Phase Space Master Integrals for Higgs Production in Gluon Fusion

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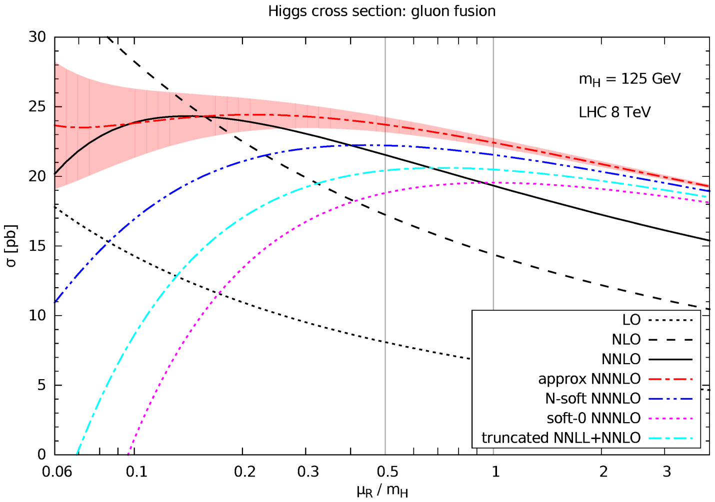
Higgs Particle and Henn's Method

- Experiment: Higgs Particle discovery [ATLAS, CMS, '12]
- Phenomenology: Total inclusive cross sections at NNNLO for $gg \to h$
  - threshold limit $s \to m_h^2$ [Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Mistlberger, '14]
  - approximations [Moch, Vogt, '05; Ball, Bonvini, Forte, Marzani, Ridolfi, '14]



$\mu_R = m_h : \frac{\text{NNNLO}}{\text{NNLO}} \sim 16\%$ [Ball, Bonvini, Forte, Marzani, Ridolfi, '14]

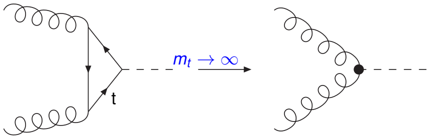
Full s-dependence of the NNNLO $gg \to h$ total inclusive cross section

Pedagogical (NLO only!) journey from real corrections to canonical Master Integrals (MIs)

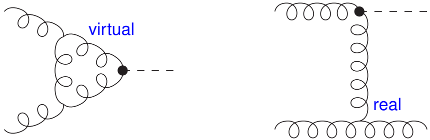
- Reversed unitarity
- Topology
- Reduction to MIs
- Differential equation method
- Canonical form

Effective Theory and Quantum Corrections

- LO



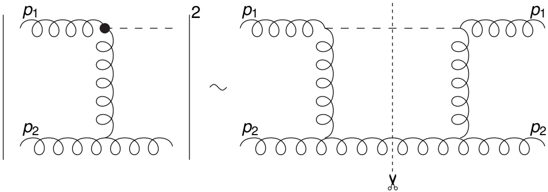
- NLO



- Calculation of total inclusive cross section demands integration over whole phase space of outgoing particles

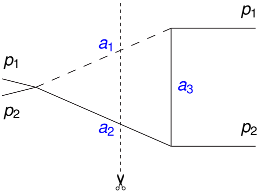
Reversed Unitarity [Anastasiou, Melnikov, ’02]

- $\int dPS$



- On shell delta functions $\to$ cut propagators
- $\delta(p^2 - m^2) \to \left(\frac{1}{p^2 - m^2}\right)_{✂} = \frac{1}{2\pi i}\left(\frac{1}{p^2 - m^2 + i0} - \frac{1}{p^2 - m^2 - i0}\right)$
- Phase space integral $\to$ loop integral
- Reduction to MIs

Topology

- $T[a_1, a_2, a_3] :=$ 

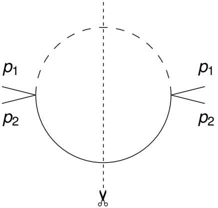
- $s = (p_1 + p_2)^2$ and $x = \frac{m_h^2}{s}$
- $T[a_1, a_2, a_3] := \int d^D k \left(\frac{1}{(p_1+p_2-k)^2+x}\right)^{a_1} \left(\frac{1}{(k)^2}\right)^{a_2} \left(\frac{1}{(p_2-k)^2}\right)^{a_3}$
- $D = 4 - 2\epsilon$

