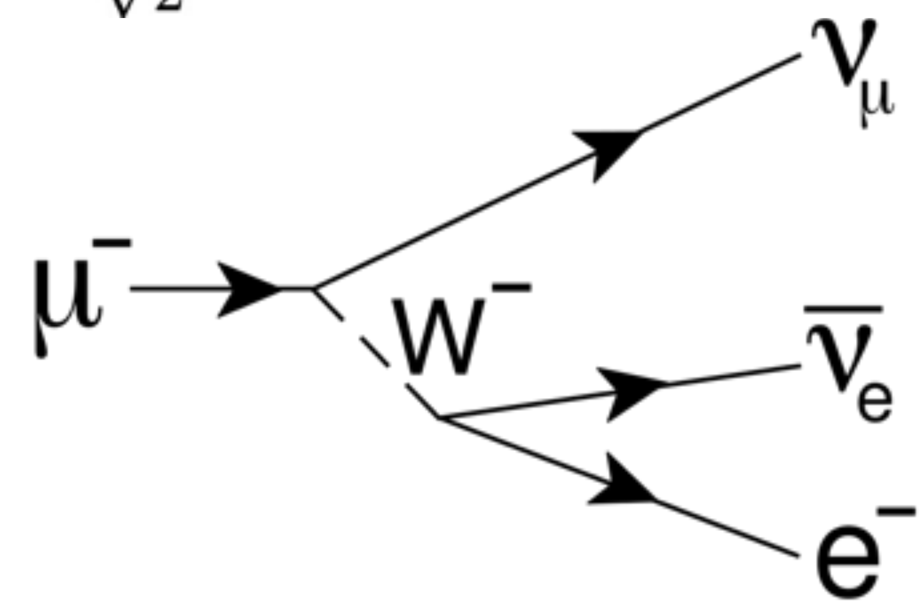
Philosophy of EFT

EFT example #1

Fermi Theory of weak interactions

See the spinor bible 0812.1594 for 2-component notation

$$\mathcal{L} = \frac{g_L}{\sqrt{2}} \left(\bar{\nu}_\mu \bar{\sigma}_\rho \mu + \bar{\nu}_e \bar{\sigma}_\rho e \right) W^+_\rho + \text{h.c.}$$



$$\mathcal{M} = \frac{g_L^2}{2} \bar{x}(k_{\nu_\mu}) \bar{\sigma}_\rho x(k_\mu) \frac{1}{q^2 - m_W^2} \bar{x}(k_e) \bar{\sigma}_\rho y(k_{\nu_e})$$

$$q = k_\mu - k_{\nu_\mu}$$

$$q^2 \leq m_\mu^2 \ll m_W^2$$

- In SM, muon decays to electrons and neutrinos are mediated by W bosons

$$\frac{d\Gamma(\mu \to e\nu\nu)}{dq^2} = \frac{g_L^4 (m_\mu^2 - q^2)^2 (m_\mu^2 + 2q^2)}{3072\pi^3 m_\mu^3 (m_W^2 - q^2)^2}$$

$$\approx \frac{g_L^4 (m_\mu^2 - q^2)^2 (m_\mu^2 + 2q^2)}{3072\pi^3 m_\mu^3 m_W^4} \left(1 + \frac{2q^2}{m_W^2} + \ldots \right)$$

Fermi Theory of weak interactions

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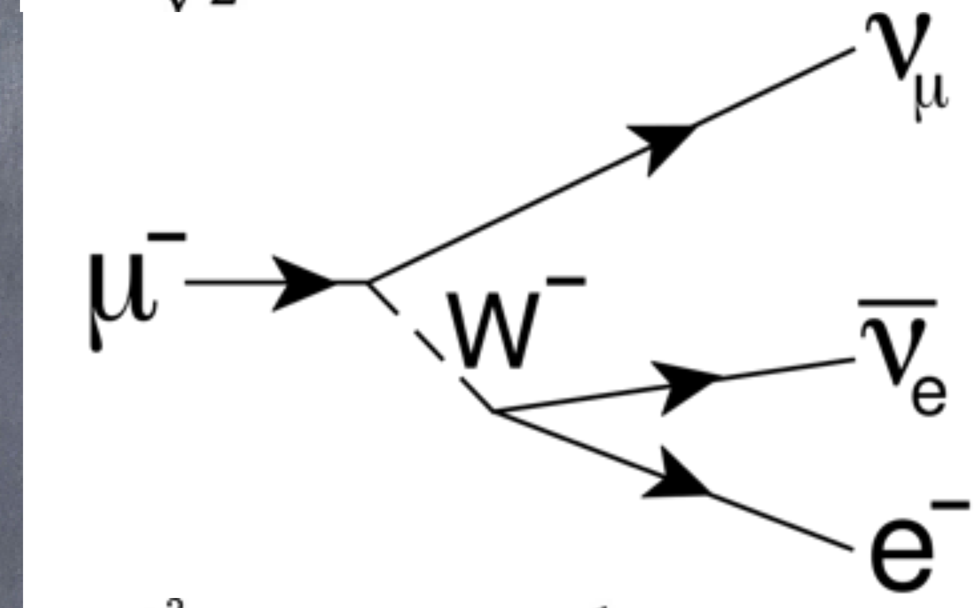


$$\mathcal{M} = \frac{g_L^2}{2} \bar{x}(k_{\nu_\mu}) \bar{\sigma}_\rho x(k_\mu) \frac{1}{q^2 - m_W^2} \bar{x}(k_e) \bar{\sigma}_\rho y(k_{\nu_e})$$

