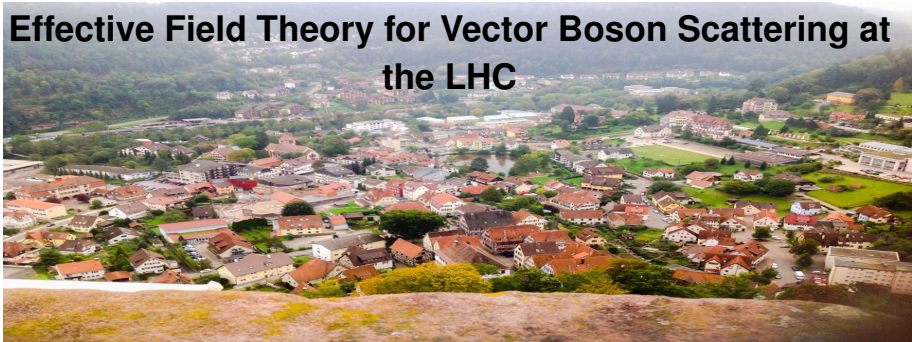




Effective Field Theory for Vector Boson Scattering at the LHC



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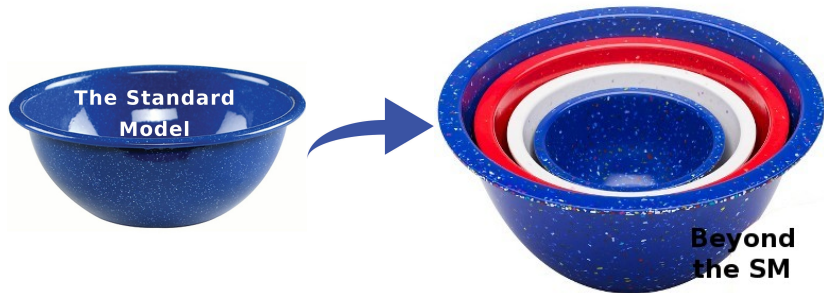
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Motivation



Motivation



Motivation: Effective Field Theory



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Parameterize new physics using the maximum information available from the SM:



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To extend the SM as an EFT:

- Fields.
- Interactions.
- How to expand the parameters:
Bottom-up description of the SM.



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Bottom-up description of the SM.



Bottom-up:

Dimensional expansion for physics above the scales of the SM, expanding in terms of an energy scale (cut-off energy where NP is expected).

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Bottom-up description of the SM.



$$\mathcal{L}_{\text{EFT}} = \mathcal{L}^{(0)} + \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \dots$$
$$\sim \Lambda^0 \quad \sim \Lambda^1 \quad \sim \Lambda^2$$

The Standard Model as an Effective Field Theory

To construct the Effective Lagrangian:

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- The EFT is constructed using the SM fields:

	Gauge Bosons $X_{\mu\nu}$	Fermions ψ	Scalars ϕ	Derivative D_μ
	$G_{\mu\nu}^A$ $W_{\mu\nu}^J$ $B_{\mu\nu}$	L_L^J e_R $Q_L^{\alpha j}$ u_R^α, d_R^α	H^j	$\partial_\mu - i\frac{g}{2}W_\mu^a\sigma^a - ig'B^\mu$
Dim	2	$\frac{3}{2}$	1	1

Building Blocks

- The SM includes all the possible dim-4 operators: $\mathcal{L}^{(0)} = \mathcal{L}_{SM}$.

The Standard Model as an Effective Field Theory

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- The SM includes all the possible dim-4 operators: $\mathcal{L}^{(0)} = \mathcal{L}_{SM}$.
- Symmetries of the SM imposed (Lorentz and gauge invariance are assumed).
- New particles are produced only at the new scale $m \sim \Lambda$: kinematically their production is not allowed before this value.
- Any \mathcal{L}^i term should have the same degrees of freedom as the SM.

