

The $\mathcal{O}(\alpha_t\alpha_s)$ -corrections to the trilinear Higgs-selfcouplings in the complex NMSSM

Hanna Ziesche in collaboration with Dao Thi Nhung and Margarete Mühlleitner

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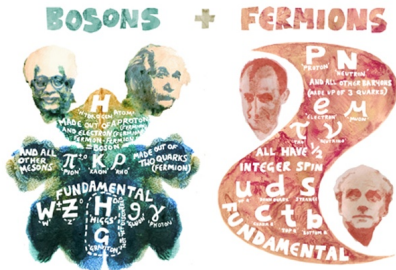
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- Introduction into the **complex NMSSM**
- Trilinear Higgs Self-Couplings
- The $\mathcal{O}(\alpha_t\alpha_s)$ -corrections
- Results

Motivations for Supersymmetry:

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- Theoretical elegance: Symmetry between **Fermions** and **Bosons**



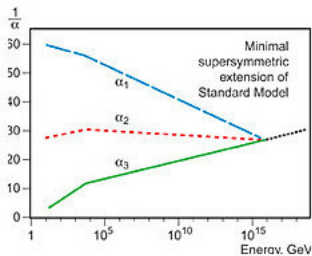
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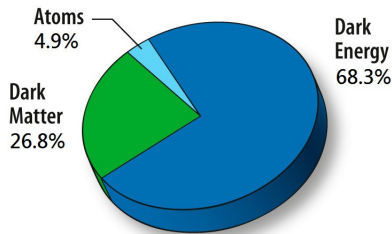
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Introduction into the **complex NMSSM**

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$$V_H = \mu^2 (H_u^\dagger H_u + H_d^\dagger H_d) + \frac{1}{2} g_2^2 (H_u^\dagger H_d)^2 + \frac{1}{8} (g_1^2 + g_2^2) (H_u^\dagger H_u - H_d^\dagger H_d)^2 + V_{soft}$$

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⇒ Solution: **NMSSM**

NMSSM: Introduction of an additional scalar singlet and its superpartner

- Higgs sector of the NMSSM:

$$H_d = \begin{pmatrix} \frac{1}{\sqrt{2}} (v_d + h_d + ia_d) \\ H_d^- \end{pmatrix} \quad H_u = e^{i\varphi_u} \begin{pmatrix} H_u^+ \\ \frac{1}{\sqrt{2}} (v_u + h_u + ia_u) \end{pmatrix}$$
$$S = e^{i\varphi_s} \frac{1}{\sqrt{2}} (v_s + h_s + ia_s)$$

⇒ Higgs potential of the NMSSM:

$$V_H = |\lambda|^2 |S|^2 \left(H_u^\dagger H_u + H_d^\dagger H_d \right) + \left| \lambda (H_u^T H_u - H_d^T H_d)^2 + \kappa S^2 \right|^2 +$$
$$V_{D,\text{MSSM}} + V_{\text{soft},\text{MSSM}} + m_S^2 |S|^2 + \left(\lambda A_\lambda (H_u^T \epsilon H_d) S + \frac{1}{3} \kappa A_\kappa (S^3 + c.c.) \right)$$

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Introduction into the **complex NMSSM**

NMSSM

- μ problem:

- Upper limit on the tree-level Higgs mass:

- CP-violation in the Higgs sector:

NMSSM

- μ problem: μ parameter is generated **dynamically**

$$V_H = (\lambda v_s)^2 (H_u^\dagger H_u + H_d^\dagger H_d) + \frac{1}{2} g_2^2 (H_u^\dagger H_d)^2 + \frac{1}{8} (g_1^2 + g_2^2) (H_u^\dagger H_u - H_d^\dagger H_d)^2 \\ + V_{soft} + \dots$$

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- Upper limit on the tree-level Higgs mass: **is relaxed**

$$m_{h_1}^2 \leq M_Z^2 \cos(2\beta)^2 + \lambda^2 v^2 \sin(2\beta)$$

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- CP-violation in the Higgs sector:
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For the NMSSM Higgs sector available:

- Corrections to the masses:
 - full 1-loop corrections in the real and complex NMSSM
[Ellwanger et al., Degrandi et al., Graf et al., Cheung et al.]
 - Corrections up to $\mathcal{O}(\alpha_s(\alpha_t + \alpha_b))$ and beyond in the real NMSSM
[Degrandi et al., Staub et al.] [Goodsell et al.]
 - $\mathcal{O}(\alpha_s\alpha_t)$ corrections in the complex NMSSM
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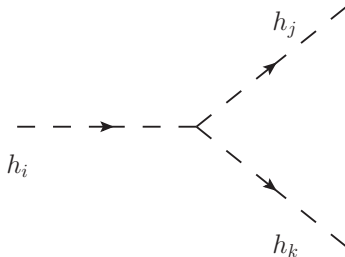
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[\[Mühlleitner, Dao, Rzehak, Walz\]](#)
- Corrections to the trilinear Higgs self-couplings
 - full 1-loop corrections to the trilinear Higgs self-couplings in the real NMSSM [\[Mühlleitner, Dao, Streicher, Walz\]](#)
 - $\mathcal{O}(\alpha_s\alpha_t)$ corrections in the complex NMSSM (this talk)
[\[Mühlleitner, Dao, HZ arXiv:1506.0332\]](#)

The $\mathcal{O}(\alpha_t\alpha_s)$ Corrections

⇒ **Consistent description of the NMSSM Higgs sector demands:**

Calculation of $\mathcal{O}(\alpha_t\alpha_s)$ corrections to the **trilinear Higgs self-couplings**

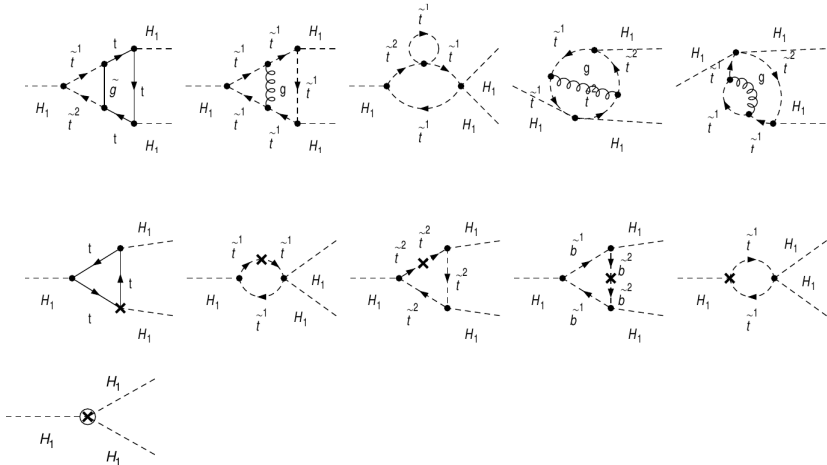


Approximations and Simplifications:

- “gaugeless limit”: $e, g_1, g_2 = 0$
- vanishing external momenta $p_{\text{ext}} = 0$

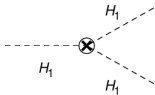
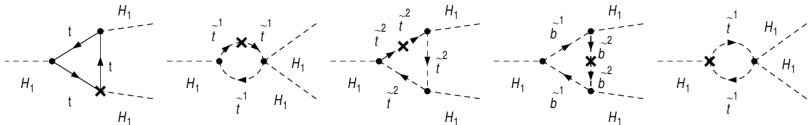
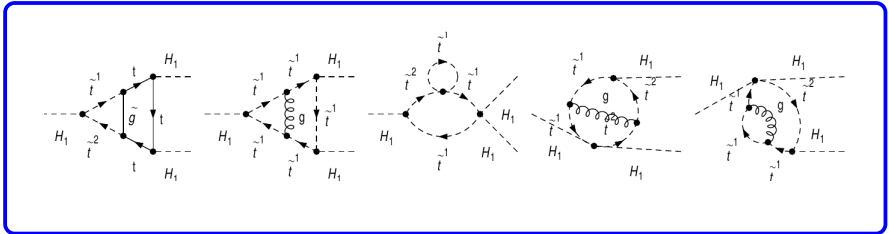
The $\mathcal{O}(\alpha_t \alpha_s)$ Corrections