$$q = k_\mu - k_{\nu_\mu}$$

- In SM, muon decays to electrons and neutrinos are mediated by W bosons

$$q^2 \leq m_\mu^2 \quad \& \quad m_\mu^2 / m_W^2 \sim 10^{-6}$$

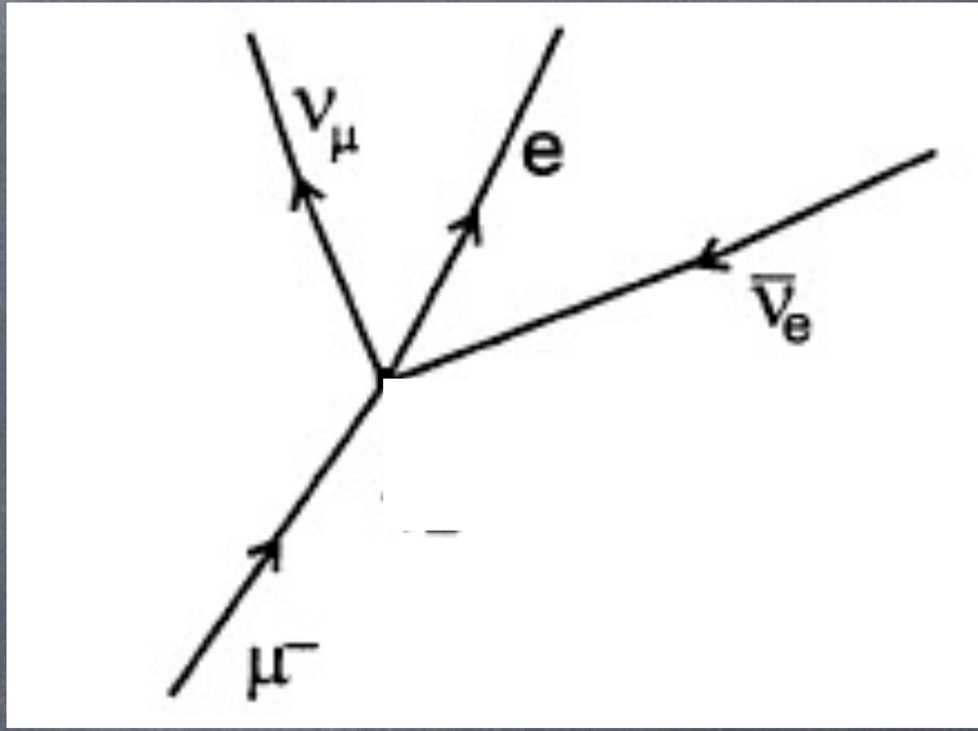
$$\frac{d\Gamma(\mu \to e\nu\nu)}{dq^2} \approx \frac{g_L^4 (m_\mu^2 - q^2)^2 (m_\mu^2 + 2q^2)}{3072 \pi^3 m_\mu^3 m_W^4}$$

Fermi Theory of weak interactions

$$\mathcal{L}_{\text{eff}} \supset \frac{c}{\Lambda^2}(\bar{\nu}_\mu \bar{\sigma}_\rho \mu)(\bar{e}\bar{\sigma}_\rho \nu_e) + \text{h.c.}$$

- In SM, muon decays to electrons and neutrinos are mediated by W bosons

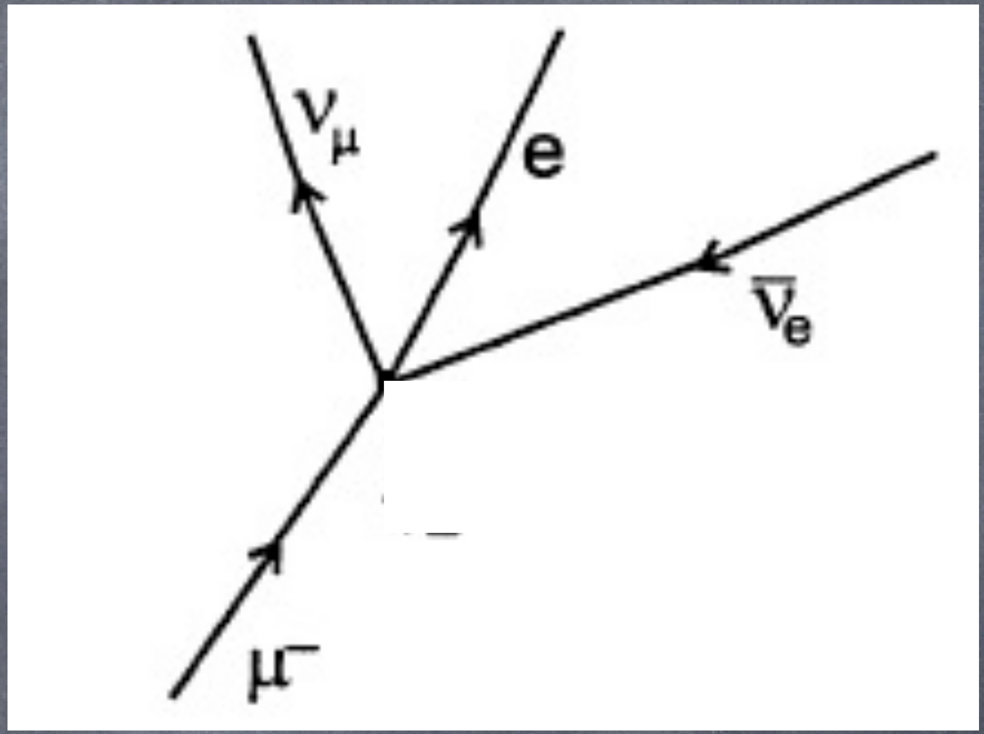
- Up to 10^-6 corrections, this process can be approximated by the Fermi theory where W boson is "integrated out" and instead there is a 4-fermion contact interaction between muon, electron, and neutrino



$$\mathcal{M} = \frac{c}{\Lambda^2}\bar{x}(k_{\nu_\mu})\bar{\sigma}_\rho x(k_\mu)\bar{x}(k_e)\bar{\sigma}_\rho y(k_{\nu_e})$$

$$\frac{d\Gamma(\mu \to e\nu\nu)}{dq^2} = \frac{c^2(m_\mu^2 - q^2)^2(m_\mu^2 + 2q^2)}{768\pi^3 m_\mu^3 \Lambda^4}$$