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- When baryon and lepton numbers are conserved, only operators with even dim can be constructed:

$$\mathcal{L}_{\text{EFT}} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{dim-6}} + \mathcal{L}_{\text{dim-8}} + \dots$$

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$$\mathcal{L}_{\text{EFT}} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda^2} \sum_i c_i \mathcal{O}_i + \frac{1}{\Lambda^4} \left[\sum_{Si} c_{Si} \mathcal{O}_{Si} + \frac{1}{\Lambda^4} \sum_{Ti} c_{Ti} \mathcal{O}_{Ti} + \frac{1}{\Lambda^4} \sum_{Mi} c_{Mi} \mathcal{O}_{Mi} \right]$$

- Restrictions on the coefficients come from high precision measurements.

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- Restrictions on the coefficients come from high precision measurements.
- The most stronger constraints come from fermionic interactions, so we concentrate on bosonic searches : electroweak precision tests are sensitive observables to constraint NP.

New Physics from Effective Field Theory

$$\mathcal{L}_{\text{EFT}} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda^2} \sum_i c_i \mathcal{O}_i + \frac{1}{\Lambda^4} \left[\sum_{Si} c_{Si} \mathcal{O}_{Si} + \frac{1}{\Lambda^4} \sum_{Ti} c_{Ti} \mathcal{O}_{Ti} + \frac{1}{\Lambda^4} \sum_{Mi} c_{Mi} \mathcal{O}_{Mi} \right]$$

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Dim-6 Operators

They affect triple and quartic vector boson couplings (ATGCs & AQGCs, respectively); therefore, the deviations from the SM values cannot be treated separately.

$$\text{e.g. } \mathcal{O}_{WWW} = \text{Tr} [W_{\mu\nu} W^{\nu\rho} W_{\rho}^{\mu}]$$

$$\frac{\mathcal{O}_{WWW}}{\quad} \left\| \begin{array}{ccc} ZWW & AWW & WWWW \\ ZZWW & ZAWW & AAWW \end{array} \right.$$

TGCs are strongly constrained
(e.g. by the LEP experiments).

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Dim-8 Operators

- An independent parameterization of QGCs is possible.
- Some NP effects appear as dim-8 operators, e.g. heavy resonances

e.g.

$$\mathcal{O}_{S0} = [(D_\mu \Phi)^\dagger D_\nu \Phi] \times [(D^\mu \Phi)^\dagger D^\nu \Phi]$$

$$\frac{\mathcal{O}_{T0}}{\parallel} \begin{array}{|l} WWWW \quad WWZZ \\ ZZZZ \end{array}$$

Dim-8 Operators for Vector Boson Scattering

The dim-8 operators can be divided into:

- Operators built from Higgs doublet and covariant derivatives only, \mathcal{O}_{S_i} . It affects only massive vector bosons (longitudinally polarized particles).

$$\mathcal{O}_{S0} = \left[(D_\mu \Phi)^\dagger D_\nu \Phi \right] \times \left[(D^\mu \Phi)^\dagger D^\nu \Phi \right] \quad (1)$$

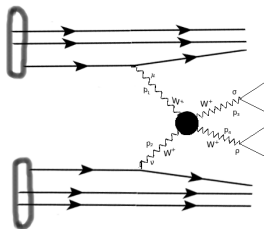
- Operators containing only field strength tensors, \mathcal{O}_{T_i} . Therefore, only transverse polarizations are of importance.

$$\mathcal{O}_{T0} = \text{Tr} [W_{\mu\nu} W^{\mu\nu}] \times \text{Tr} [W_{\alpha\beta} W^{\alpha\beta}] \quad (2)$$

- Operators built from field strength tensors, Higgs doublet and covariant derivatives, \mathcal{O}_{M_i} . A mixture between transverse and longitudinal polarization.

$$\mathcal{O}_{M0} = \text{Tr} [W_{\mu\nu} W^{\mu\nu}] \times \left[(D_\beta \Phi)^\dagger D_\beta \Phi \right] \quad (3)$$