Typical diagrams:



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Typical diagrams:



The $\mathcal{O}(\alpha_t \alpha_s)$ Corrections: genuine 2-loop diagrams

After dimensional reduction, we encounter integrals like:

$$C^2 \int d^d l_1 d^d l_2 \frac{(l_1^2)^{\alpha_1} (l_2^2)^{\alpha_2} (l_1 \cdot l_2)^{\alpha_3}}{(l_1^2 - m_1^2)^{\nu_1} (l_2^2 - m_2^2)^{\nu_2} ((l_1 - l_2)^2 - m_3^2)^{\nu_3} (l_1^2 - m_4^2)^{\nu_4} (l_2^2 - m_5^2)^{\nu_5}}$$
$$\alpha_j, \nu_j = 0, 1, 2 \quad d = 4 - 2\epsilon \quad C = \frac{\mu^{2\epsilon}}{(2\pi)^d}$$

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Simplification with the Mathematica package TARCER [Mertig,Scharf arxiv:9801383] implemented in FeynCalc [Bohm,Denner,Mertig Comput.Phys.Commun 64,345 (1991)]

- TARCER reduces integrals like

$$C^2 \int d^d l_1 d^d l_2 \frac{(l_1)^{\alpha_1} (l_2)^{\alpha_2} (l_1 \cdot l_2)^{\alpha_3} (l_1 \cdot p)^{\alpha_4} (l_2 \cdot p)^{\alpha_5}}{D_1^{\nu_1} D_2^{\nu_2} D_3^{\nu_3} D_4^{\nu_4} D_5^{\nu_5}}$$

$$D_j = ((l_j - p_j)^2 - m_j^2)$$

to a set of “master integrals” (via Tarasov Algorithmus [Tarasov arxiv:9606018,arxiv:9703319])

The $\mathcal{O}(\alpha_t \alpha_s)$ Corrections: genuine 2-loop diagrams

Here: all integrals reducible to **1-loop vacuum function**

$$A_0(m_1^2) = C \int d^d l_1 \frac{1}{l_1^2 - m_1^2}$$

und **2-loop vacuum function** [Davydchev, Tausk (1993)]

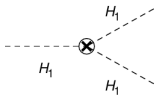
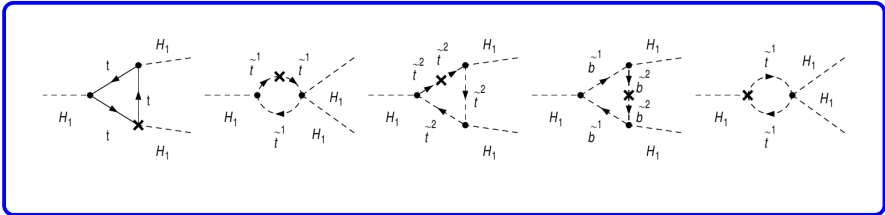
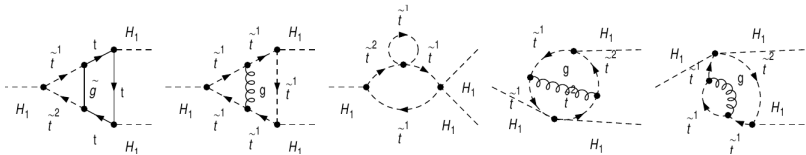
$$K_0(m_1^2, m_2^2, m_3^2) = C^2 \int d^d l_1 d^d l_2 \frac{1}{(l_1^2 - m_1^2)(l_2^2 - m_2^2)((l_1 - l_2)^2 - m_3^2)}$$

A_0 and K_0 analytically known and expandable in a series in ϵ

⇒ UV divergencies of the 2-loop integrals appear as:

- simple poles $\propto \frac{1}{\epsilon}$
- double poles $\propto \frac{1}{\epsilon^2}$

Die $\mathcal{O}(\alpha_t \alpha_s)$ -Korrekturen: Renormierung des Stop-Top-Sektors



The $\mathcal{O}(\alpha_t\alpha_s)$ Corrections: Renormalization of the stop-top sector