Fermi Theory of weak interactions



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$$\mathcal{L}_{\text{eff}} \supset \frac{c}{\Lambda^2}(\bar{\nu}_\mu \bar{\sigma}_\rho \mu)(\bar{e}\bar{\sigma}_\rho \nu_e) + \text{h.c.}$$

Matching effective theory amplitude to SM one at leading order in q^2/mW^2

$$\Lambda = m_W$$

$$c = -\frac{g_L^2}{2}$$

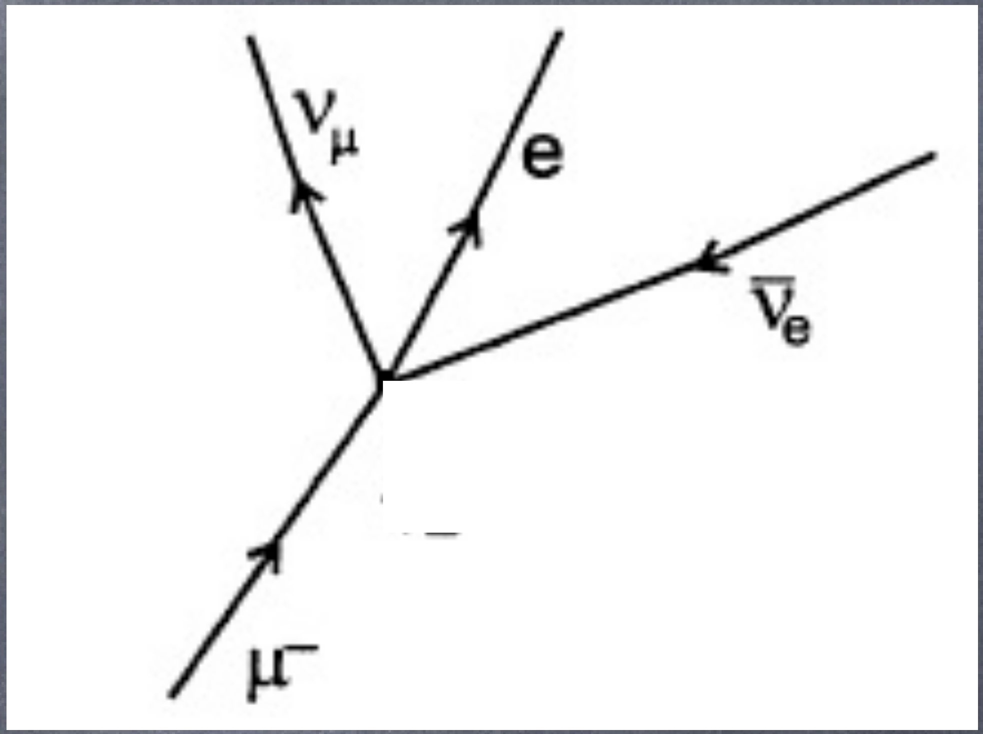
$$\frac{|c|}{\Lambda^2} = \frac{2}{v^2} \equiv 2\sqrt{2}G_F \approx \frac{1}{(174\text{GeV})^2}$$

Fermi Theory of weak interactions

- In SM, muon decays to electrons and neutrinos are mediated by W bosons

- Up to 10^-6 corrections, this process can be approximated by the Fermi theory where W boson is "integrated out" and instead there is a 4-fermion contact interaction between muon, electron, and neutrino

- One can systematically "improve" Fermi theory by adding higher-order operators suppressed by more powers of Λ



$$\mathcal{L}_{\text{eff}} \supset \frac{c}{\Lambda^2}(\bar{\nu}_\mu \bar{\sigma}_\rho \mu)(\bar{e}\bar{\sigma}_\rho \nu_e) + \frac{c_8}{\Lambda^4}(\bar{\nu}_\mu \bar{\sigma}_\rho \mu)\Box(\bar{e}\bar{\sigma}_\rho \nu_e) + \cdots + \text{h.c.}$$

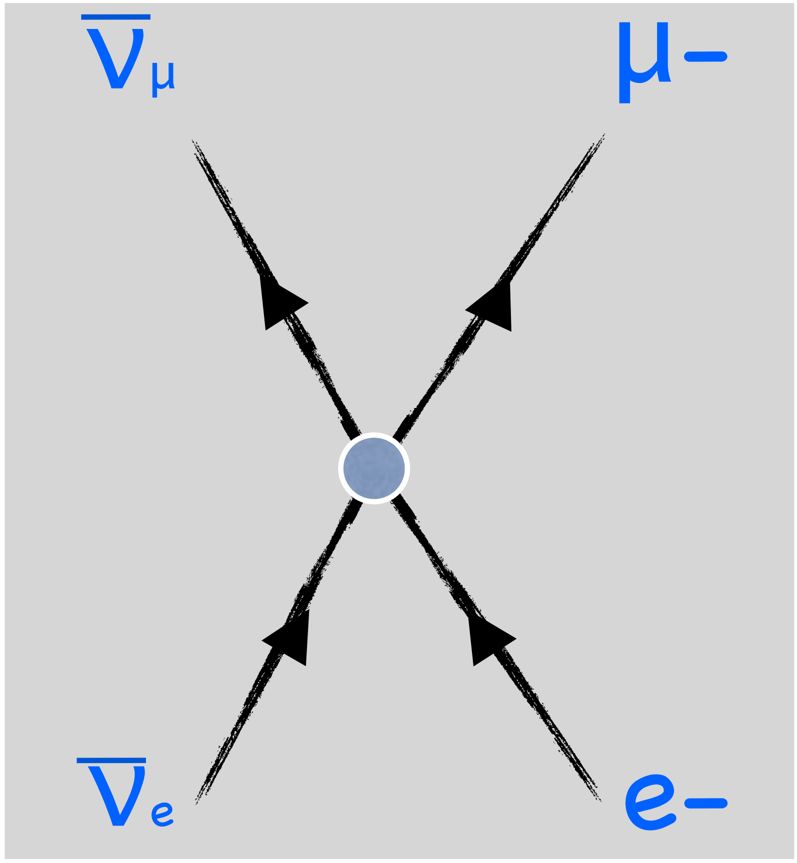
$$\Lambda = m_W$$

$$c = -\frac{g_L^2}{2}$$

$$\frac{|c|}{\Lambda^2} = \frac{2}{v^2} \equiv 2\sqrt{2}G_F \approx \frac{1}{(174\text{GeV})^2}$$

