A visualization of a particle collision event, likely from the Large Hadron Collider. It shows a central point from which numerous lines radiate outwards, representing particle tracks. The lines are colored in shades of red, orange, yellow, and green, suggesting different particle types or energies. The background is white with a thin horizontal line passing through the center of the event.

QCD and Event Generation for the Large Hadron Collider

Bryan Webber
Cavendish Laboratory
University of Cambridge

Outline

- QCD Basics
 - ★ Factorization, PDFs, running coupling
- QCD and Higgs boson production
 - ★ Inclusive production cross sections
 - ★ Differential cross sections, resummation
- QCD and Higgs boson decays
 - ★ Quark masses
 - ★ Uncertainties and prospects
- Monte Carlo event generation
 - ★ Perturbative and non-perturbative components
 - ★ Improvements: matching and merging
 - ★ Survey of results

References

- R.K. Ellis, W.J. Stirling & B.R. Webber, “QCD and Collider Physics” (C.U.P. 1996)
- A. Buckley et al., “General-purpose event generators for LHC physics”, Phys.Rept. 504 (2011) 145 (MCNET-11-01, arXiv:1101.2599)
- M.H. Seymour & M. Marx, “Monte Carlo Event Generators”, MCNET-13-05, arXiv:1304.6677
- A. Siódmok, “LHC event generation with general-purpose Monte Carlo tools”, Acta Phys. Polon. B44 (2013) 1587

QCD

Basics

QCD Factorization

$$\sigma_{pp \rightarrow X}(s) = \sum_{i,j} \int_0^1 dx_1 dx_2 f_i(x_1, \mu_F^2) f_j(x_2, \mu_F^2) \hat{\sigma}_{ij \rightarrow X}(x_1 x_2 s, \alpha_S(\mu_R^2), \mu_F^2, \mu_R^2)$$

momentum fractions parton distribution functions at scale μ_F hard process cross section, renormalised at scale μ_R

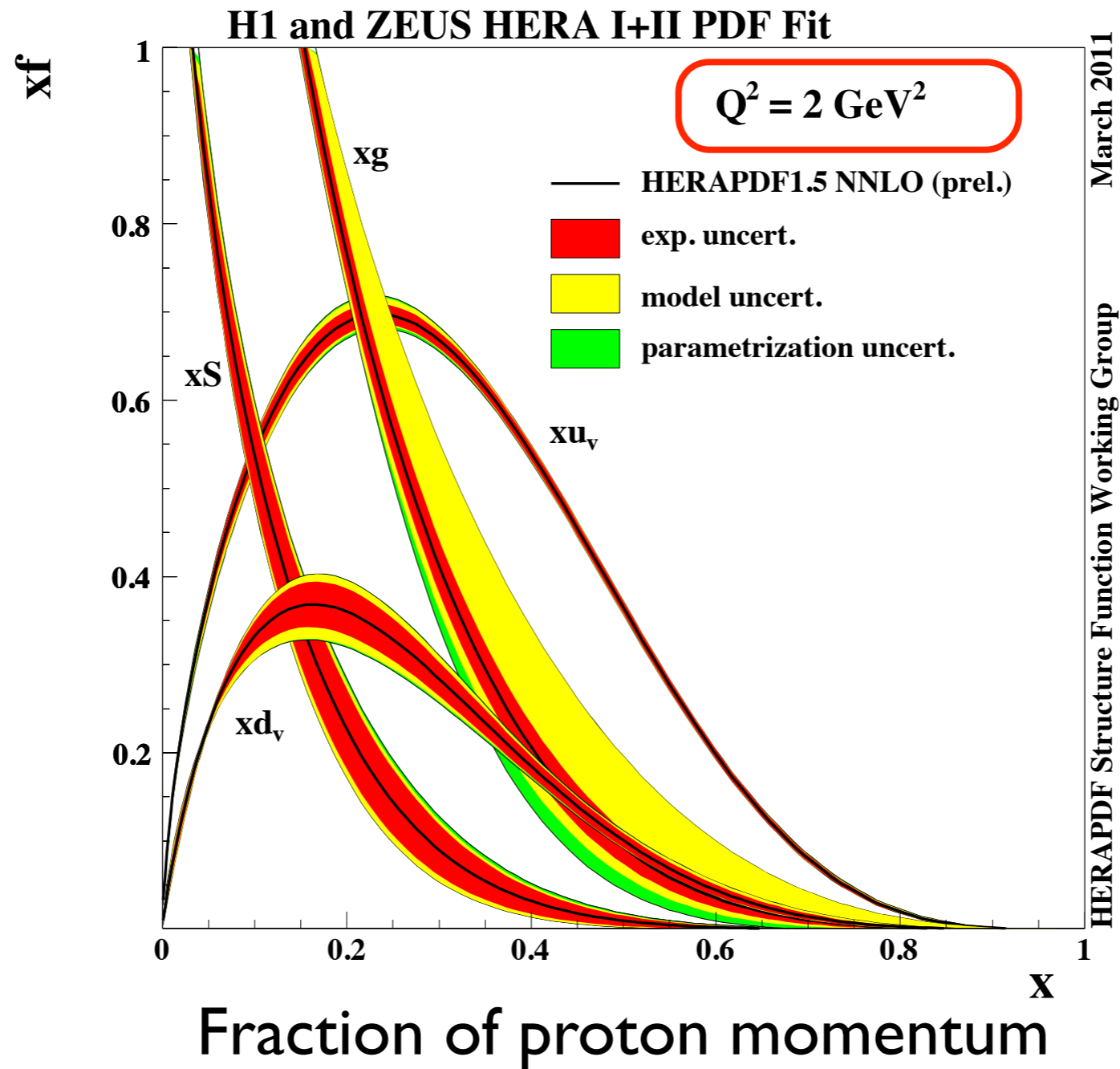
- Non-perturbative physics takes place over a much longer time scale, with unit probability
- Hence it cannot change the cross section
- Scale dependences of parton distribution functions and hard process cross section are perturbatively calculable, and cancel order by order

PDF Evolution

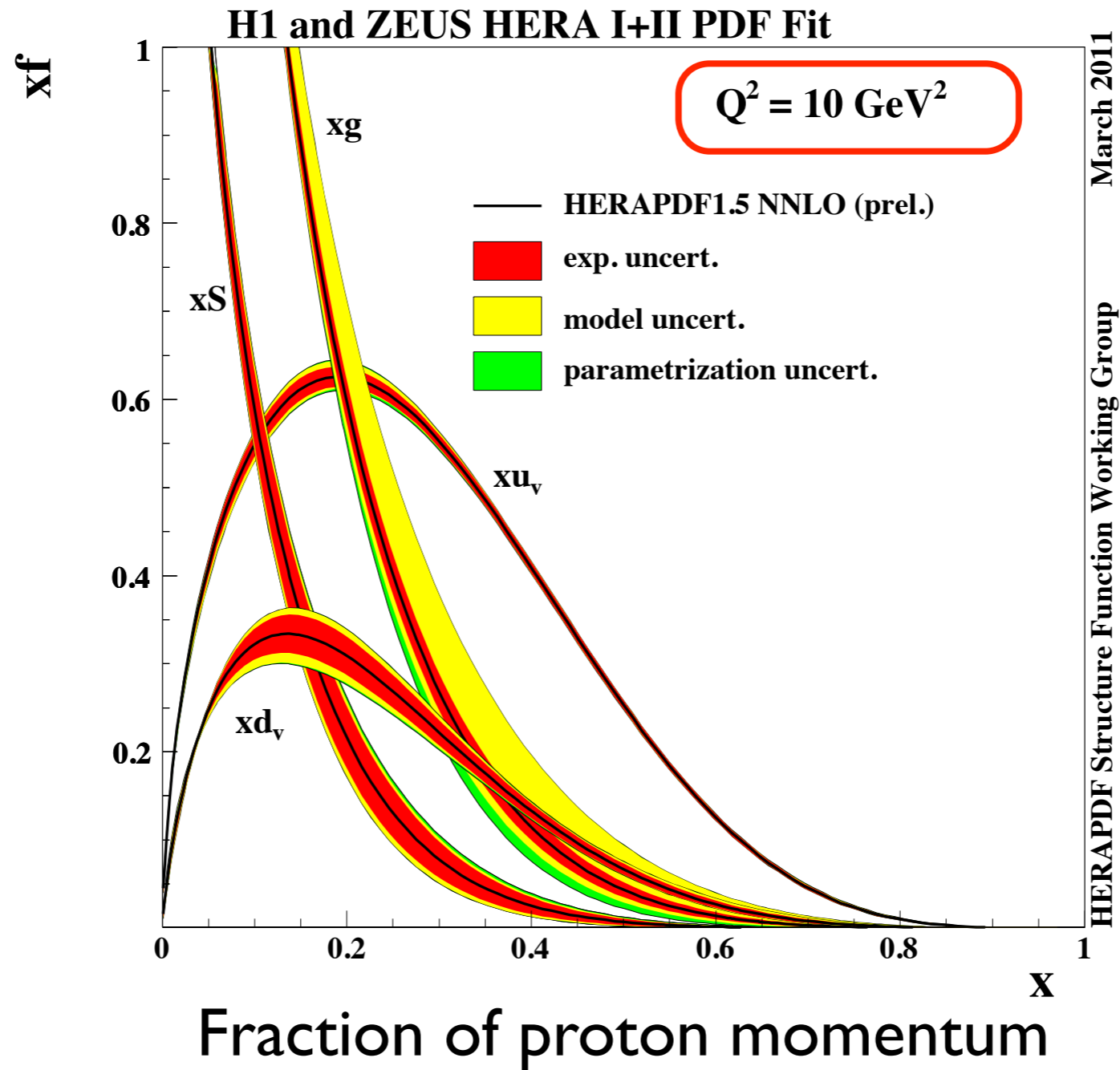
$$\mu^2 \frac{\partial}{\partial \mu^2} f_i(x, \mu^2) = \sum_j \int_x^1 \frac{d\xi}{\xi} P_{ij} \left(\frac{x}{\xi}, \alpha_S(\mu^2) \right) f_j(\xi, \mu^2)$$

- PDFs measured in various processes at various scales
- Global fits satisfying evolution equations give PDF sets
- Generally done at NNLO nowadays