- **Top sector:**

1 parameter: $m_t \rightarrow m_t + \delta m_t$

- **Stop sector:**

3 parameters: $m_{\tilde{t}_1} \rightarrow m_{\tilde{t}_1} + \delta m_{\tilde{t}_1}$, $m_{\tilde{t}_2} \rightarrow m_{\tilde{t}_2} + \delta m_{\tilde{t}_2}$, $A_t \rightarrow A_t + \delta A_t$

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Different renormalization schemes possible:

- On-Shell scheme (OS):

- $\overline{\text{DR}}$ -Schema:

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$\delta m_t, \delta m_{\tilde{t}_1}, \delta m_{\tilde{t}_2}$: fixed via propagator pole condition
 δA_t : fixed via unmixing condition for on-shell stops

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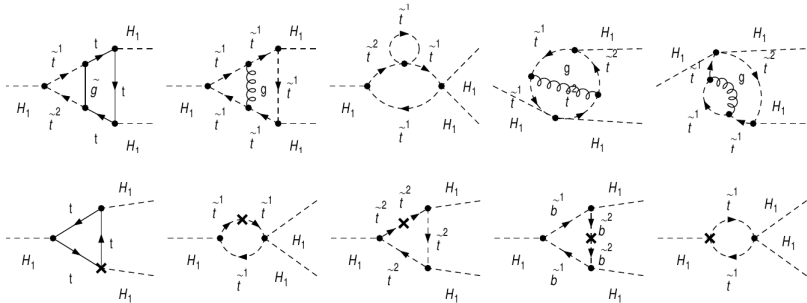
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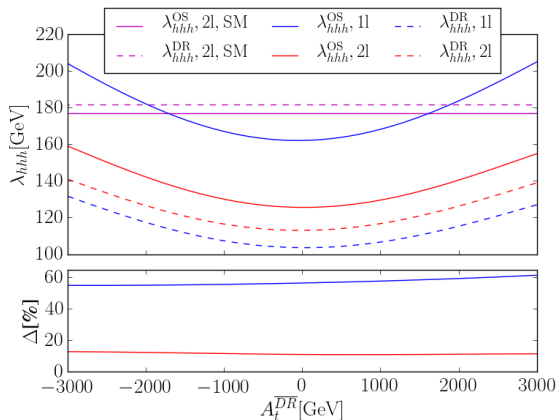
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- $\overline{\text{DR}}$ -Schema:

$\delta m_t, \delta m_{\tilde{t}_1}, \delta m_{\tilde{t}_2}, \delta A_t$: contain only divergent terms

The $\mathcal{O}(\alpha_t \alpha_s)$ Corrections: Renormierung des Higgs sectors





Size of corrections:

$$\frac{|\lambda_{hhh}^{(n)} - \lambda_{hhh}^{(n-1)}|}{\lambda_{hhh}^{(n-1)}}$$

1-loop: 140% (OS), 24% (\overline{DR})

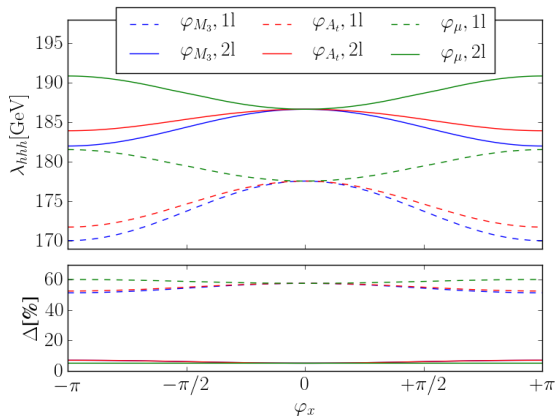
2-loop: 74% (OS), 9% (\overline{DR})

Estimation of theoretical uncertainty:

$$\Delta^{(n)} = \frac{|\lambda_{hhh}^{\overline{DR},(n)} - \lambda_{hhh}^{\text{OS},(n)}|}{\lambda_{hhh}^{\overline{DR},(n)}}$$