Example: $WW \rightarrow WW$ Scattering



$$\begin{aligned} & \mathcal{O}_{S0}, \mathcal{O}_{S1} \\ & \mathcal{O}_{T0}, \mathcal{O}_{T1}, \mathcal{O}_{T2} \\ & \mathcal{O}_{M0}, \mathcal{O}_{M1}, \mathcal{O}_{M6}, \mathcal{O}_{M7} \end{aligned}$$

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \frac{c_{S0}}{\Lambda^4} \mathcal{O}_{S0}$$

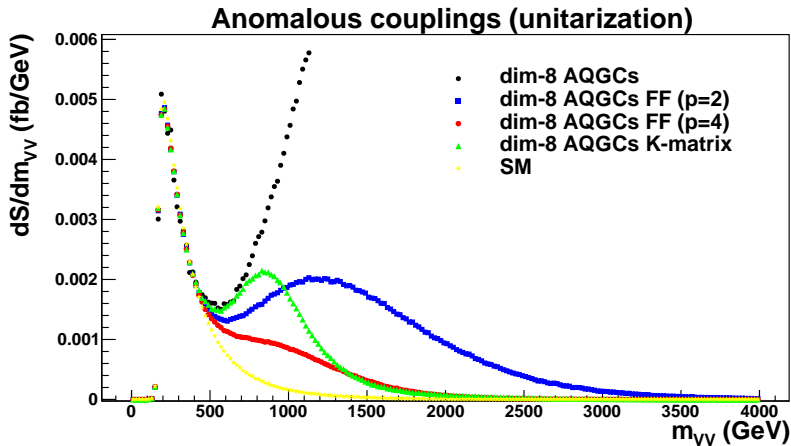
The matrix element can be written as,

$$\mathcal{M} = J_{13}^{\mu}(q_1) J_{24}^{\mu}(q_2) (m_{\mu\nu, \text{SM}} + m_{\mu\nu, \text{dim-8}}) \quad (4)$$

For this process, the amplitude given by equation (1) is

$$m_{S0} \propto \epsilon(q_1, \lambda_1) \cdot \epsilon(q_2, \lambda_2) \epsilon^*(q_3, \lambda_3) \cdot \epsilon^*(q_4, \lambda_4) \quad (5)$$

When these anomalous couplings are considered, the cross section falls off asymptotically with a slower rate than expected.



The S-Matrix for Vector Boson Scattering

Recall: $2 \rightarrow 2$ scattering.

The transition amplitude for an initial state $|n\rangle$ to a final state $|m\rangle$ is given by $\langle m|S|n\rangle$, where S is a linear operator, called the S-matrix, and it is unitary.

Properties of the S-matrix

- The S-matrix can be decomposed usefully into two parts, where T describes a non-trivial scattering:

$$S = 1 + i T \quad (6)$$

- As S have to be unitary:

$$SS^\dagger = 1 \quad (7)$$

therefore,

$$T^\dagger T = -i (T - T^\dagger) \quad (8)$$

For two particle scattering: $p_1 p_2 \rightarrow p_3 p_4$

The T-matrix is related to the scattering amplitude $\mathcal{M}(p_1 p_2 \rightarrow p_3 p_4)$

$$\langle p_3, p_4 | T | p_1, p_2 \rangle = (2\pi)^4 \delta^{(4)}(p_1 + p_2 - p_3 - p_4) \mathcal{M}. \quad (9)$$

The differential cross section can be written as,

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} |\mathcal{M}|^2. \quad (10)$$

The equation (7) sets bounds on T , and therefore on the cross sections.

The Unitary problem:

Partial Wave Decomposition

Using the partial wave decomposition (simplified version), it is possible to write the scattering amplitude $\mathcal{M}(s, \theta)$ as

$$\mathcal{M}(s, \theta) = 16\pi \sum_j (2j + 1) P_j(\cos \theta) \mathcal{A}_j \quad (11)$$

where, \mathcal{A}_j is the partial wave and $P_j(\cos \theta)$ Legendre polynomials.