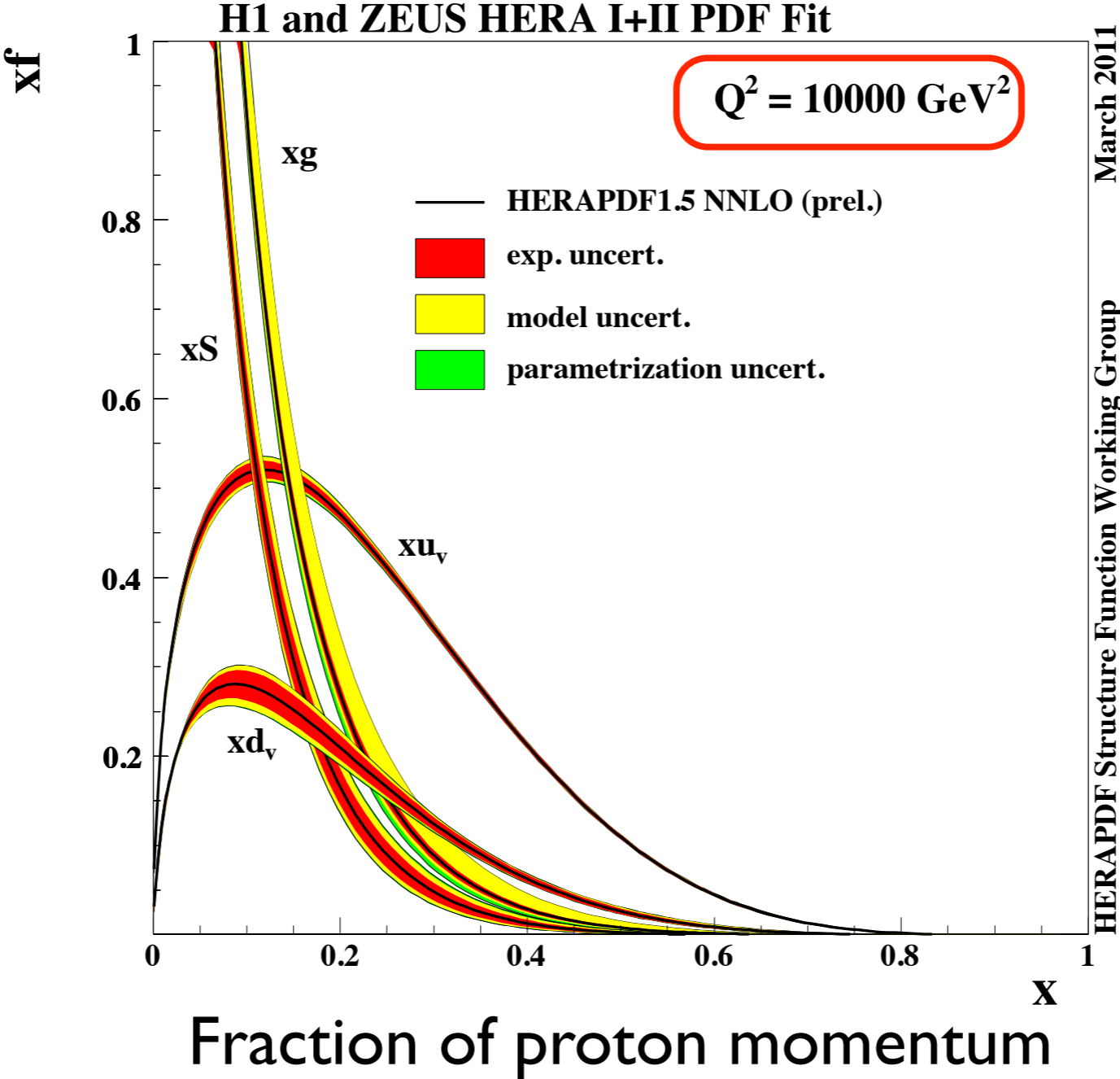
PDF Evolution



PDF Evolution

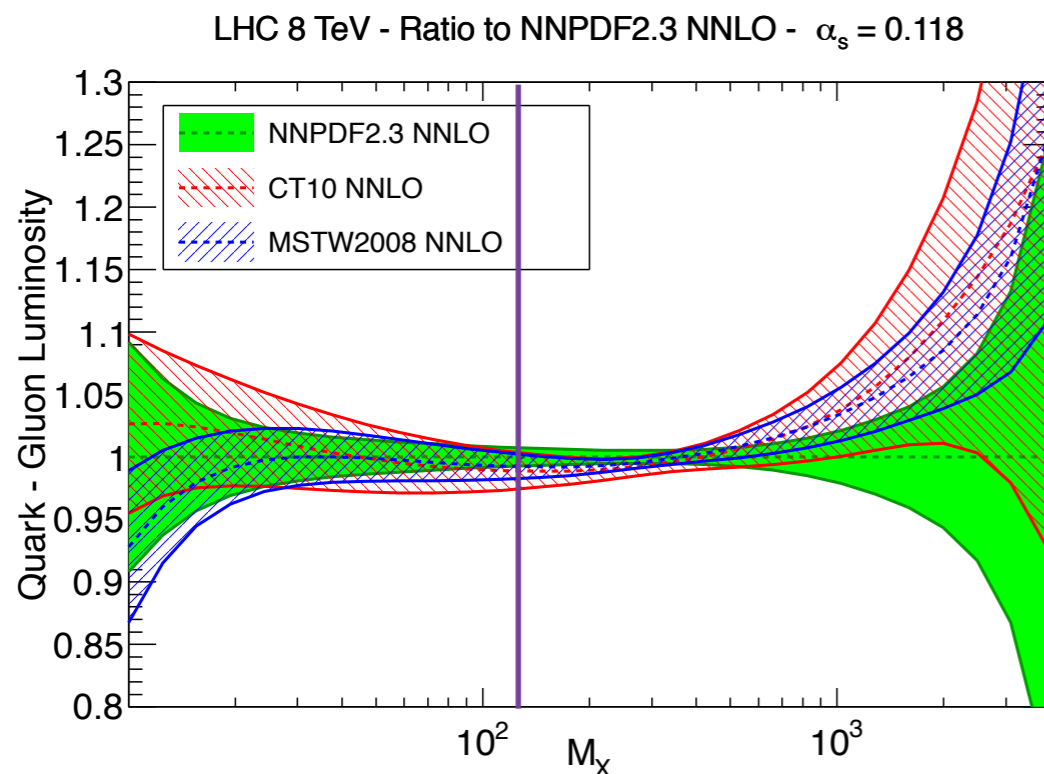
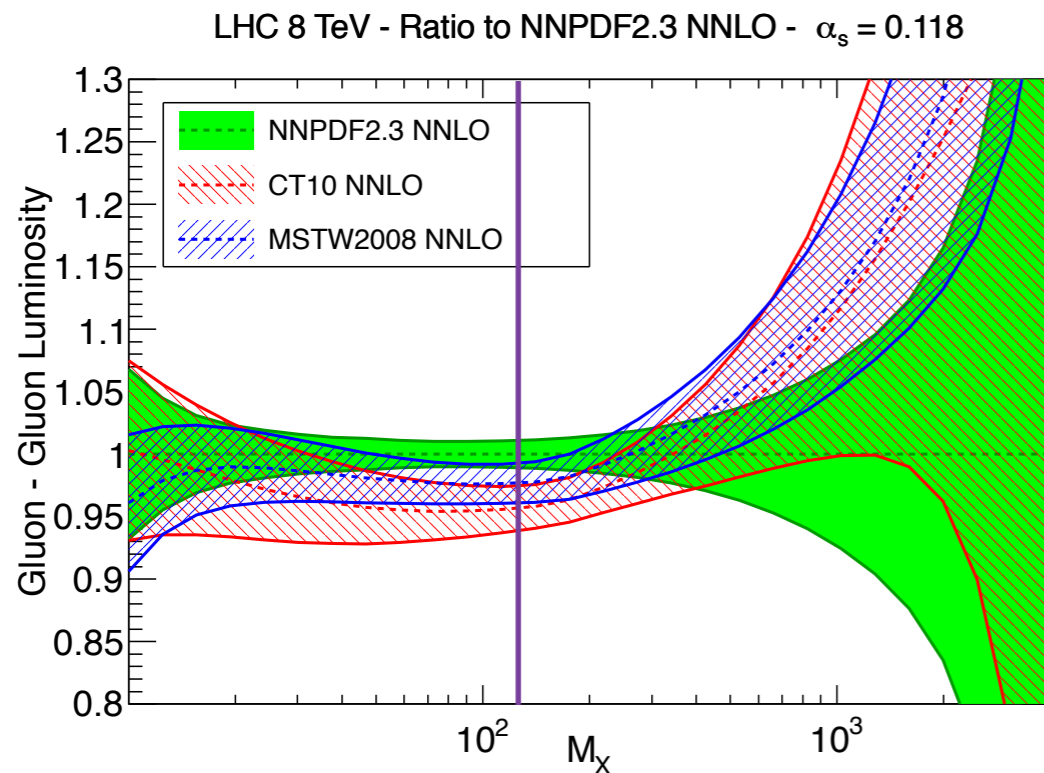


PDF Evolution



PDF Uncertainties

Ball et al., 1211.5142



- Parton luminosity $\mathcal{L}_{ij}(M_X^2, s) = \int dx_1 dx_2 f_i(x_1, M_X^2) f_j(x_2, M_X^2) \delta(x_1 x_2 s - M_X^2)$

- Relevant PDFs (relatively) well known at $x \sim M_H/\sqrt{s}$
- Some disagreement with CT10 \mathcal{L}_{gg}
- Remains true at 13 TeV
- Can be improved (in principle)

QCD Running Coupling

- Consider a dimensionless quantity R depending on a single hard scale Q
 - ★ Dependence on Q can only be via Q/μ
 - ★ But μ is arbitrary, so overall dependence on it must vanish

$$\rightarrow \mu^2 \frac{d}{d\mu^2} R(Q^2/\mu^2, \alpha_S) \equiv \left[\mu^2 \frac{\partial}{\partial \mu^2} + \mu^2 \frac{\partial \alpha_S}{\partial \mu^2} \frac{\partial}{\partial \alpha_S} \right] R = 0$$

- Define $t = \ln \left(\frac{Q^2}{\mu^2} \right)$, $\beta(\alpha_S) = \mu^2 \frac{\partial \alpha_S}{\partial \mu^2}$

$$\rightarrow \left[-\frac{\partial}{\partial t} + \beta(\alpha_S) \frac{\partial}{\partial \alpha_S} \right] R(e^t, \alpha_S) = 0$$

- Introduce $\alpha_S(Q^2)$ such that $t = \int_{\alpha_S}^{\alpha_S(Q^2)} \frac{dx}{\beta(x)}$, $\alpha_S(\mu^2) \equiv \alpha_S$.

- Then solution is $R(1, \alpha_S(Q^2))$

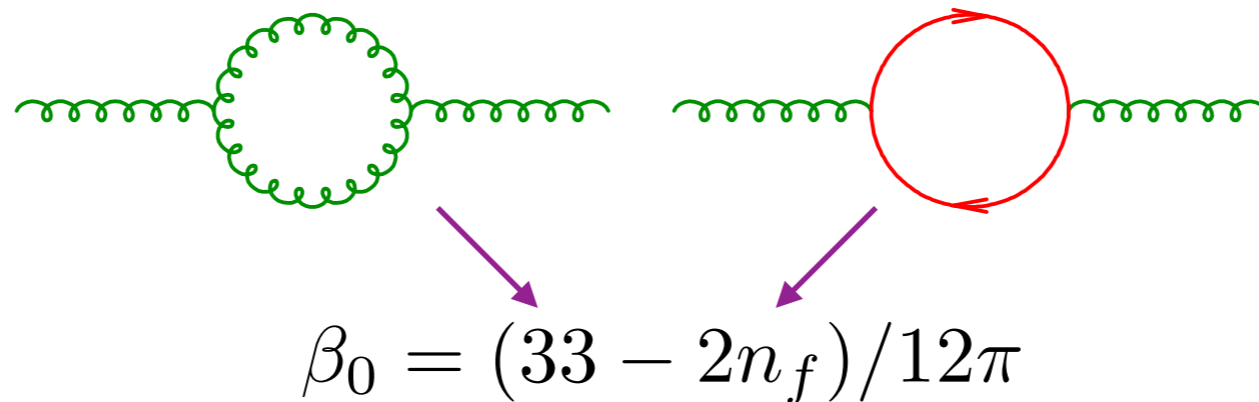
- ★ All scale dependence is absorbed in **running coupling** $\alpha_S(Q^2)$

QCD Running Coupling

$$\ln \left(\frac{Q^2}{\mu^2} \right) = \int_{\alpha_S(\mu^2)}^{\alpha_S(Q^2)} \frac{d\alpha_S}{\beta(\alpha_S)}, \quad \beta(\alpha_S) = -\alpha_S^2 (\beta_0 + \beta_1 \alpha_S + \dots)$$