1-loop: $\Delta^{(1)} \approx 60\%$

2-loop: $\Delta^{(2)} \approx 10\%$



$$\mu = \mu_{\text{eff}} = v_s \lambda$$

Influence of the phases:

$$\delta^\varphi = \frac{|\lambda_{hhh}^{(2)}(\pi) - \lambda_{hhh}^{(2)}(0)|}{\lambda_{hhh}^{(2)}(0)}$$

$$\delta^{\varphi_{A_t}} = 1.6\%$$

$$\delta^{\varphi_\mu} = 2.2\%$$

$$\delta^{\varphi_{M_3}} = 2.7\%$$

Size of corrections:

$$\Delta^{(n)} = \frac{|\lambda_{hhh}^{(n)} - \lambda_{hhh}^{(n-1)}|}{\lambda_{hhh}^{(n-1)}}$$

1-loop: $\Delta^{(1)} \approx 50 - 60\%$

2-loop: $\Delta^{(2)} \approx 5 - 8\%$

- The $\mathcal{O}(\alpha_t, \alpha_s)$ corrections to the trilinear Higgs self-couplings in the complex NMSSM have been calculated.
- The renormalization of the stop-top sector in the OS- and the \overline{DR} scheme allows an estimation of the theoretical uncertainty.
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Thank you for your interest!

Scenario 1:

$$\begin{aligned}
 m_{\tilde{u}_R, \tilde{c}_R} &= m_{\tilde{d}_R, \tilde{s}_R} = m_{\tilde{Q}_{1,2}} = m_{\tilde{L}_{1,2}} = m_{\tilde{e}_R, \tilde{\mu}_R} = 3 \text{ TeV}, \quad m_{\tilde{t}_R} = 1909 \text{ GeV}, \\
 m_{\tilde{Q}_3} &= 2764 \text{ GeV}, \quad m_{\tilde{b}_R} = 1108 \text{ GeV}, \quad m_{\tilde{L}_3} = 472 \text{ GeV}, \quad m_{\tilde{\tau}_R} = 1855 \text{ GeV}, \\
 |A_{u,c,t}| &= 1283 \text{ GeV}, \quad |A_{d,s,b}| = 1020 \text{ GeV}, \quad |A_{e,\mu,\tau}| = 751 \text{ GeV}, \\
 |M_1| &= 908 \text{ GeV}, \quad |M_2| = 237 \text{ GeV}, \quad |M_3| = 1966 \text{ GeV}, \\
 \varphi_{A_{d,s,b}} &= \varphi_{A_{e,\mu,\tau}} = \varphi_{A_{u,c,t}} = \pi, \quad \varphi_{M_1} = \varphi_{M_2} = \varphi_{M_3} = 0
 \end{aligned}$$

$$\begin{aligned}
 |\lambda| &= 0.374, \quad |\kappa| = 0.162, \quad |A_\kappa| = 178 \text{ GeV}, \quad |\mu_{\text{eff}}| = 184 \text{ GeV}, \\
 \varphi_\lambda &= \varphi_\kappa = \varphi_{\mu_{\text{eff}}} = \varphi_u = 0, \quad \varphi_{A_\kappa} = \pi, \quad \tan \beta = 7.52, \quad M_{H^\pm} = 1491 \text{ GeV}.
 \end{aligned}$$

Scenario 2:

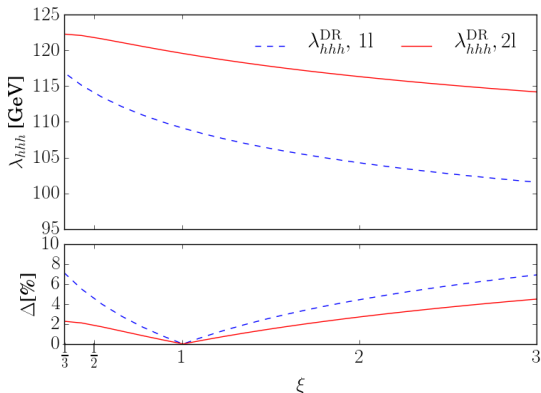
$$\begin{aligned}
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 m_{\tilde{Q}_3} &= 1336 \text{ GeV}, \quad m_{\tilde{b}_R} = 1029 \text{ GeV}, \quad m_{\tilde{L}_3} = 2465 \text{ GeV}, \quad m_{\tilde{\tau}_R} = 301 \text{ GeV} \\
 |A_{u,c,t}| &= 1824 \text{ GeV}, \quad |A_{d,s,b}| = 1539 \text{ GeV}, \quad |A_{e,\mu,\tau}| = 1503 \text{ GeV}, \\
 |M_1| &= 862.4 \text{ GeV}, \quad |M_2| = 201.5 \text{ GeV}, \quad |M_3| = 2285 \text{ GeV} \\
 \varphi_{A_{d,s,b}} &= \varphi_{A_{e,\mu,\tau}} = \pi, \quad \varphi_{A_{u,c,t}} = \varphi_{M_1} = \varphi_{M_2} = \varphi_{M_3} = 0
 \end{aligned}$$