Exercise: calculate c8 by matching to SM amplitude at O(Λ^-4)

Fermi Theory of weak interactions



$$\mathcal{L}_{\text{eff}} \supset -\frac{2}{v^2}(\bar{\nu}_\mu \bar{\sigma}_\rho \mu)(\bar{e}\bar{\sigma}_\rho \nu_e)$$

- Same Fermi theory can be used to describe related processes, e.g. high-energy neutrino inelastic scattering

$$\mathcal{M}(\bar{\nu}_e e^- \to \bar{\nu}_\mu e^+) = -\frac{2}{v^2}[\bar{x}(k_\mu)\bar{\sigma}_\rho y(k_{\nu_\mu})][\bar{y}(k_{\nu_e})\bar{\sigma}_\rho x(k_e))]$$

In the limit all fermions are massless, only 1 helicity amplitude is non-zero:

$$\mathcal{M}(+- \to +-) = (1+\cos\theta)\frac{2s}{v^2}$$

In Fermi theory, amplitudes grow indefinitely with scattering energy!

At some point, s-wave amplitude violates unitarity bound

$$\mathcal{M}^0(+- \to +-) = \frac{s}{8\pi v^2}$$

$$\mathcal{M}^J = \frac{1}{32\pi}\int_{-1}^{1} d\cos\theta P_J(\cos\theta)\mathcal{M}$$

$$\frac{1}{4} = [\mathrm{Re}\mathcal{M}^J]^2 + \left[\mathrm{Im}\mathcal{M}^J - \frac{1}{2}\right]^2$$

Perturbative Unitarity $\Rightarrow \mathrm{Re}\mathcal{M}^J \lesssim 1$

!

Fermi theory does not make sense
(at least perturbatively)
for E $\gtrsim$ 4 π v ~ few TeV

Fermi Theory of weak interactions

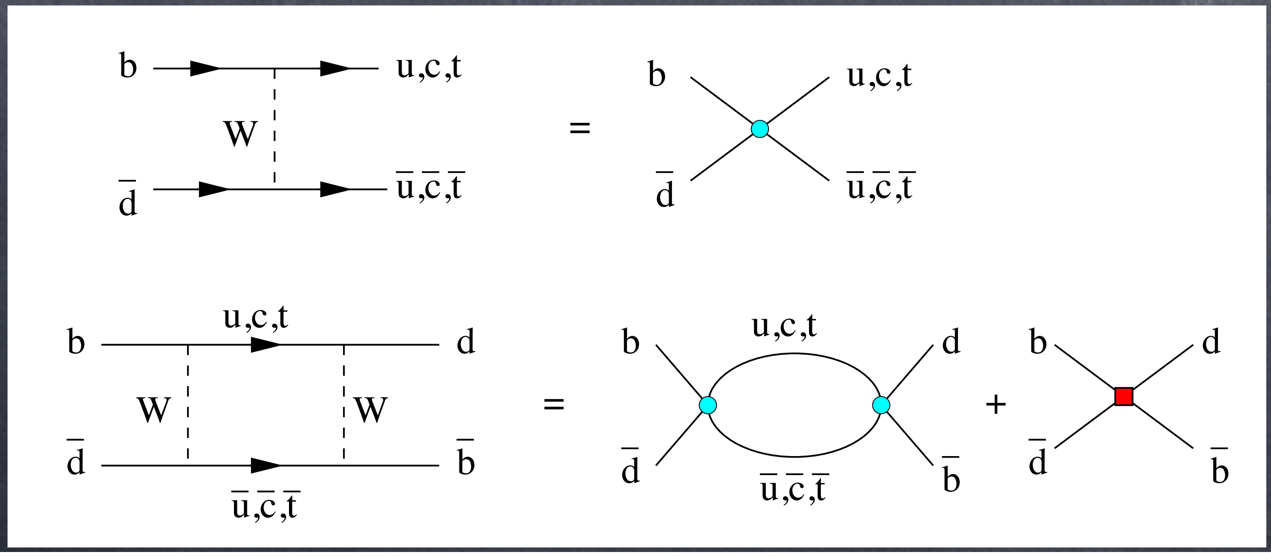
Lessons learned:

- EFT can be a great and simple tool to study low-energy consequences of more complete theories as long as $E \ll \Lambda$, where Λ is the mass scale of the UV theory.
- It predicts correlations between rates of different processes (in our example processes related by crossing symmetry, but it is less trivial in other examples)
- However, EFT has limited validity range. It stops making sense as a perturbative theory for $E \gtrsim 4 \pi \Lambda/g$, where g is the coupling strength in the UV theory
- In reality, as one approaches $E = \Lambda$ from EFT side, higher-dimensional operators become more and more relevant, and expansion in $1/\Lambda$ becomes impractical. For $E \sim \Lambda$ resonance in UV theory can be resolved and EFT description becomes useless.