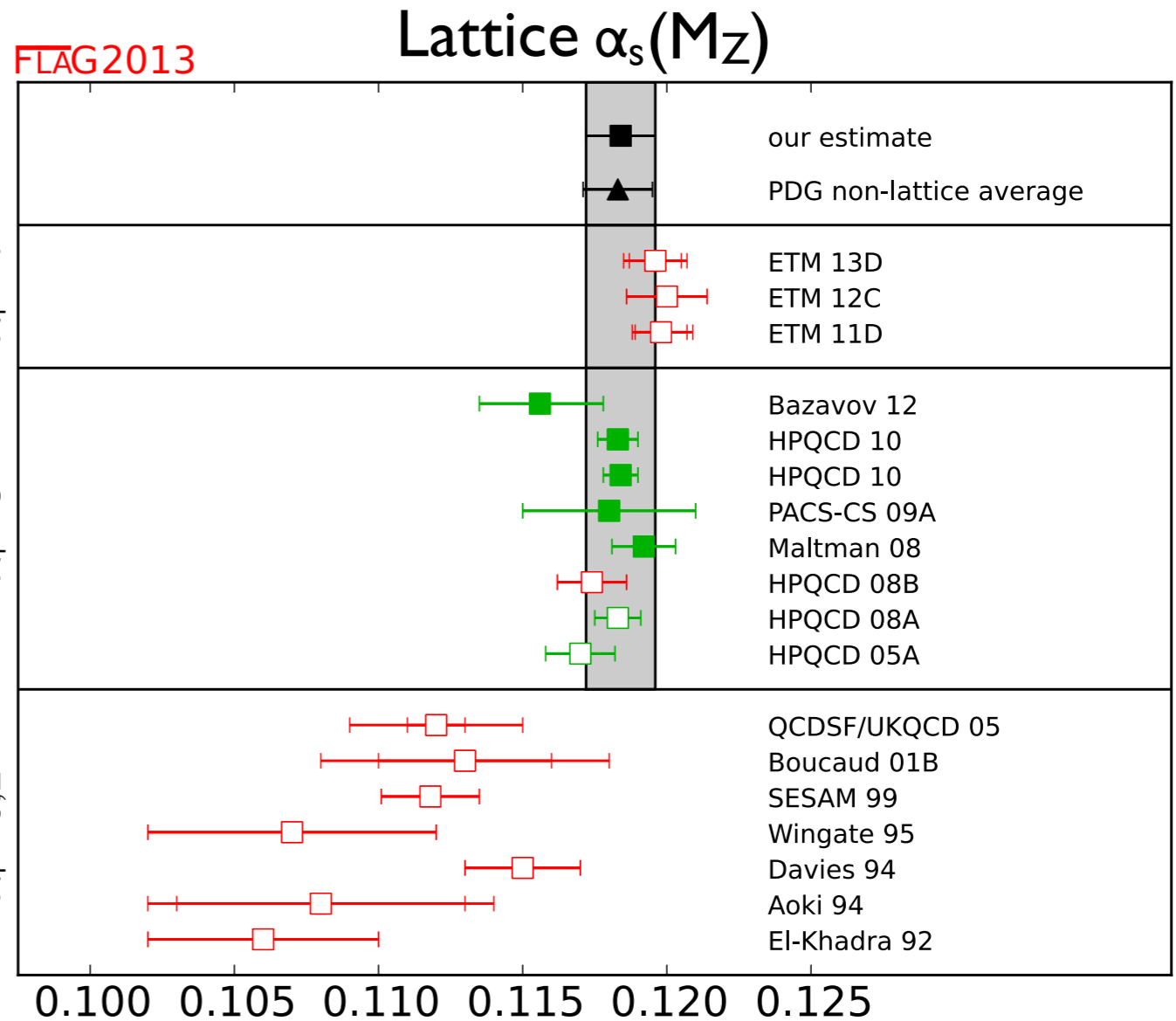
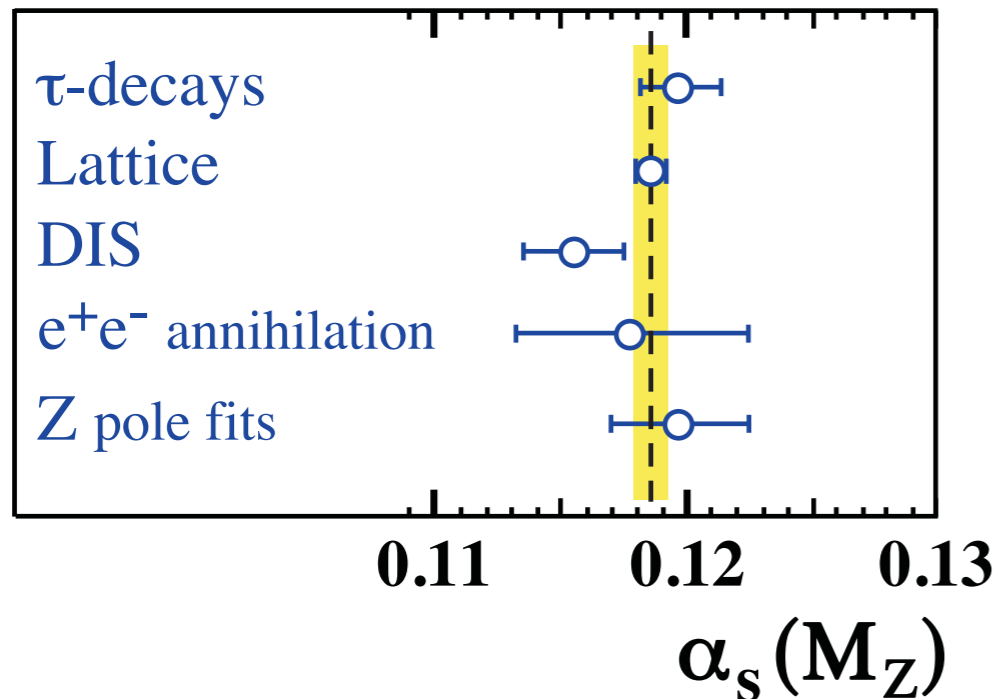
$$\rightarrow \ln \left(\frac{Q^2}{\mu^2} \right) = \frac{1}{\beta_0} \left[\frac{1}{\alpha_S(Q^2)} - \frac{1}{\alpha_S(\mu^2)} \right] + \dots$$

$$\rightarrow \alpha_S(Q^2) = \frac{\alpha_S(\mu^2)}{1 + \beta_0 \alpha_S(\mu^2) \ln(Q^2/\mu^2)} + \dots$$



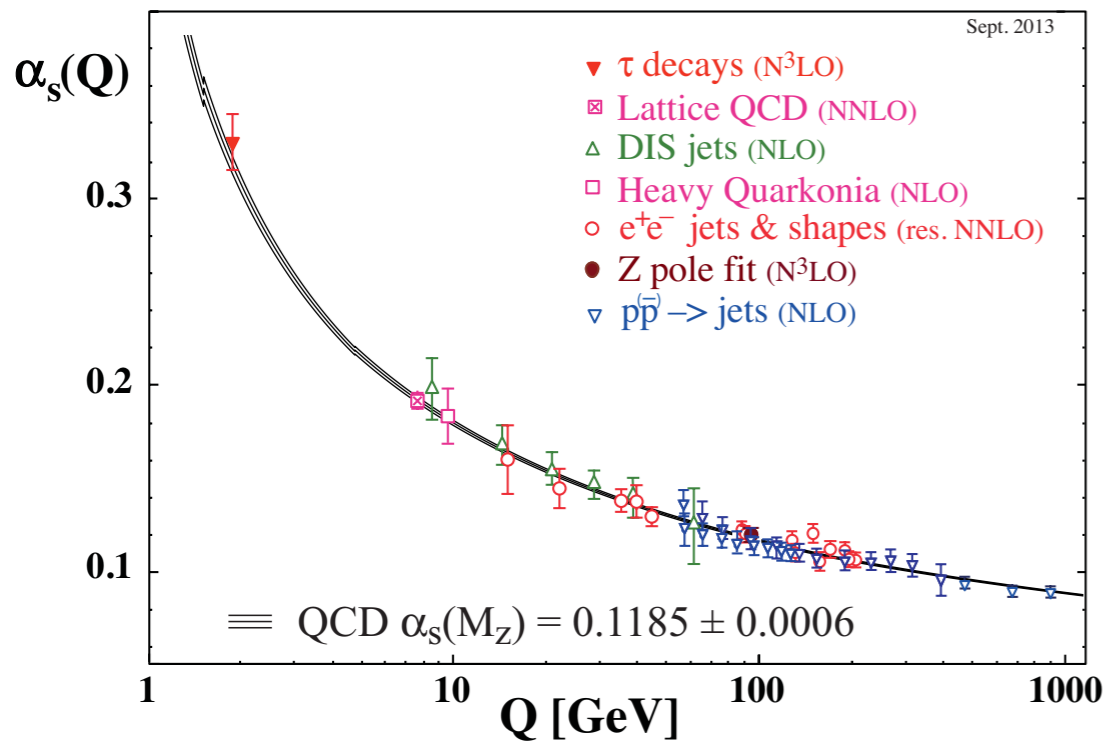
- $\beta_0 > 0$ means **asymptotic freedom**
- β -function known to 4 loops (β_3)

QCD Running Coupling



FLAG WG: Aoki et al., 1310.8555

$\alpha_s(M_Z)=0.1185(5)$ [lattice]
 $\alpha_s(M_Z)=0.1183(12)$ [non-lattice]