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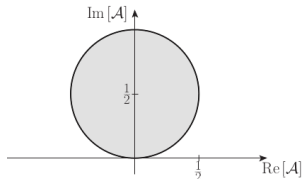
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$$| \mathcal{A}_j - \frac{i}{2} | \leq \frac{1}{2} \quad (12)$$

$$\Rightarrow \text{Re}(\mathcal{A}_j) < \frac{1}{2} \quad (13)$$



The Argand Circle, with radius $\frac{1}{2}$ and center $\frac{i}{2}$.

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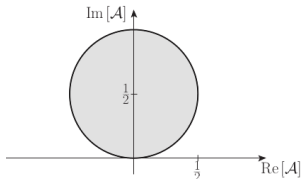
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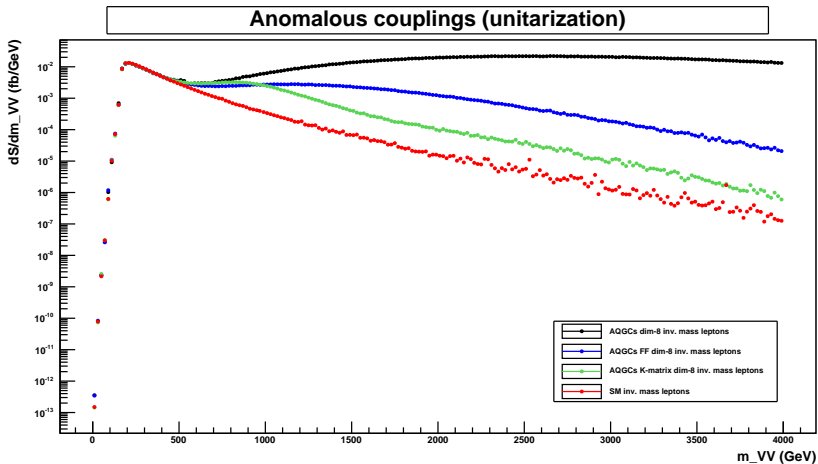
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Scattering amplitudes are not allowed to grow proportional with energy, because the σ becomes unphysical (it violates unitarity).



To obtain a valid prediction, which does not violate any fundamental law, a unitarization scheme is needed to reconstruct the amplitudes for the anomalous couplings.

Restoring Unitarity

Unitarity considerations have been used to spot the energy region where underlying new physics could arise: an *incomplete description of the physics* symptom.

Some unitarization schemes:

Form Factor unitarization

A function $\mathcal{F}(s)$ can be applied to ensure unitarity at high energies: it is multiplied to the couplings, to suppress the tail of the amplitudes arising from the operators.

K-matrix formalism

The real scattering amplitudes are projected onto the Argand circle to restore unitarity, using a Cayley transform of the S-matrix: the K-matrix.

$$S = \frac{1 + iK/2}{1 - iK/2} \quad (14)$$

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Unitarity bounds are derived from the partial wave analysis of the on-shell $2 \rightarrow 2$ scattering amplitudes, but the LHC processes are off-shell:

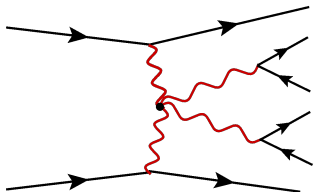
How do we restore unitarity for off-shell processes?

Implementing the Unitarization

VBFNLO - A parton level Monte Carlo for processes with electroweak bosons

J. Baglio et al. [arXiv:1107.4038]

«A fully flexible parton level Monte Carlo program for the simulation of vector boson fusion, double and triple vector boson production in hadronic collisions at next to leading order in the strong coupling constant. It includes Higgs and vector boson decays with full spin correlations and all off-shell effects.»



$$\mathcal{M} = J_{13}^{\mu}(q_1) J_{24}^{\mu}(q_2) (m_{\mu\nu, \text{SM}} + m_{\mu\nu, \text{dim-8}})$$