EFT example #2

Weak meson decays

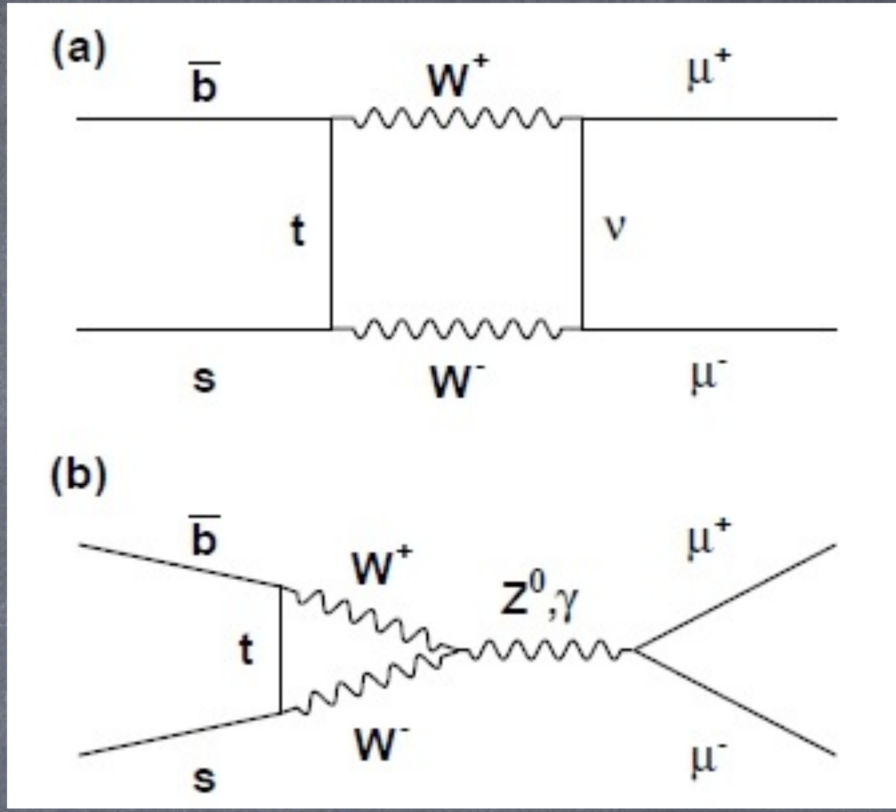
- At low energies below W mass, W boson can be integrated out, leading to effective theory with 4-fermion interactions
- Some flavor violating operators are loops and CKM suppressed, therefore their coefficients are suppressed by more than heavy mass scale



Weak meson decays

- Consider one particular process of dilepton B-meson decays, $B_s \to \mu^+\mu^-$
- At quark level, in SM decay is mediated by loops of heavy particles such as W boson and top quark with mass scale ~100 GeV
- On the other hand, B-meson decay is process happening at scale ~5 GeV. Thus it can be described in EFT, where expansion parameter is $(m_b/m_W)^2 \sim 0.5\%$
- In this approach, integrating out W and t at 1-loop produces a 4-fermion operator in EFT
- Then this EFT can be more easily matched onto the Heavy Quark Effective Theory, which is a theory of mesons

Exercise: compute branching fraction of Bs to $\mu\mu$ mediated by $bs\mu\mu$ 4-fermion operators with other tensor structures



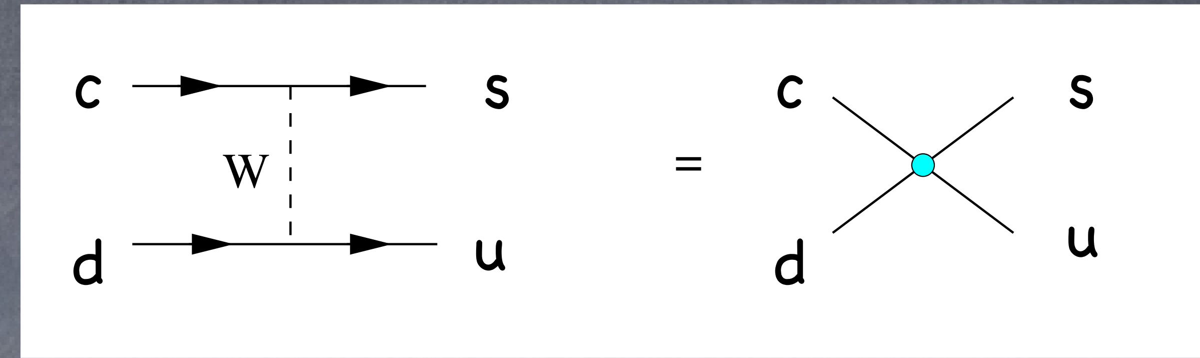
$$\mathcal{L}_{eff} \supset -4.1\frac{\alpha V_{ts}^* V_{tb}}{2\pi v^2}(\bar{s}_L\gamma_\mu b_L)(\bar{\ell}\gamma_\mu\gamma_5\ell)$$
$$\approx -\frac{1}{(17\mathrm{TeV})^2}(\bar{s}_L\gamma_\mu b_L)(\bar{\ell}\gamma_\mu\gamma_5\ell)$$

$$\langle 0|\bar{s}\gamma_\mu\gamma_5 b|B_s(p)\rangle = i p_\mu f_{B_s}$$
$$f_{B_s} \approx 0.25\mathrm{GeV}$$

$$\Gamma(B_s \to \mu\mu) \approx \frac{f_{B_s}^2 m_{B_s} m_\mu^2}{8\pi}\frac{1}{(17\mathrm{TeV})^4}$$