Bethke, Dissertori, Salam, RPP 2013

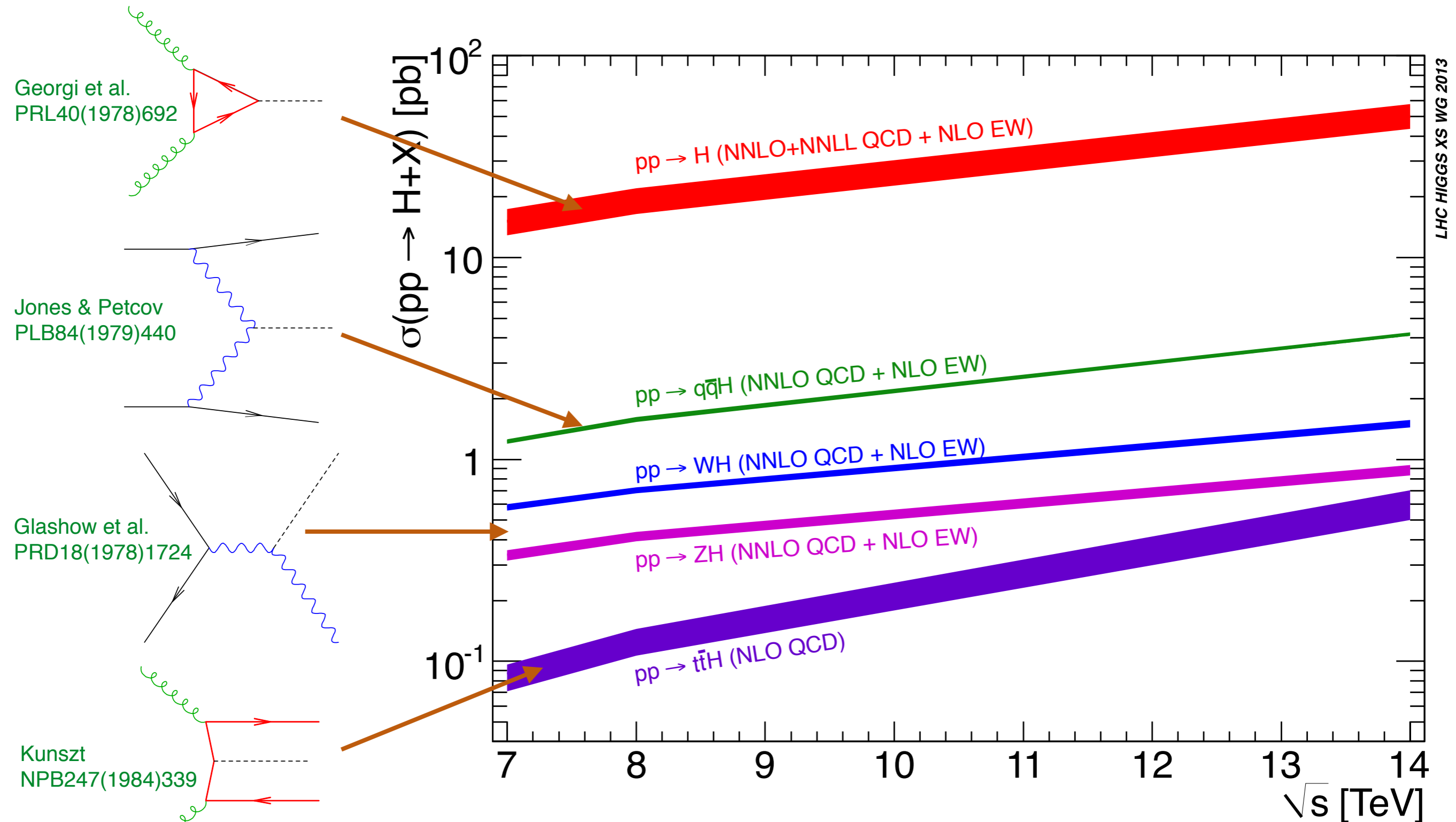
Lattice QCD Coupling

FLAG WG: Aoki et al., 1310.8555

Collaboration	Ref.	N_f	publication status	renormalisation scale	perturbative behaviour	continuum extrapolation	$\alpha_{\overline{\text{MS}}}(M_Z)$	Method	Table
ETM 13D	[544]	2+1+1	A	○	○	■	0.1196(4)(8)(16)	gluon-ghost vertex	37
ETM 12C	[545]	2+1+1	A	○	○	■	0.1200(14)	gluon-ghost vertex	37
ETM 11D	[546]	2+1+1	A	○	○	■	0.1198(9)(5)($^{+0}_{-5}$)	gluon-ghost vertex	37
Bazavov 12	[503]	2+1	A	○	○	○	0.1156($^{+21}_{-22}$)	$Q-\bar{Q}$ potential	33
HPQCD 10	[73]	2+1	A	○	○	○	0.1183(7)	current two points	36
HPQCD 10	[73]	2+1	A	○	★	★	0.1184(6)	Wilson loops	35
PACS-CS 09A	[486]	2+1	A	★	★	○	0.118(3) [#]	Schrödinger functional	32
Maltman 08	[517]	2+1	A	○	○	○	0.1192(11)	Wilson loops	35
HPQCD 08B	[85]	2+1	A	■	■	■	0.1174(12)	current two points	36
HPQCD 08A	[514]	2+1	A	○	★	★	0.1183(8)	Wilson loops	35
HPQCD 05A	[513]	2+1	A	○	○	○	0.1170(12)	Wilson loops	35
QCDSF/UKQCD 05	[518]	0, 2 \rightarrow 3	A	★	■	★	0.112(1)(2)	Wilson loops	35
Boucaud 01B	[539]	2 \rightarrow 3	A	○	○	■	0.113(3)(4)	gluon-ghost vertex	37
SESAM 99	[519]	0, 2 \rightarrow 3	A	★	■	■	0.1118(17)	Wilson loops	35
Wingate 95	[520]	0, 2 \rightarrow 3	A	★	■	■	0.107(5)	Wilson loops	35
Davies 94	[521]	0, 2 \rightarrow 3	A	★	■	■	0.115(2)	Wilson loops	35
Aoki 94	[522]	2 \rightarrow 3	A	★	■	■	0.108(5)(4)	Wilson loops	35
El-Khadra 92	[523]	0 \rightarrow 3	A	★	○	○	0.106(4)	Wilson loops	35

QCD and the Higgs Boson

Higgs production cross sections



The challenges:

(taken from [R. Tanaka, talk at Aspen Higgs WS 03/13])

ggF, VBF, WH/ZH, ttH, BSM Higgs

Higgs Cross Sections
(inclusive/exclusive)

Differential K-factors
(effect of jet-veto etc.)

QCD correction $N^k\text{LO} + N^m\text{LL}$
EW correction, Mixed QCD-EW

Heavy Higgs Line Shape
SM Backgrounds & Interferences

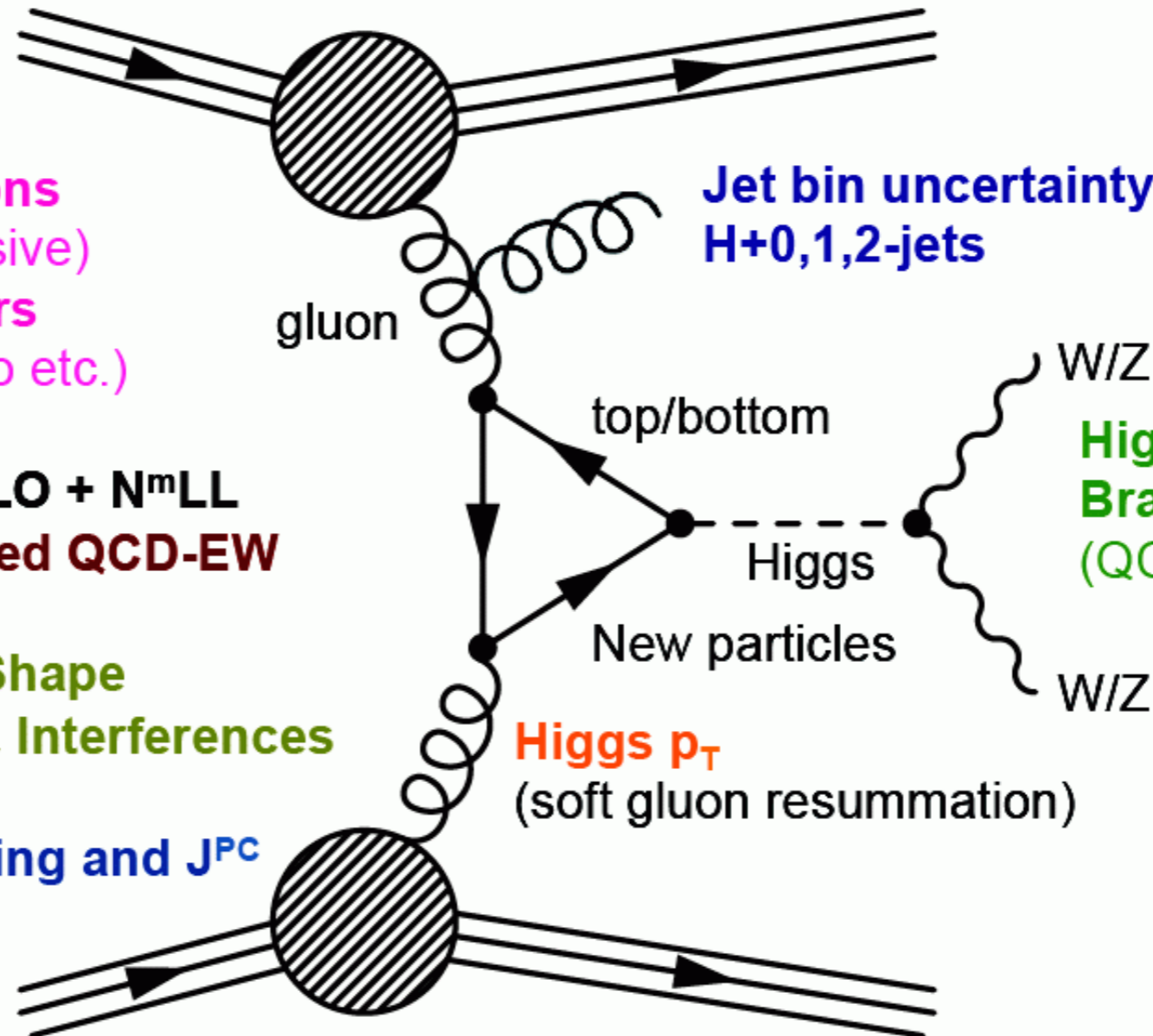
Higgs Mass, Coupling and J^{PC}

Jet bin uncertainty
H+0,1,2-jets

Higgs decay
Branching ratios
(QCD/EW corr.)

Higgs p_T
(soft gluon resummation)

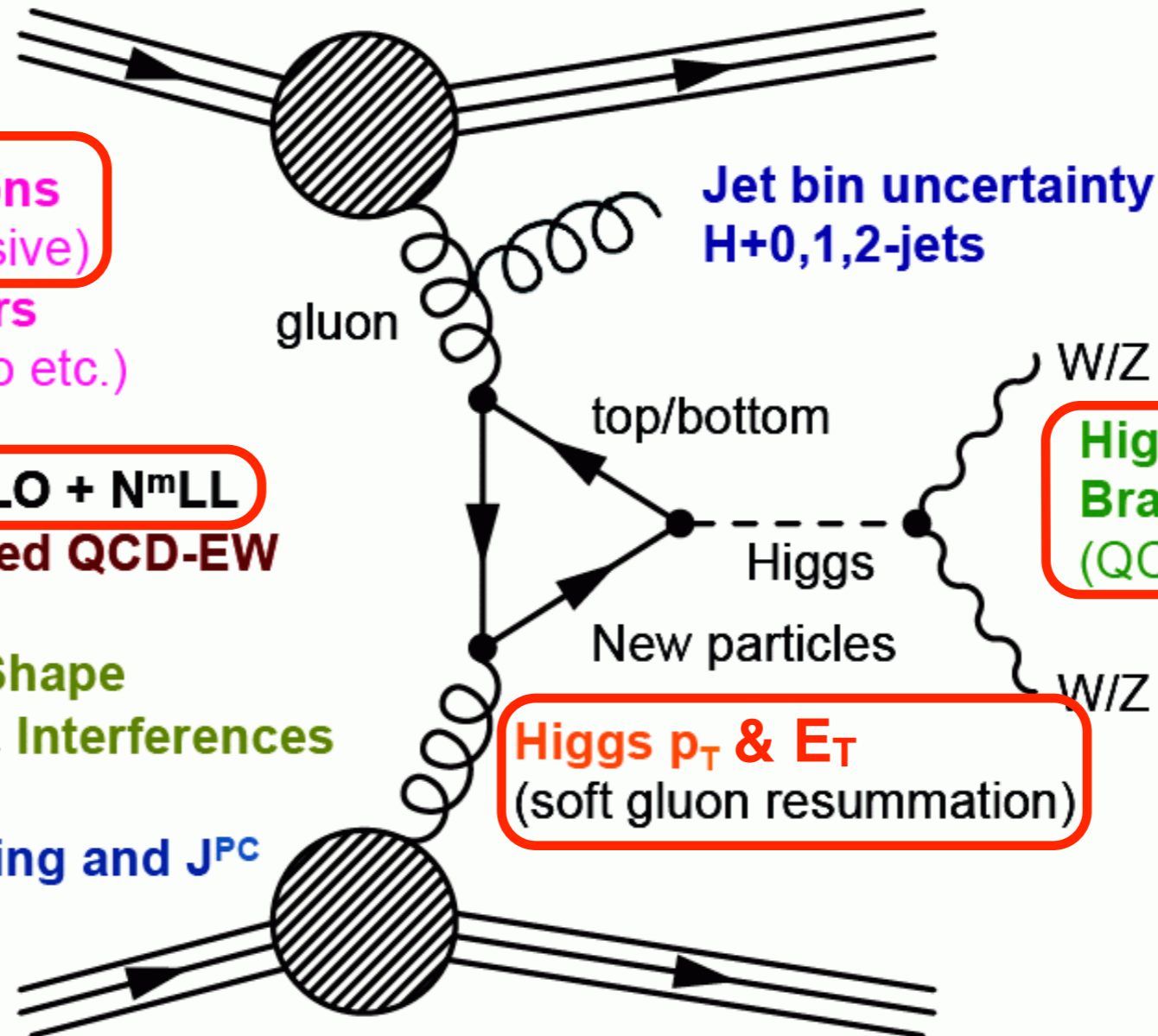
PDF+ α_s uncertainties
Renormalization/Factorization scale dependence



The challenges:

(taken from [R. Tanaka, talk at Aspen Higgs WS 03/13])

ggF, VBF, WH/ZH, ttH, BSM Higgs



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EW correction, Mixed QCD-EW

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SM Backgrounds & Interferences