$$\begin{aligned}
 |\lambda| &= 0.629, \quad |\kappa| = 0.208, \quad |A_\kappa| = 179.7 \text{ GeV}, \quad |\mu_{\text{eff}}| = 173.7 \text{ GeV}, \\
 \varphi_\lambda &= \varphi_{\mu_{\text{eff}}} = \varphi_U = 0, \quad \varphi_\kappa = \pi, \quad \tan \beta = 4.02, \quad M_{H^\pm} = 788 \text{ GeV}.
 \end{aligned}$$

Scenario 3:

$$\begin{aligned}
 m_{\tilde{u}_R, \tilde{c}_R} &= m_{\tilde{d}_R, \tilde{s}_R} = m_{\tilde{Q}_{1,2}} = m_{\tilde{L}_{1,2}} = m_{\tilde{e}_R, \tilde{\mu}_R} = 3 \text{ TeV}, \quad m_{\tilde{t}_R} = 1940 \text{ GeV}, \\
 m_{\tilde{Q}_3} &= 2480 \text{ GeV}, \quad m_{\tilde{b}_R} = 1979 \text{ GeV}, \quad m_{\tilde{L}_3} = 2667 \text{ GeV}, \quad m_{\tilde{\tau}_R} = 1689 \text{ GeV}, \\
 |A_{u,c,t}| &= 1192 \text{ GeV}, \quad |A_{d,s,b}| = 685 \text{ GeV}, \quad |A_{e,\mu,\tau}| = 778 \text{ GeV}, \\
 |M_1| &= 517 \text{ GeV}, \quad |M_2| = 239 \text{ GeV}, \quad |M_3| = 1544 \text{ GeV}, \\
 \varphi_{A_{d,s,b}} &= \varphi_{A_{e,\mu,\tau}} = 0, \quad \varphi_{A_{u,c,t}} = \pi, \quad \varphi_{M_1} = \varphi_{M_2} = \varphi_{M_3} = 0
 \end{aligned}$$

$$\begin{aligned}
 |\lambda| &= 0.267, \quad |\kappa| = 0.539, \quad |A_\kappa| = 810 \text{ GeV}, \quad |\mu_{\text{eff}}| = 104 \text{ GeV}, \\
 \varphi_\lambda &= \varphi_\kappa = \varphi_{\mu_{\text{eff}}} = \varphi_u = 0, \quad \varphi_{A_\kappa} = \pi, \quad \tan \beta = 8.97, \quad M_{H^\pm} = 613 \text{ GeV}
 \end{aligned}$$



theoretische Unsicherheit:
 Variation der Renormierungs-
 -Skala μ_r um zentrale Skala
 $\mu_0 = M_{\text{SUSY}} = \sqrt{m_{\tilde{Q}_3} m_{\tilde{t}_R}}$

$$\xi = \frac{\mu_r}{\mu_0}$$

$$\Delta^{(n)} = \frac{\lambda_{hhh}^{(n)}(\mu_r) - \lambda_{hhh}^{(n)}(\mu_0)}{\lambda_{hhh}^{(n)}(\mu_0)}$$