Reduction

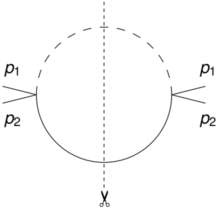
- Integration by Parts Identities [Chetyrkin, Tkachov, ’81]
- Relate all integrals to a small set of MIs with low indices [Laporta, ’01]
- $I = \sum_i r_i(x,\epsilon)\mathrm{MI}_i$
- e.g. $\mathrm{T}[2,2,2] = r(x,\epsilon)\ \mathrm{T}[1,1,0]$

Basis (Master) Integral

Rational function of x and ϵ

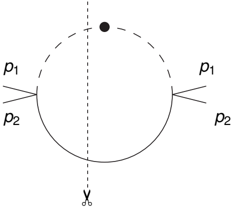


Differential Equation [Kotikov, '91; Bern, Dixon, Kosower, '94; Remiddi, '97; Gehrmann, Remiddi, '00]

- $\frac{\partial}{\partial x}$ 

$$= \frac{\partial}{\partial x} \int d^D k \, \frac{1}{(p_1+p_2-k)^2+x} \frac{1}{(k)^2}$$

$$= - \int d^D k \left(\frac{1}{(p_1+p_2-k)^2+x} \right)^2 \frac{1}{(k)^2}$$

$= -$ 

- $\frac{\partial}{\partial x} \mathrm{T}[1,1,0] = -\mathrm{T}[2,1,0] = \frac{-1+2\epsilon}{1-x} \mathrm{T}[1,1,0]$

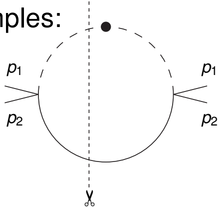
Reduction

Canonical Basis [Henn,'13]

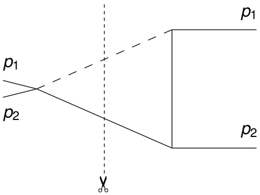
- A basis exists, with $\frac{\partial}{\partial x}\mathrm{T}[b_1, b_2, b_3] = \epsilon A(x)\mathrm{T}[b_1, b_2, b_3]$

  independent of ϵ

- Examples:



$$\frac{\partial}{\partial x}\mathrm{T}[2, 1, 0] = \frac{2\epsilon}{1-x}\mathrm{T}[2, 1, 0]$$



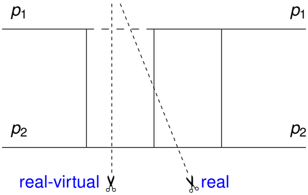
$$\frac{\partial}{\partial x}\mathrm{T}[1, 1, 1] = \frac{2\epsilon}{1-x}\mathrm{T}[1, 1, 1]$$

Algebraic Solution

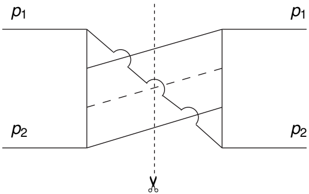
- Ansatz: $\mathrm{T}[2,1,0] = f(x,\epsilon) = f_0(x) + \epsilon f_1(x) + \epsilon^2 f_2(x) + \ldots$
- $\frac{\partial}{\partial x}\left[f_0(x) + \epsilon f_1(x) + \epsilon^2 f_2(x)\right] = \frac{2\epsilon}{1-x}\left[f_0(x) + \epsilon f_1(x) + \epsilon^2 f_2(x)\right]$
  - $f_0'(x) = 0 \;\Rightarrow\; f_0(x) = c_0$
  - $f_1'(x) = \frac{2c_0}{1-x} \;\Rightarrow\; f_1(x) = c_1 - 2c_0 \ln(1-x)$
  - $f_2'(x) = \frac{2[c_1 - 2c_0 \ln(1-x)]}{1-x} \;\Rightarrow\; f_2(x) = c_2 - 2c_1 \ln(1-x) + 2c_0 \ln^2(1-x)$
- Solution given by iteratively integrated logarithms
- Integration constants c_i fixed by limit $x \to 1$

Beyond NLO [arXiv:1407.4049]

- Finding a Canonical Basis
  - Known guiding principles
  - New algorithm for coupled MIs



- Complete NNLO
  - 17 real and 6 real-virtual MIs
  - $\frac{\partial}{\partial x}\vec{f} = \epsilon A(x)\vec{f}$
  - $A(x) = \frac{a}{x} + \frac{b}{1-x} + \frac{c}{1+x}$
- Exemplary NNNLO sea snake topology
  - 11 real MIs



Thank you for your attention

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