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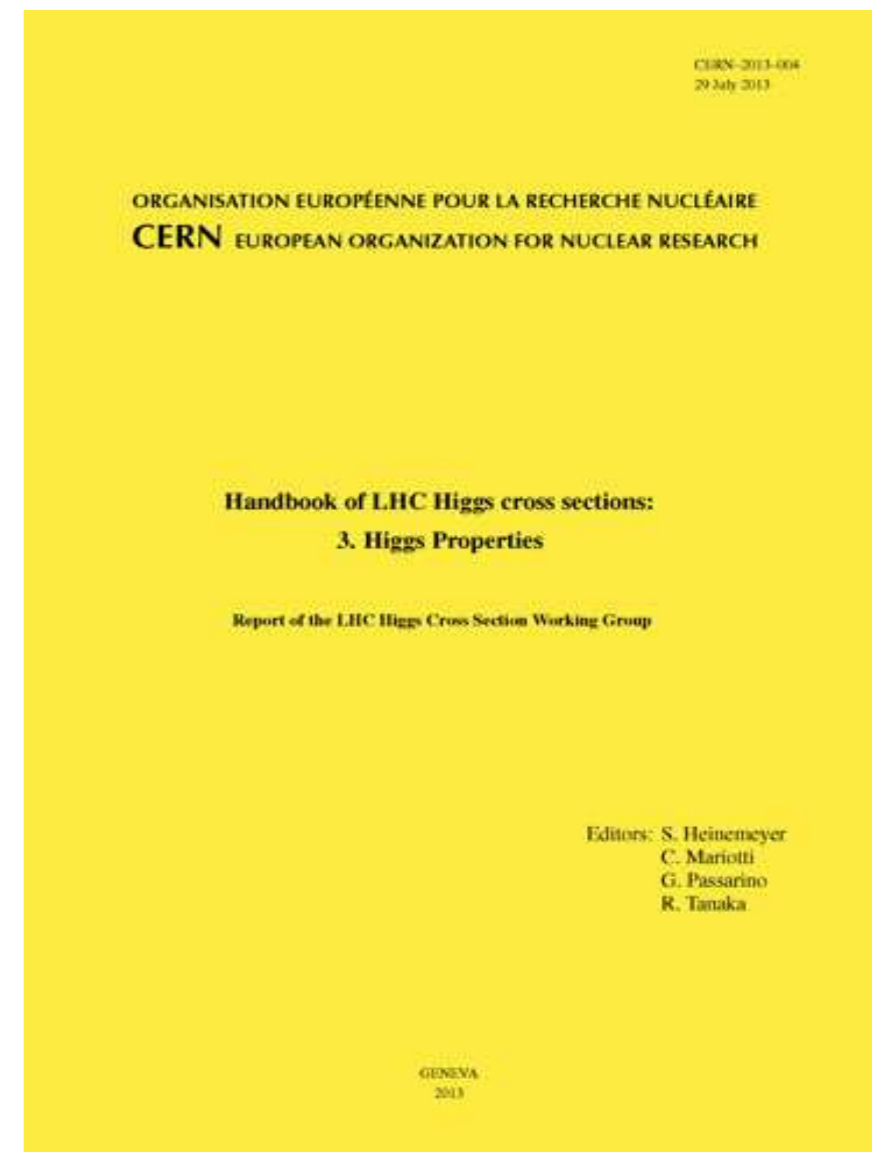
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(soft gluon resummation)

PDF+ α_s uncertainties
Renormalization/Factorization scale dependence

LHC Higgs Cross Section Working Group



arXiv:1101.0593, 1201.3084, 1307.1347

Higgs Production Cross Section

Inclusive (QCD) Cross Section

$$\sigma_H(s) = \sum_{i,j} \int dx_1 dx_2 f_i(x_1, \mu_F^2) f_j(x_2, \mu_F^2) \hat{\sigma}_{ij}(x_1 x_2 s, m_H^2, \alpha_S(\mu_R^2), \mu_F^2, \mu_R^2)$$

Table B.10: ggF cross sections at the LHC at 8 TeV and corresponding scale and PDF+ α_s uncertainties computed according to the PDF4LHC recommendation.

HXSWG vol.3

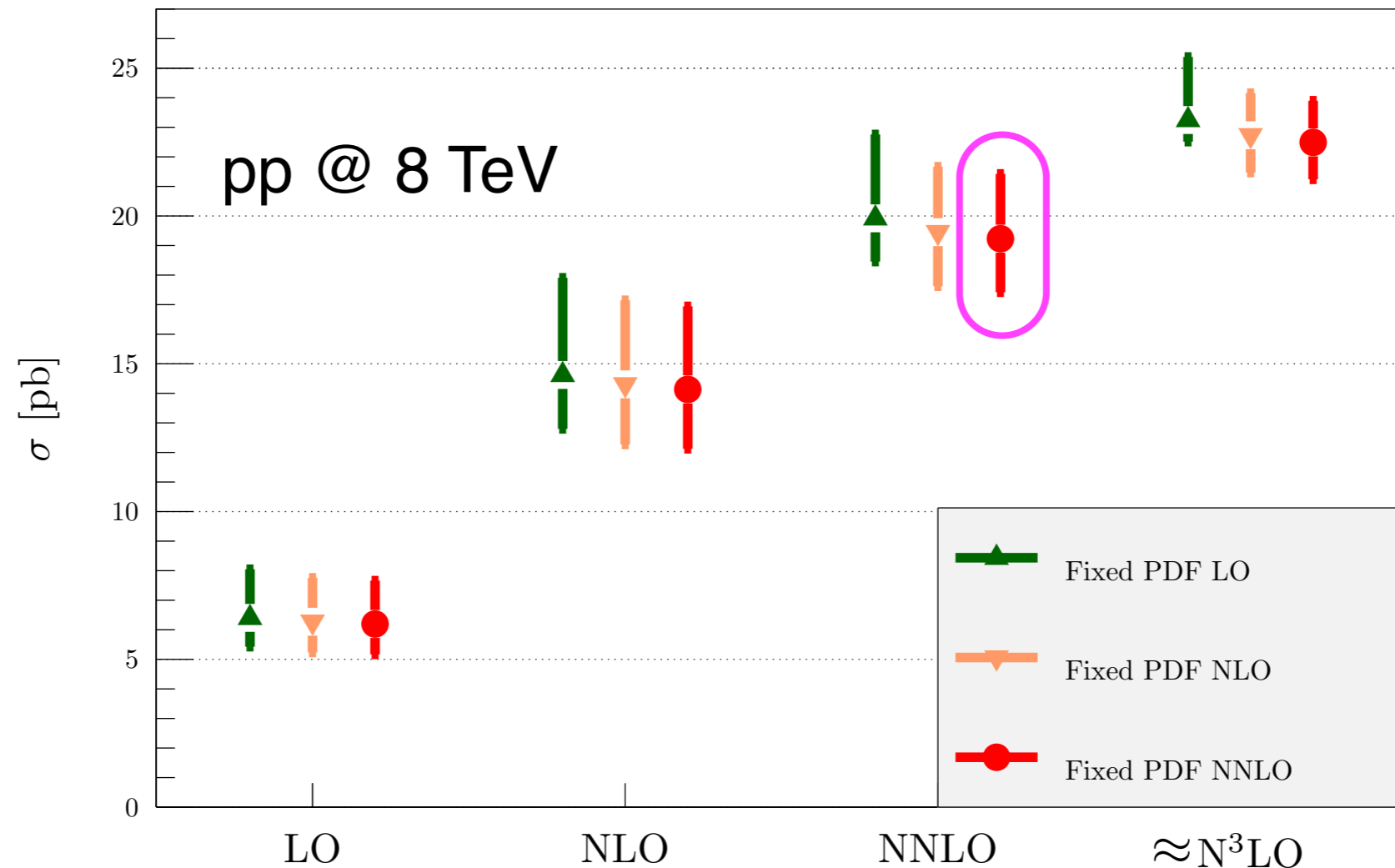
M_H [GeV]	σ [pb]	QCD Scale [%]		PDF+ α_s [%]	
124.4	19.45	+7.2	-7.9	+7.5	-6.9
124.5	19.42	+7.2	-7.9	+7.5	-6.9
124.6	19.39	+7.2	-7.9	+7.5	-6.9
124.7	19.36	+7.2	-7.9	+7.5	-6.9
124.8	19.33	+7.2	-7.8	+7.5	-6.9
124.9	19.30	+7.2	-7.8	+7.5	-6.9
125.0	19.27	+7.2	-7.8	+7.5	-6.9
125.1	19.24	+7.2	-7.8	+7.5	-6.9
125.2	19.21	+7.2	-7.8	+7.5	-6.9
125.3	19.18	+7.2	-7.8	+7.5	-6.9
125.4	19.15	+7.2	-7.8	+7.5	-6.9
125.5	19.12	+7.2	-7.8	+7.5	-6.9
125.6	19.09	+7.2	-7.8	+7.5	-6.9
125.7	19.06	+7.2	-7.8	+7.5	-6.9
125.8	19.03	+7.2	-7.8	+7.5	-6.9
125.9	19.00	+7.2	-7.8	+7.5	-6.9
126.0	18.97	+7.2	-7.8	+7.5	-6.9
126.1	18.94	+7.2	-7.8	+7.5	-6.9
126.2	18.91	+7.2	-7.8	+7.5	-6.9
126.3	18.88	+7.2	-7.8	+7.5	-6.9
126.4	18.85	+7.2	-7.8	+7.5	-6.9
126.5	18.82	+7.2	-7.8	+7.5	-6.9

NNLO \rightarrow $\sigma_{\text{ggF}}(8 \text{ TeV}) = 19.1 \pm 2.0 \text{ pb}$

Inclusive (QCD) Cross Section

Ball et al., 1303.3590

Forte, Isgro, Vita, 1312.6688



- Higher-order effects are larger than x2 scale variation estimates

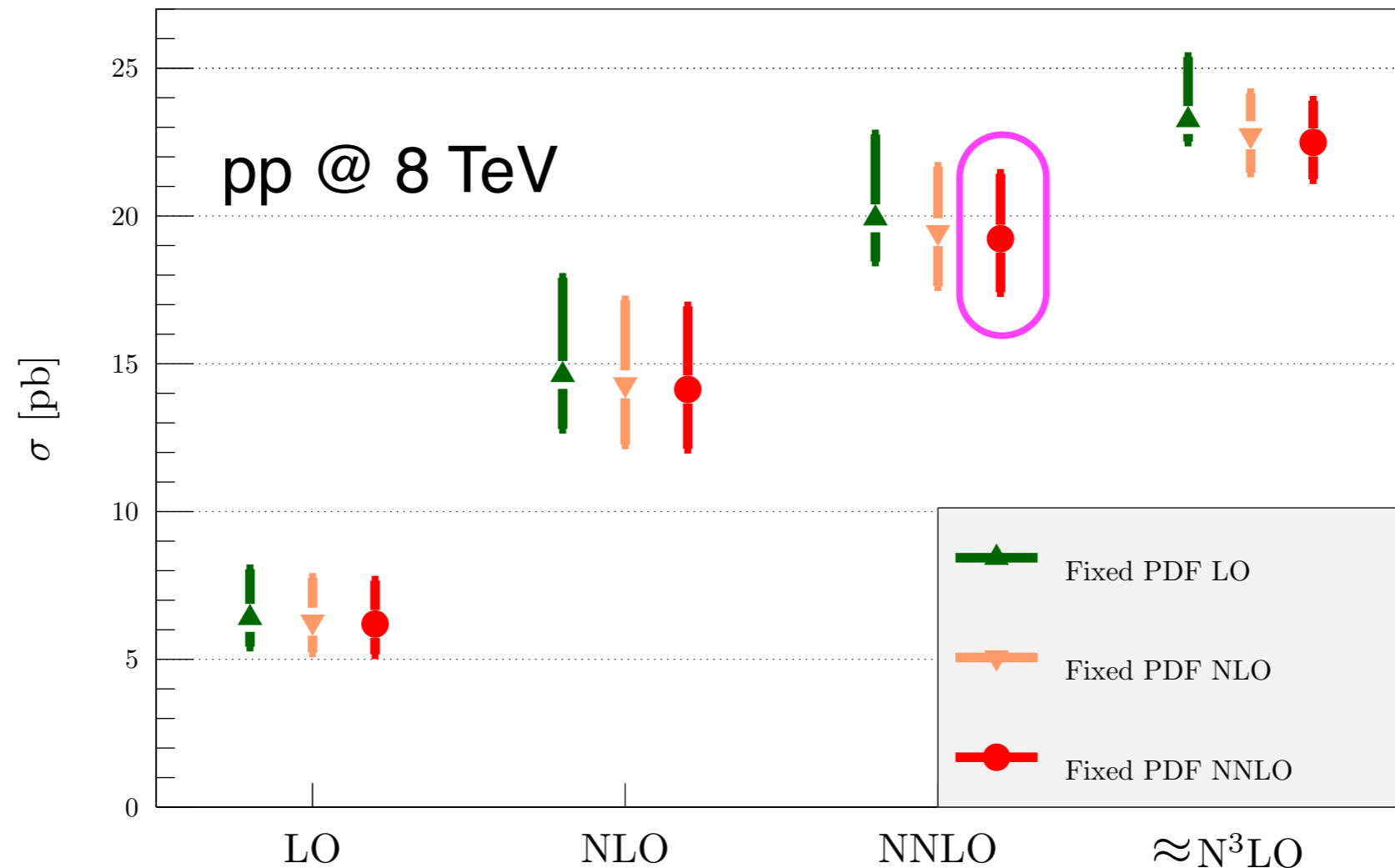
- $\sigma_{ggF} \approx \sigma_{LO} [1 + \kappa \{ \lambda \alpha_S + (\lambda \alpha_S)^2 + (\lambda \alpha_S)^3 + \dots \}]$