1-loop: $\Delta^{(1)} = 7\%$

2-loop: $\Delta^{(2)} = 2 - 5\%$

Näherungsformel für Skalenvariation über Relation:

$$p^{\text{OS}} = p^{\overline{\text{DR}}}(\mu_1) - \delta p^{\text{fin}}(\mu_1) = p^{\overline{\text{DR}}}(\mu_2) - \delta p^{\text{fin}}(\mu_2)$$

$$\Rightarrow p^{\overline{\text{DR}}}(\mu_2) = p^{\overline{\text{DR}}}(\mu_1) - \delta p^{\text{fin}}(\mu_1) + \delta p^{\text{fin}}(\mu_2)$$

Zusammenhang zwischen Parametern in unterschiedlichen Schemata:

$$p^0(Q_1) = p^{\text{OS}} + \delta p^{\text{OS}}(Q_1) = p^{\overline{\text{DR}}}(Q_1) + \delta p^{\overline{\text{DR}}}$$

$$p^{\text{OS}} = p^{\overline{\text{DR}}}(Q_1) + \underbrace{\delta p^{\overline{\text{DR}}} - \delta p^{\text{OS}}(Q_1)}_{-\delta p^{\text{fin}}(Q_1)} = p^{\overline{\text{DR}}}(Q_2) + \underbrace{\delta p^{\overline{\text{DR}}} - \delta p^{\text{OS}}(Q_2)}_{-\delta p^{\text{fin}}(Q_2)}$$

$$\Rightarrow p^{\overline{\text{DR}}}(Q_2) = p^{\overline{\text{DR}}}(Q_1) - \delta p^{\text{fin}}(Q_1) + \delta p^{\text{fin}}(Q_2)$$

Skalenvariation auf 1-Schleifen Niveau

Auf 1-Schleifen-Niveau besteht $\delta p^{\text{fin}}(Q)$ aus A_0 - und B_0 -Funktionen:

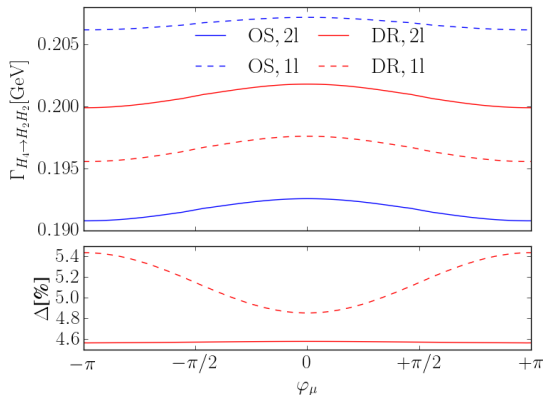
$$A_0(m_0^2) = m_0^2 \left(\Delta - \ln \left(\frac{m_0^2}{Q^2} \right) + \underbrace{1}_{\text{konstanter Beitrag}} \right) + \mathcal{O}(\epsilon)$$

$$B_0(p_1^2, m_0^2, m_1^2) = \Delta + \ln \left(\frac{Q^2}{m_0^2} \right) + \underbrace{2 + f_B(x_+) + f_B(x_-)}_{\text{konstanter Beitrag}} + \mathcal{O}(\epsilon)$$

konstanter Term fällt bei Differenzbildung heraus

Koeffizient c vor Skalenabhängigem Anteil gleich dem Koeffizient vor der Divergenz Δ :

$$\Rightarrow p^{\overline{\text{DR}}}(Q_2) = p^{\overline{\text{DR}}}(Q_1) + c \ln \left(\frac{Q_2^2}{Q_1^2} \right)$$



trilinear Higgs couplings relevant in Higgs-to-Higgs-decays

Size of corrections:

1-loop: 21% (OS), 6,5% (\overline{DR})
2-loop: 7% (OS), 2% (\overline{DR})

Estimation of theoretical uncertainty:

$$\Delta^{(n)} = \frac{|\lambda_{hhh}^{\overline{DR},(n)} - \lambda_{hhh}^{\text{OS},(n)}|}{\lambda_{hhh}^{\overline{DR},(n)}}$$

Mühlleitner, Dao, HZ arXiv:1506.0332

$$\Gamma(H_i \rightarrow H_j H_k) = \frac{\lambda^{1/2}(M_{H_i}^2, M_{H_j}^2, M_{H_k}^2)}{16\pi f M_{H_i}^3} |\mathcal{M}_{H_i \rightarrow H_j H_k}|^2$$