The on-shell/off-shell approach

$$m_{\mu\nu, \text{dim}-8} = D_{\mu\alpha}^{V_1}(q_1) D_{\nu\beta}^{V_2}(q_2) m_{AGCs}^{\alpha\beta\gamma\delta} (V_1 V_2 \rightarrow V_3 V_4) D_{\gamma\xi}^{V_3}(q_3) D_{\delta\rho}^{V_4}(q_4) j_{56}^{\xi} j_{78}^{\rho} \quad (15)$$

Using the properties of the polarization vectors,

$$\sum_{\text{polarizations}} \epsilon^{*\mu}(p_k) \epsilon^{\nu}(p_j) = -g^{\mu\nu} + N(p_k, p_j) p_k^{\mu} p_j^{\nu}$$

$$\text{So, } D_{\mu\alpha}^{V_k}(q_k) = d_k \sum_{\lambda} \epsilon_{\lambda, \mu}^*(q_k) \epsilon_{\lambda, \alpha}(q_k) \quad (\text{with } d_k = \frac{i}{q_k^2 - m_{V_k}^2}).$$

$$\therefore m_{\mu\nu} = d_1 d_2 \sum_{\lambda_1, \lambda_2} \epsilon_{\lambda_1, \mu}^*(q_1) \epsilon_{\lambda_2, \nu}^*(q_2) m_{\text{on/off}}(\lambda_1, \lambda_2) \quad (16)$$

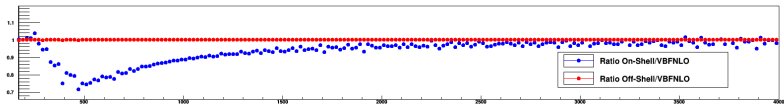
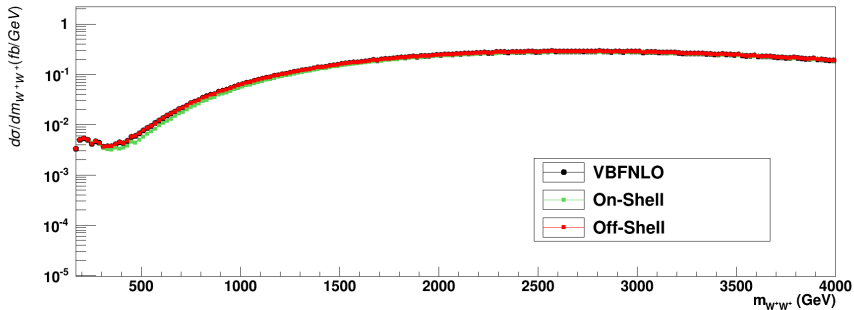
$$m_{\text{on/off}}(\lambda_1, \lambda_2) = \sum_{\lambda_3, \lambda_4} A_{QC}(\lambda_1, \lambda_2, \lambda_3, \lambda_4) Z_3(\lambda_3) Z_4(\lambda_4) \quad (17)$$

where,
and

$$Z_k(\lambda_k) = d_k \epsilon_{\lambda_k, i}(q_k) j^i$$

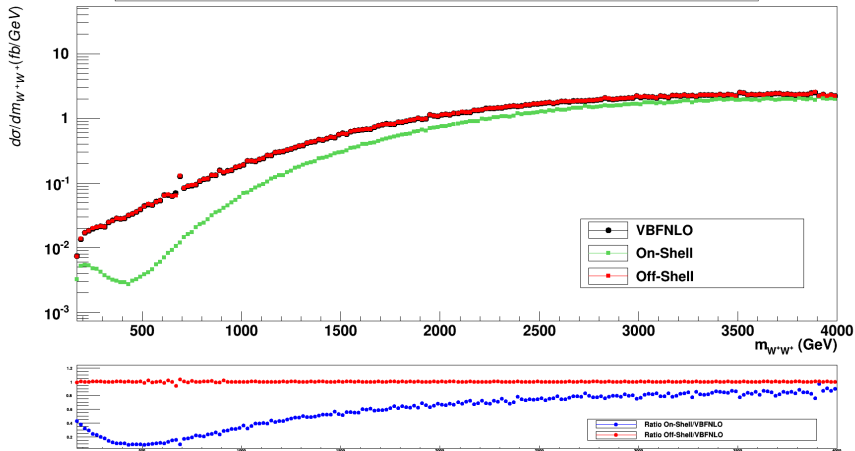
$$A_{QC}(\lambda_1, \lambda_2, \lambda_3, \lambda_4) \propto \mathbf{f}_{S_0} \cdot \epsilon(q_1, \lambda_1) \cdot \epsilon(q_2, \lambda_2) \epsilon^*(q_3, \lambda_3) \cdot \epsilon^*(q_4, \lambda_4)$$