$\lambda \approx 5.6$

Inclusive (QCD) Cross Section

Ball et al., 1303.3590

Forte, Isgro, Vita, 1312.6688



- Higher-order effects are larger than x2 scale variation estimates

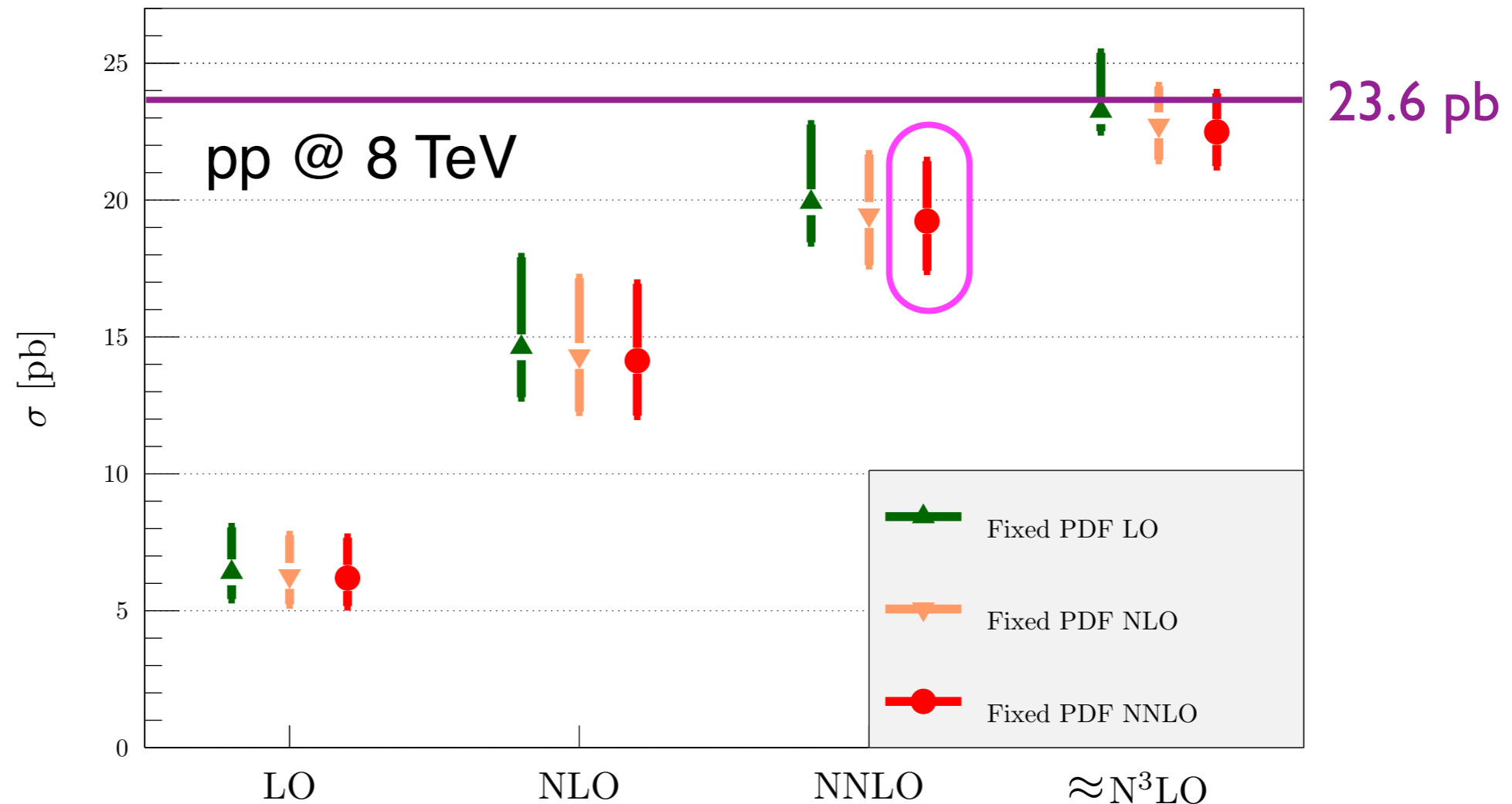
- $$\sigma_{ggF} \approx \sigma_{LO} \left[1 + \kappa \{ \lambda \alpha_S + (\lambda \alpha_S)^2 + (\lambda \alpha_S)^3 + \dots \} \right] \approx \sigma_{LO} \left[1 - \kappa + \frac{\kappa}{1 - \lambda \alpha_S} \right]$$

$\lambda \approx 5.6$

Inclusive (QCD) Cross Section

Ball et al., 1303.3590

Forte, Isgro, Vita, 1312.6688

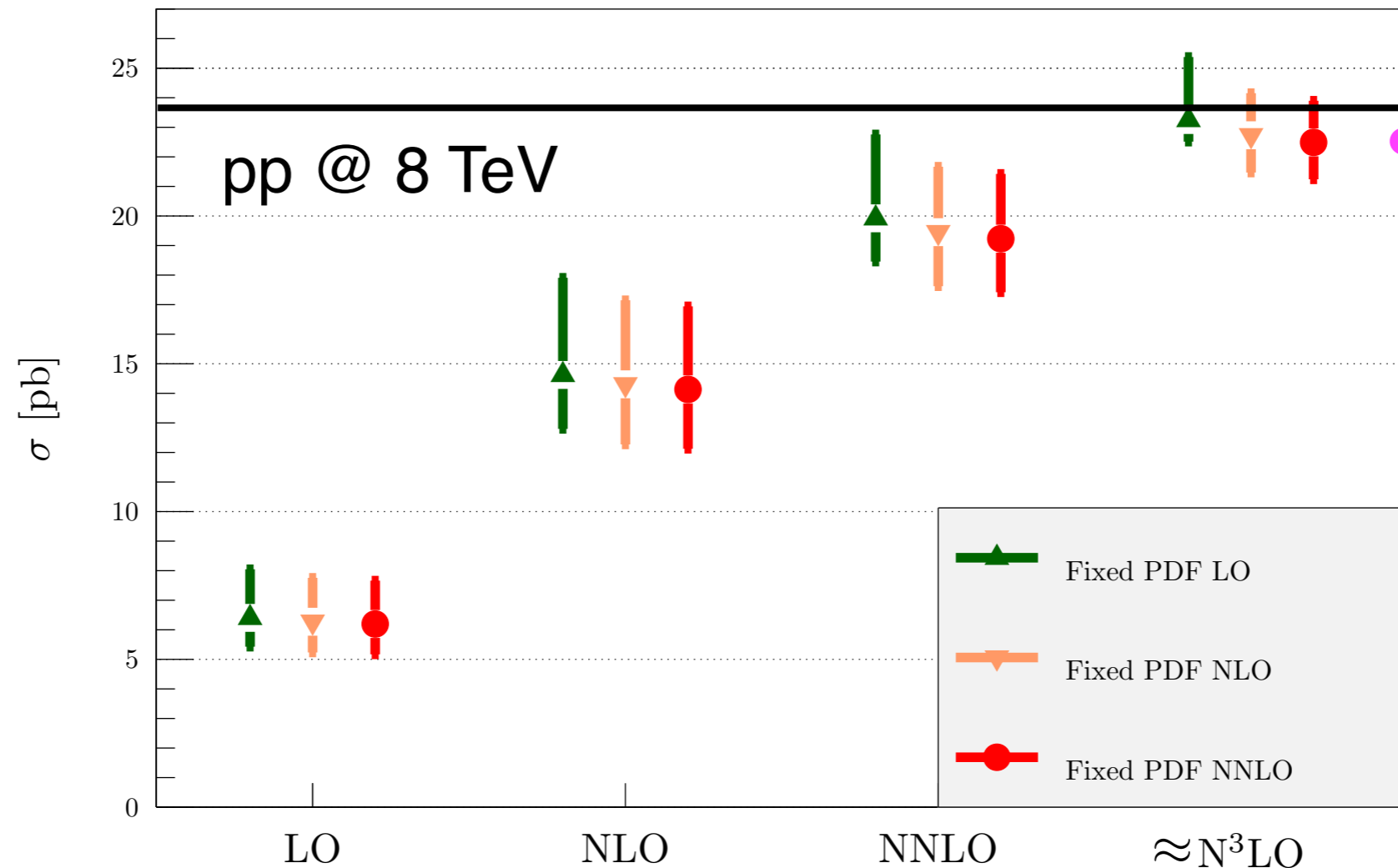


- Higher-order effects are larger than x2 scale variation estimates

- $$\sigma_{ggF} \approx \sigma_{LO} [1 + \kappa\{\lambda\alpha_S + (\lambda\alpha_S)^2 + (\lambda\alpha_S)^3 + \dots\}] \approx \sigma_{LO} \left[1 - \kappa + \frac{\kappa}{1 - \lambda\alpha_S} \right]$$

$\lambda \approx 5.6$

Inclusive (QCD) Cross Section



NNLO \rightarrow $\sigma_{ggF}(8 \text{ TeV}) = 19.1 \pm 2.0 \text{ pb}$

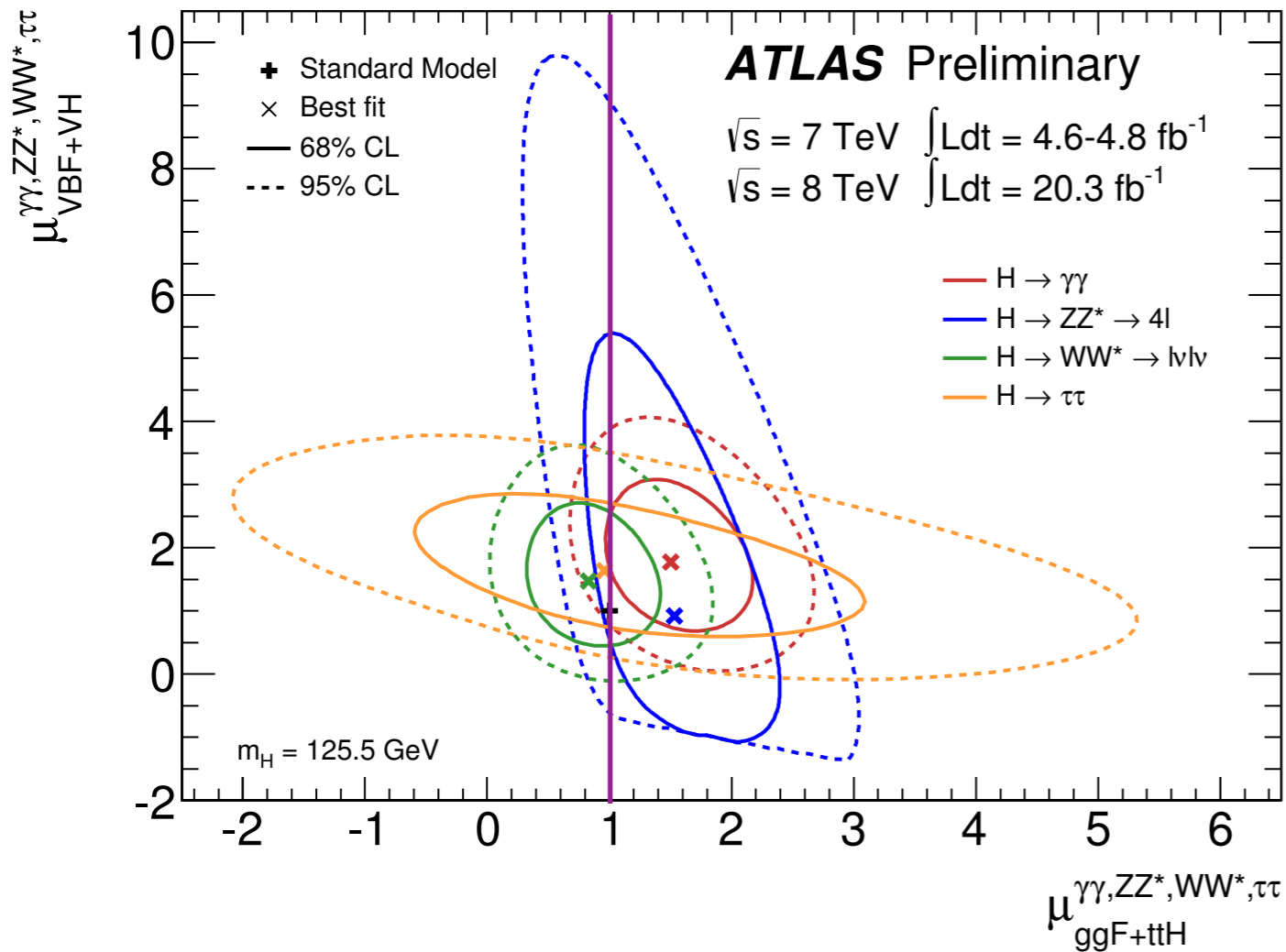
Series extrapolation: $23.6 \pm ?? \text{ pb}$