$$\mathrm{Br}(B_s \to \mu\mu) \approx 3.7\times 10^{-9}$$

Weak meson decays

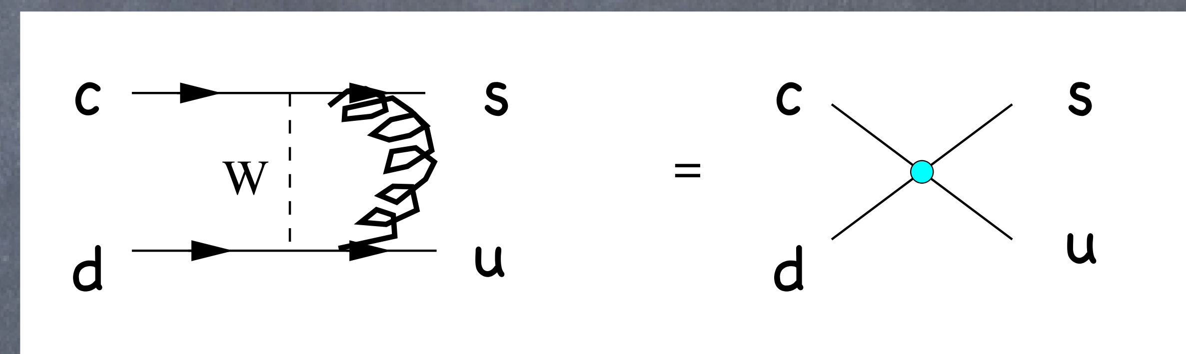


$$O_1 = -\frac{2}{v^2}(\bar{s}_a \bar{\sigma}_\mu c_a)(\bar{u}_b \bar{\sigma}_\mu d_b)$$

$$O_2 = -\frac{2}{v^2}(\bar{s}_a \bar{\sigma}_\mu c_b)(\bar{u}_b \bar{\sigma}_\mu d_a)$$

- Another example is effective 4-quark operators responsible, e.g. for D→K π decays

- Matching SM to EFT at tree level, only one color structure is generated (color flows from c to s, and from d to u)



- However, matching at 1-loop, another color structure appears due to gluons in loops

- At 1 loop the amplitudes display logarithmic corrections, O(αs/π Log(mW/μ)). These effects can be resummed using renormalization group techniques

$$\frac{dC_i}{d\log\mu} = \gamma_{ij} C_j$$

$$\gamma_{ij} = \frac{\alpha_s}{2\pi}\begin{pmatrix} -1 & 3 \\ 3 & -1 \end{pmatrix}$$

Buchalla et al,
hep-ph/9512380

Weak meson decays

Lessons learned:

- In case of meson decays and mixing, EFT becomes an essential tool to compare SM predictions with experimental observations

- Moreover, this formalism facilitates dealing with physics at vastly different scales. In this case, 5 GeV B-meson decays depend on electroweak physics at 100 GeV. In loop calculations large logarithms Log(mW/mb) appear. This is much simpler to treat within the EFT, where one can run the 4-fermion operators from mW to mb, which corresponds to resumming the large logs

EFT example #3

Chiral perturbation theory

- ChPT describes low energy interactions of pions.
- Underlying theory - QCD - is known, but coefficients of EFT operators cannot be calculated analytically.
- Approximate symmetries inherited from QCD provide a method to write down possible pion interactions in a systematic expansion

Chiral perturbation theory

- QCD has two nearly massless quarks: up and down. In massless limit, QCD Lagrangian has SU(2)LxSU(2)R symmetry corresponding to separate rotations of left-handed and right-handed components

$$\mathcal{L} = i\bar{u}\bar{\sigma}_\mu\partial_\mu u + i\bar{d}\bar{\sigma}_\mu\partial_\mu d + iu^c\sigma_\mu\partial_\mu\bar{u}^c + id^c\sigma_\mu\partial_\mu\bar{d}^c$$

$$\mathcal{L}_{mass} = -m_u u u^c - m_d d d^c + \text{h.c.}$$

- This symmetry is explicitly and completely broken by quark masses
- However, there's larger source of symmetry breaking due to QCD vacuum condensate, <u u^c> = < d d^c>
- This spontaneously breaks SU(2)LxSU(2)R down to diagonal SU(2) that rotates left-handed and right-handed quarks in the same way
- Therefore, there should 3 light Goldstone boson states (identified with pions), 1 for each spontaneously broken generator of symmetry

Chiral perturbation theory

- Low energy theory of pions should inherit symmetries of QCD

- This means the theory should have non-linearly realized SU(2)LxSU(2)R symmetry such that diagonal (vector) part is linearly realized, and under axial part pions transform under shift symmetry

- Effective Lagrangian can then be written in derivative expansion

- Lowest order term that one can write has 2 derivatives. It describes kinetic terms of pions, but also infinite series of 2-derivative pion interaction terms

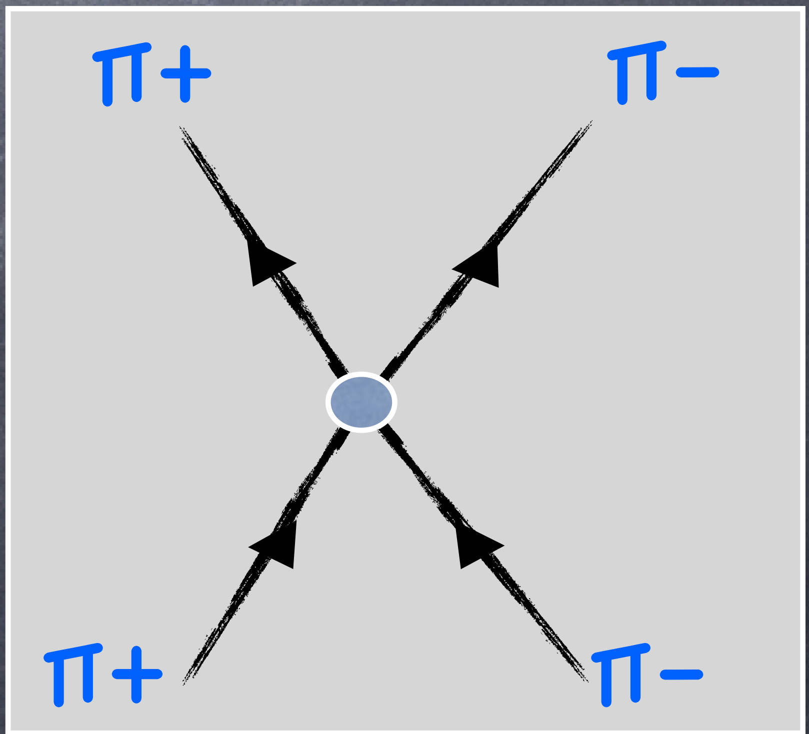
- These interactions can be tested in pion-pion scattering, which allows one to fit f≈93 MeV