AQCs W^+W^- scattering, FS0=480 TeV⁴



$$A(\lambda_1, \lambda_2, \lambda_3, \lambda_4) \propto f_{S0} [\varepsilon(q_1, \lambda_1) \cdot \varepsilon(q_2, \lambda_2) \varepsilon^*(q_3, \lambda_3) \cdot \varepsilon^*(q_4, \lambda_4)]$$

AQCs W^+W^+ scattering, FT0=480 TeV⁴



$$\begin{aligned}
 A(\lambda_1, \lambda_2, \lambda_3, \lambda_4) \propto f_{T0} \{ & [(q_3 \cdot \varepsilon(q_2, \lambda_2)) (q_2 \cdot \varepsilon^*(q_3, \lambda_3)) - (\varepsilon^*(q_3, \lambda_3) \cdot \varepsilon(q_2, \lambda_2)) (q_3 \cdot q_2)] \\
 & [(q_4 \cdot \varepsilon(q_1, \lambda_1)) (q_1 \cdot \varepsilon^*(q_4, \lambda_4)) - (\varepsilon^*(q_4, \lambda_4) \cdot \varepsilon(q_1, \lambda_1)) (q_4 \cdot q_1)] \\
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 \end{aligned}$$

The analytic K-Matrix

Using the partial wave decomposition, taking into account the helicities of the states

$$A_{QC}(\lambda_1, \lambda_2, \lambda_3, \lambda_4) = 16\pi \sum_{j=0}^2 (2j+1) a^j(\lambda_1, \lambda_2, \lambda_3, \lambda_4) d_{\alpha\beta}^j(\theta) \quad (18)$$

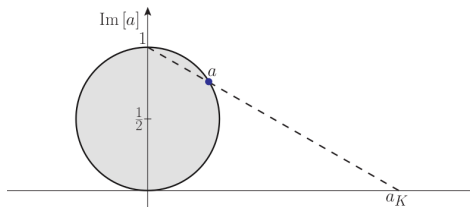
where, $d_{\alpha\beta}^j(\theta)$ are the Wigner d-matrix, and $a^j(\lambda_1, \lambda_2, \lambda_3, \lambda_4)$ the amplitudes to be unitarized.

So, using the bounds given by the Argand Circle, we look for the unitarize amplitudes, using the eigenvalue of the Cayley-transform K-matrix:

$$a_k = \frac{a}{1 + ia} \quad (19)$$

The unitarize amplitudes are then determined by,

$$a = \frac{a_k}{1 - ia_k} \quad (20)$$



Conclusions

- The effective Lagrangian formalism is a good theoretical way to describe NP: it allows to describe new physics at some scale m^2 , without knowing the detailed dynamics of the system. We can rewrite NP with a convenient parameterization in terms of the SM fields.

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- The dim-8 operators within the sensitivity level at the LHC violates unitarity of the S-matrix.
- Unitarity bounds can be calculated using the partial wave decomposition for the amplitudes of the vector boson scattering.
- We use the analytic K-matrix formalism, to project the amplitudes onto the unitarity circle (Argand circle) and restore unitarity. However, this introduce a model dependence.

Thank you

Backup Slides