David & Passarino, 1307.1843: $22.5 \pm 2.6 \text{ pb}$

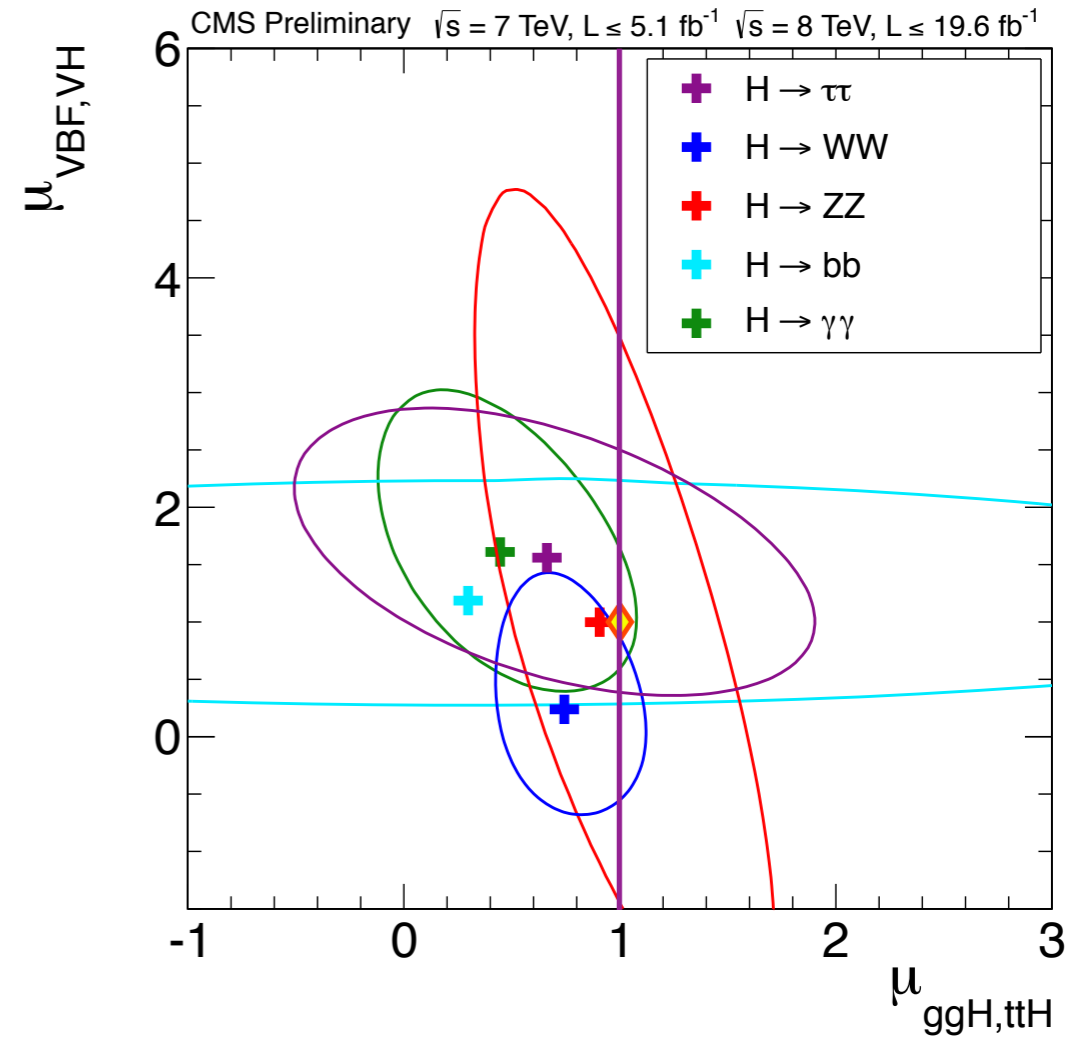
Full N^3LO coming soon (Anastasiou et al.)

Inclusive (QCD) Cross Section

- ATLAS, but not CMS, find ggF excess in $\gamma\gamma$ and ZZ^* channels



ATLAS-CONF-2014-009 (Moriond EW)



CMS PAS HIG-13-005

Cross Sections at 13 TeV

HXSWG 05/04/2014

$\sqrt{s} = 13.0 \text{ TeV}$

gluon-gluon Fusion Process

- All cross sections are in complex-pole-scheme from the dFG program. They are computed at NNLL QCD and NLO EW.

m_H (GeV)	Cross Section (pb)	+QCD Scale %	-QCD Scale %	+(PDF+ α_s) %	-(PDF+ α_s) %
125.0	43.92	+7.4	-7.9	+7.1	-6.0
125.5	43.62	+7.4	-7.9	+7.1	-6.0
126.0	43.31	+7.4	-7.9	+7.1	-6.0

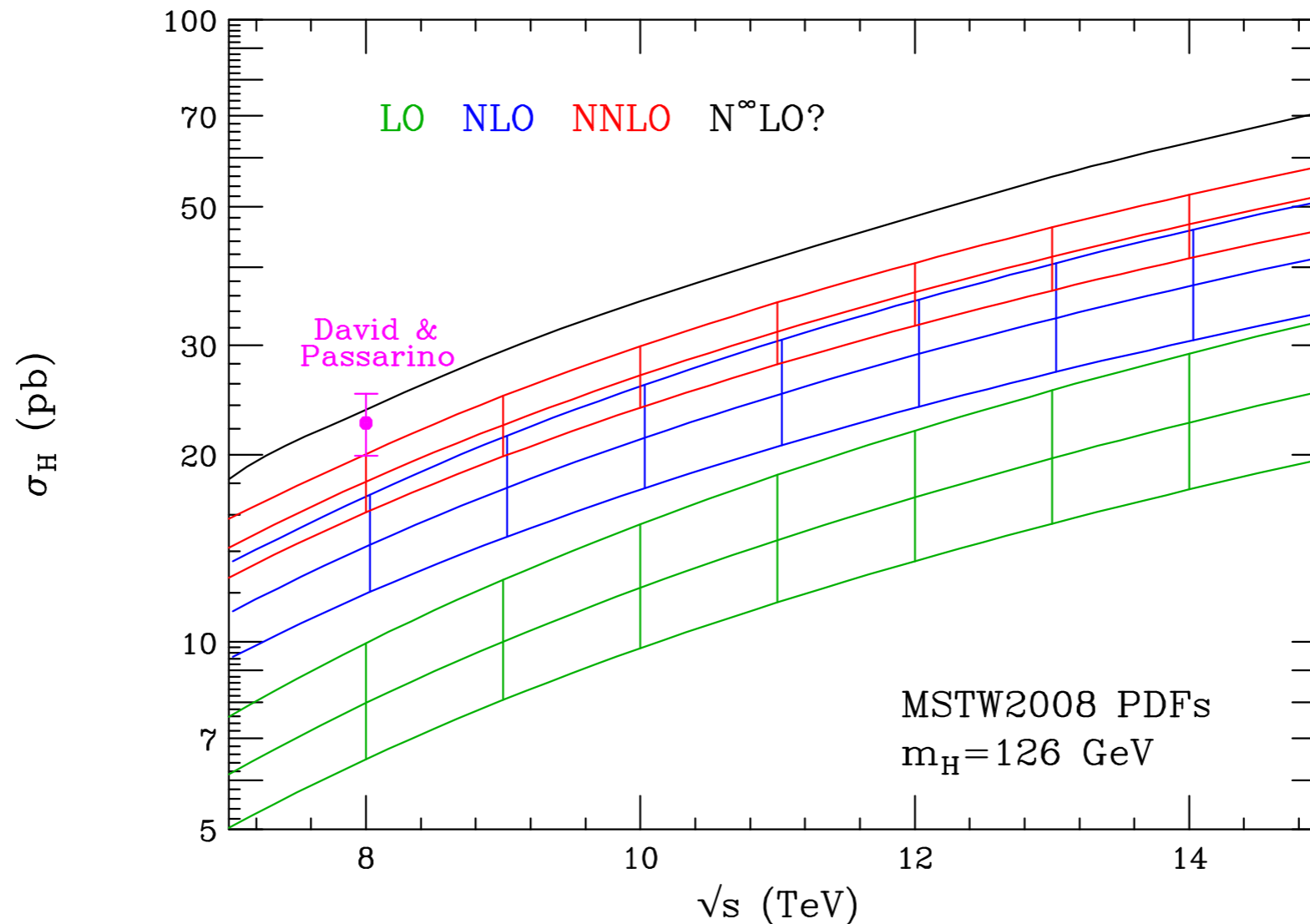
VBF Process

- At NNLO QCD and NLO EW. All cross sections are in complex-pole-scheme.

m_H (GeV)	Cross Section (pb)	+QCD Scale %	-QCD Scale %	+(PDF+ α_s) %	-(PDF+ α_s) %
125.0	3.748	+0.7	-0.7	+3.2	-3.2
125.5	3.727	+1.0	-0.7	+3.4	-3.4
126.0	3.703	+1.3	-0.6	+3.1	-3.1

Cross Section vs Energy

<http://theory.fi.infn.it/grazzini/hcalculators.html>



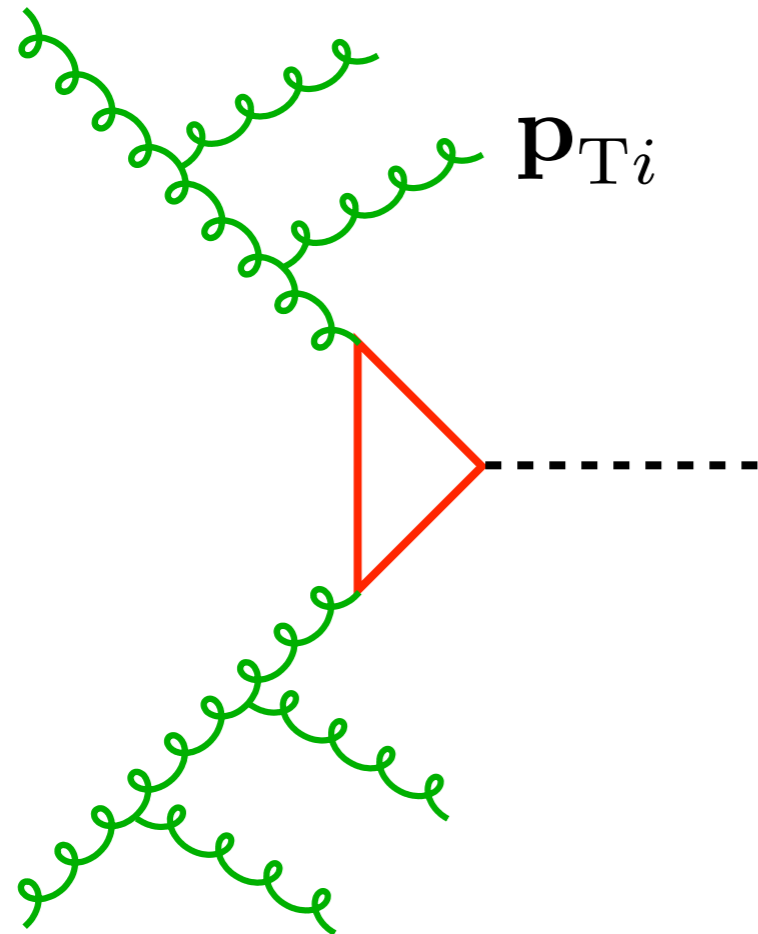
NNLO → $\sigma_{ggF}(13 \text{ TeV}) = 43.3 \pm 4.3 \text{ pb}$