$$U = \exp(i\pi^a\sigma^a/f) = \exp\left[\frac{i}{f}\begin{pmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{pmatrix}\right]$$

$$U \to LUR^\dagger, \qquad L, R \in SU(2)$$

$$\mathcal{L}^{(2)}_{\rm eff} = \frac{f^2}{4}\mathrm{Tr}[\partial_\mu U^\dagger \partial_\mu U]$$

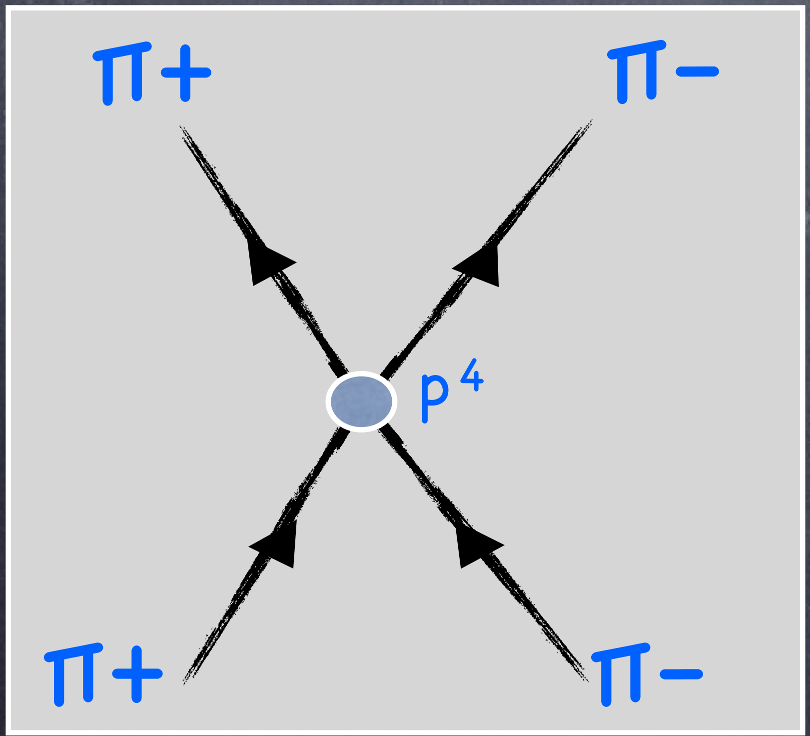
$$\mathcal{L}^{(2)}_{\rm eff} = \partial_\mu\pi^+\partial_\mu\pi^- + \frac{1}{2}\partial_\mu\pi^0\partial_\mu\pi^0 + \frac{1}{2f^2}(\partial_\mu\pi^+\pi^- + \partial_\mu\pi^-\pi^+ + \partial_\mu\pi^0\pi^0)^2 + \ldots$$

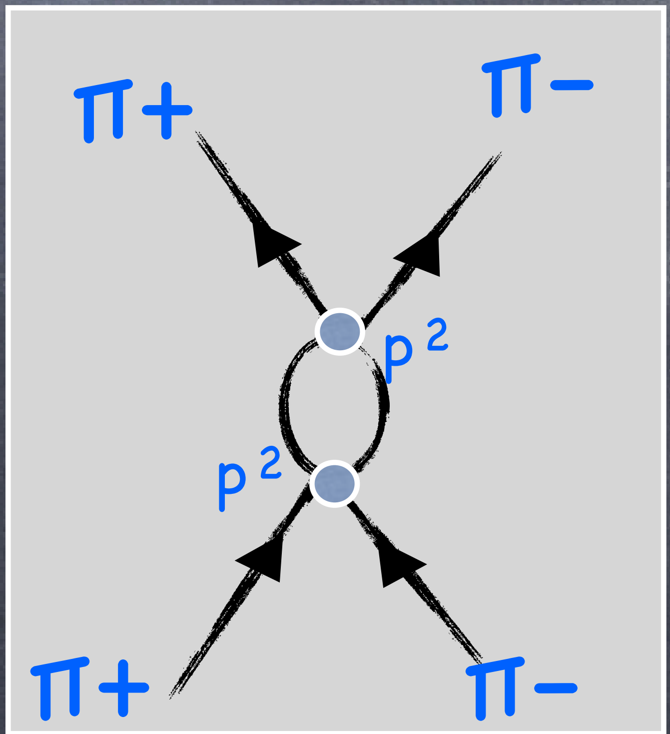


Chiral perturbation theory

$$\mathcal{L}_{\text{eff}}^{(4)} = L_1 \left(\text{Tr}[\partial_\mu U^\dagger \partial_\mu U]\right)^2 + L_2 \text{Tr}[\partial_\mu U^\dagger \partial_\nu U]\text{Tr}[\partial_\mu U^\dagger \partial_\nu U] + L_3 \text{Tr}[\partial_\mu U^\dagger \partial_\mu U \partial_\nu U^\dagger \partial_\nu U]$$

- ChPT theory can be extended to 4-derivative level. This produces 4-derivative interactions terms of pions, in addition to 2-derivative ones
- By studying momentum dependence of pion scattering one can fit the parameters L1, L2, L3
- Note that in this case 1-loop diagrams with 2-derivative vertices have to included together with tree-level diagrams with 4-derivative vertices. In ChPT, derivative expansion is intimately tied to loop expansion.





Scherer, hep-ph/0210398

Coefficient	Empirical Value
L_1^r	0.4 ± 0.3
L_2^r	1.35 ± 0.3
L_3^r	-3.5 ± 1.1

(In units of 10^-3, at scale mρ)

Chiral perturbation theory

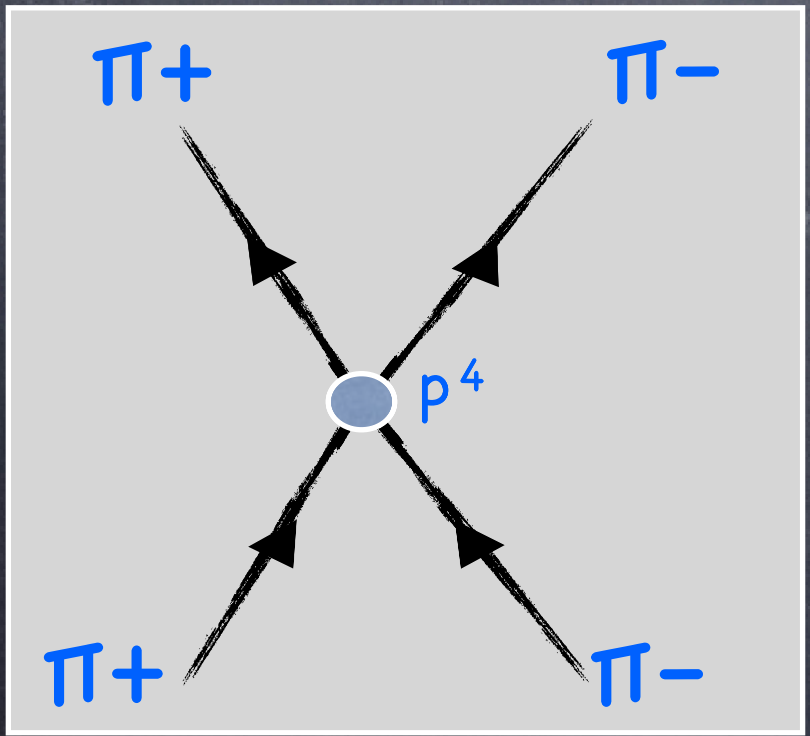
$$\mathcal{L}^{(4)}_{\text{eff}} = L_1 \left(\text{Tr}[\partial_\mu U^\dagger \partial_\mu U]\right)^2 + L_2 \text{Tr}[\partial_\mu U^\dagger \partial_\nu U]\text{Tr}[\partial_\mu U^\dagger \partial_\nu U] + L_3 \text{Tr}[\partial_\mu U^\dagger \partial_\mu U \partial_\nu U^\dagger \partial_\nu U]$$

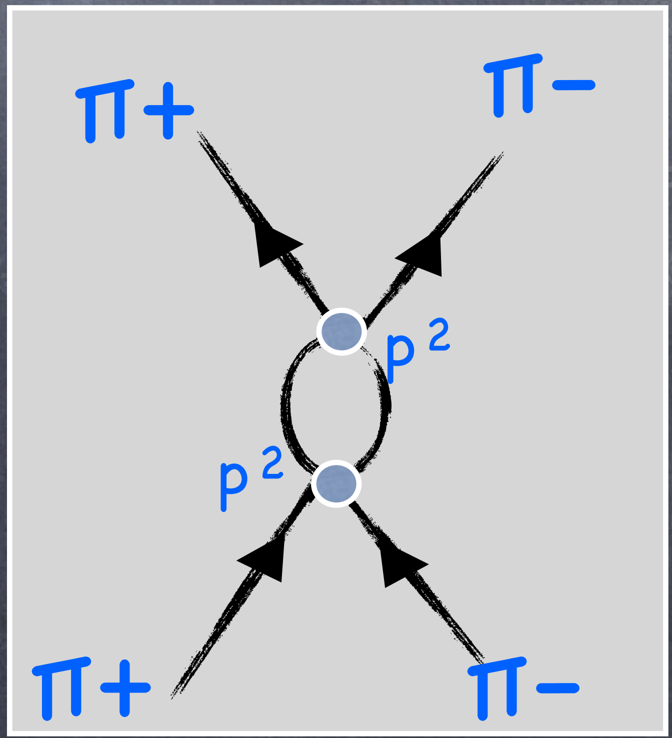
? $\text{Tr}[\partial^2 U^\dagger \partial^2 U]$

- Operators that can be eliminated or traded for other by equations of motion are not included in effective Lagrangian

- This is because they are redundant - all their effect on on-shell amplitudes can be described by other terms

- In this case, in the limit of massless pions, equation of motion is $\Box U = 0$, so the new term above does not contribute to on-shell amplitudes at all





Chiral perturbation theory

Lessons learned:

- It is often advantageous to work with EFT even when matching with UV theory cannot be calculated. Then one needs to write down all possible non-redundant interaction terms consistent with EFT symmetries in some systematic expansion, and determine their coefficients from experiment

- EFT is not renormalizable, therefore it formally has infinite number of parameter. However, at a fixed order in EFT expansion it is renormalizable. As soon as all coefficients are fixed at a given order from experiment, other observables can be predicted at that order

Other EFT examples

- Non-relativistic QED. Describes bound states of electrons, positrons, muons, etc.
- Soft-collinear effective theory. Describes light-like interaction of light quarks.
- Nuclear effective theory. Describes interactions of protons, neutrons, deuterons, etc.

Summary of 1st part

- EFTs emerge naturally in particle physics and elsewhere, at vastly different scales and kinematical regimes
- Even when UV theory is known, and matching to IR EFT is calculable, EFT is important tool for calculations (simplicity, resummation of large logs)
- When IR Lagrangian cannot be calculated, EFT framework is important tool to organize physics description of low energy theory.
- We expect Standard Model is low-energy effective theory to some yet unknown UV theory