Series extrapolation: $\sigma_{ggF}(13 \text{ TeV}) = 55.9 \pm ?? \text{ pb}$

My best guess: $\sigma_{ggF}(13 \text{ TeV}) = 53 \pm 8 \text{ pb}$

Higgs Differential Cross Sections

Higgs q_T & E_T



- Higgs transverse momentum

$$q_T = - \sum \mathbf{p}_{Ti}$$

Bozzi et al. 0705.3887

Mantry & Petriello, 0911.4135

Catani & Grazzini, 1011.3918

de Florian et al. 1109.2109

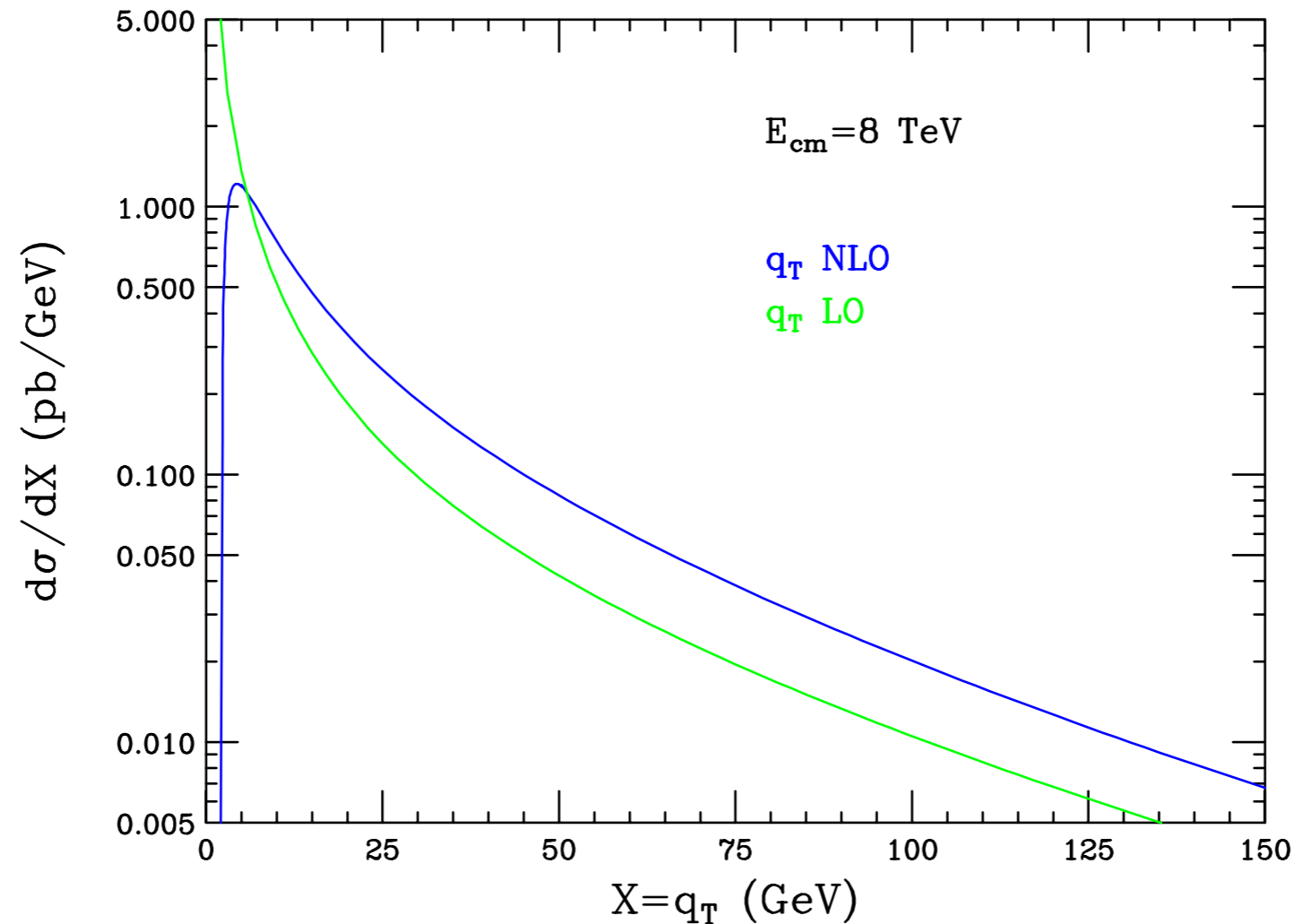
- Radiated transverse energy

$$E_T = \sum |\mathbf{p}_{Ti}|$$

Papaefstathiou, Smillie, BW, 1002.4375

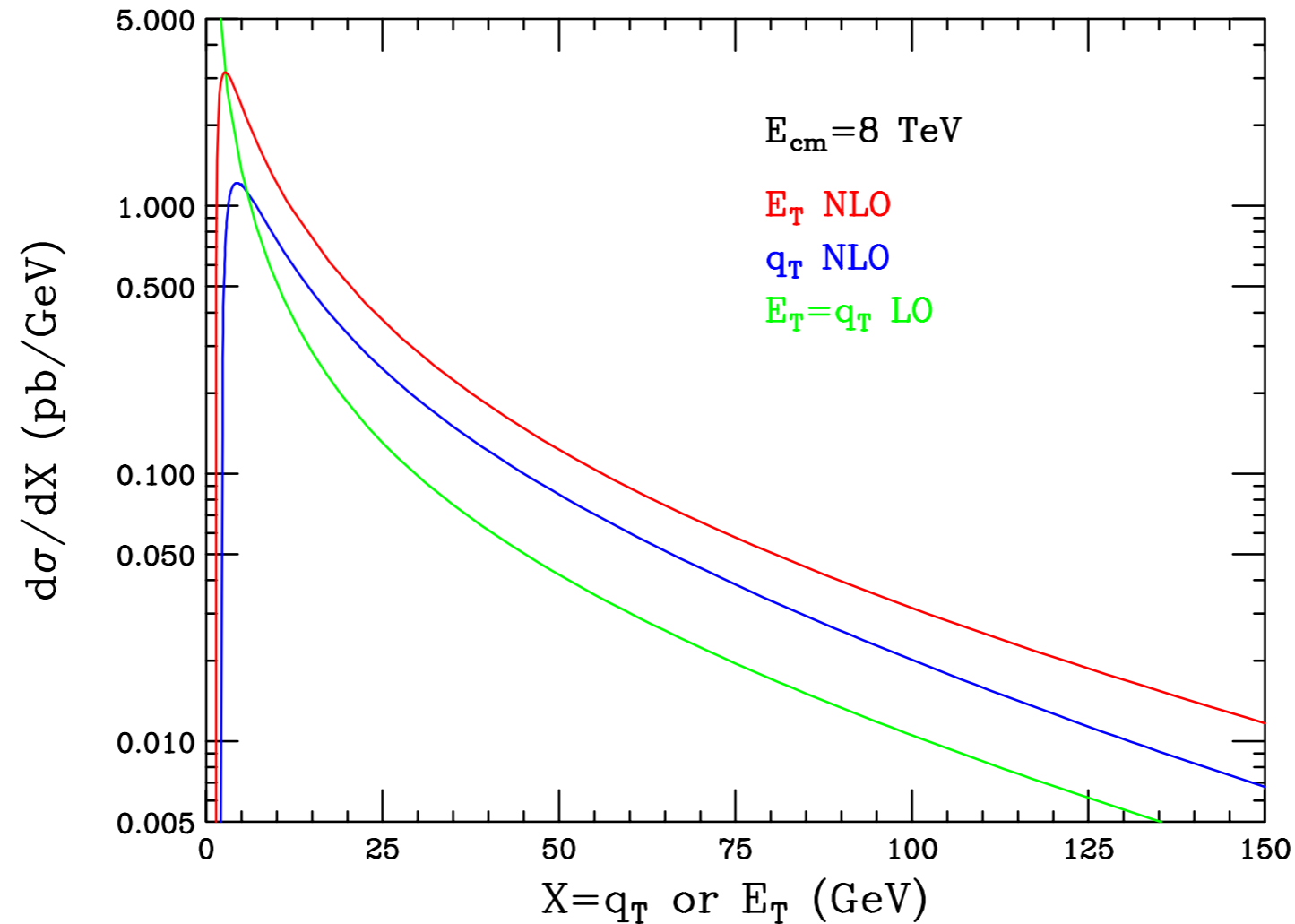
+Grazzini, 1403.3394

Higgs q_T (fixed order)



- $(\text{N})\text{LO} \xrightarrow{q_T \rightarrow 0} (-)\infty$
- Large logs of m_H^2/q_T^2 need resummation

Higgs q_T & E_T (fixed order)



- $(\text{N})\text{LO} \xrightarrow{E_T \rightarrow 0} (-)\infty$
- Large logs of m_H^2/E_T^2 need resummation

Resummation of Higgs q_T

$$d\sigma = \int dx_1 dx_2 f_a(x_1, \mu) f_b(x_2, \mu) d\hat{\sigma}_{ab}(x_1 x_2 s, \mu, \dots)$$

$$\frac{1}{\hat{\sigma}_{gg}} \frac{d^2 \hat{\sigma}_{gg}}{d\mathbf{q}_T^2} \sim \delta^2(\mathbf{q}_T) + \alpha_S \int d^2 \mathbf{p}_T \left[\frac{A_g}{\mathbf{p}_T^2} \ln \frac{m_H^2}{\mathbf{p}_T^2} + \frac{B_g}{\mathbf{p}_T^2} \right]_+ \delta^2(\mathbf{q}_T + \mathbf{p}_T) + \dots$$

Resummation of Higgs q_T

$$d\sigma = \int dx_1 dx_2 f_a(x_1, \mu) f_b(x_2, \mu) d\hat{\sigma}_{ab}(x_1 x_2 s, \mu, \dots)$$

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$$\sim \int \frac{d^2 \mathbf{b}}{(2\pi)^2} e^{i\mathbf{b} \cdot \mathbf{q}_T} \left\{ 1 + \alpha_S \int d^2 \mathbf{p}_T \left[\frac{A_g}{\mathbf{p}_T^2} \ln \frac{m_H^2}{\mathbf{p}_T^2} + \frac{B_g}{\mathbf{p}_T^2} \right] (e^{i\mathbf{b} \cdot \mathbf{p}_T} - 1) + \dots \right\}$$

Resummation of Higgs q_T

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$$\frac{1}{\hat{\sigma}_{gg}} \frac{d^2 \hat{\sigma}_{gg}}{d\mathbf{q}_T^2} \sim \delta^2(\mathbf{q}_T) + \alpha_S \int d^2 \mathbf{p}_T \left[\frac{A_g}{\mathbf{p}_T^2} \ln \frac{m_H^2}{\mathbf{p}_T^2} + \frac{B_g}{\mathbf{p}_T^2} \right]_+ \delta^2(\mathbf{q}_T + \mathbf{p}_T) + \dots$$

$$\sim \int \frac{d^2 \mathbf{b}}{(2\pi)^2} e^{i\mathbf{b} \cdot \mathbf{q}_T} \left\{ 1 + \alpha_S \int d^2 \mathbf{p}_T \left[\frac{A_g}{\mathbf{p}_T^2} \ln \frac{m_H^2}{\mathbf{p}_T^2} + \frac{B_g}{\mathbf{p}_T^2} \right] (e^{i\mathbf{b} \cdot \mathbf{p}_T} - 1) + \dots \right\}$$

$$\sim \int \frac{d^2 \mathbf{b}}{(2\pi)^2} e^{i\mathbf{b} \cdot \mathbf{q}_T} \exp \left\{ \alpha_S \int d^2 \mathbf{p}_T \left[\frac{A_g}{\mathbf{p}_T^2} \ln \frac{m_H^2}{\mathbf{p}_T^2} + \frac{B_g}{\mathbf{p}_T^2} \right] (e^{i\mathbf{b} \cdot \mathbf{p}_T} - 1) \right\}$$

Resummation & matching of Higgs q_T

$$d\sigma = \int dx_1 dx_2 f_a(x_1, \mu) f_b(x_2, \mu) d\hat{\sigma}_{ab}(x_1 x_2 s, \mu, \dots)$$

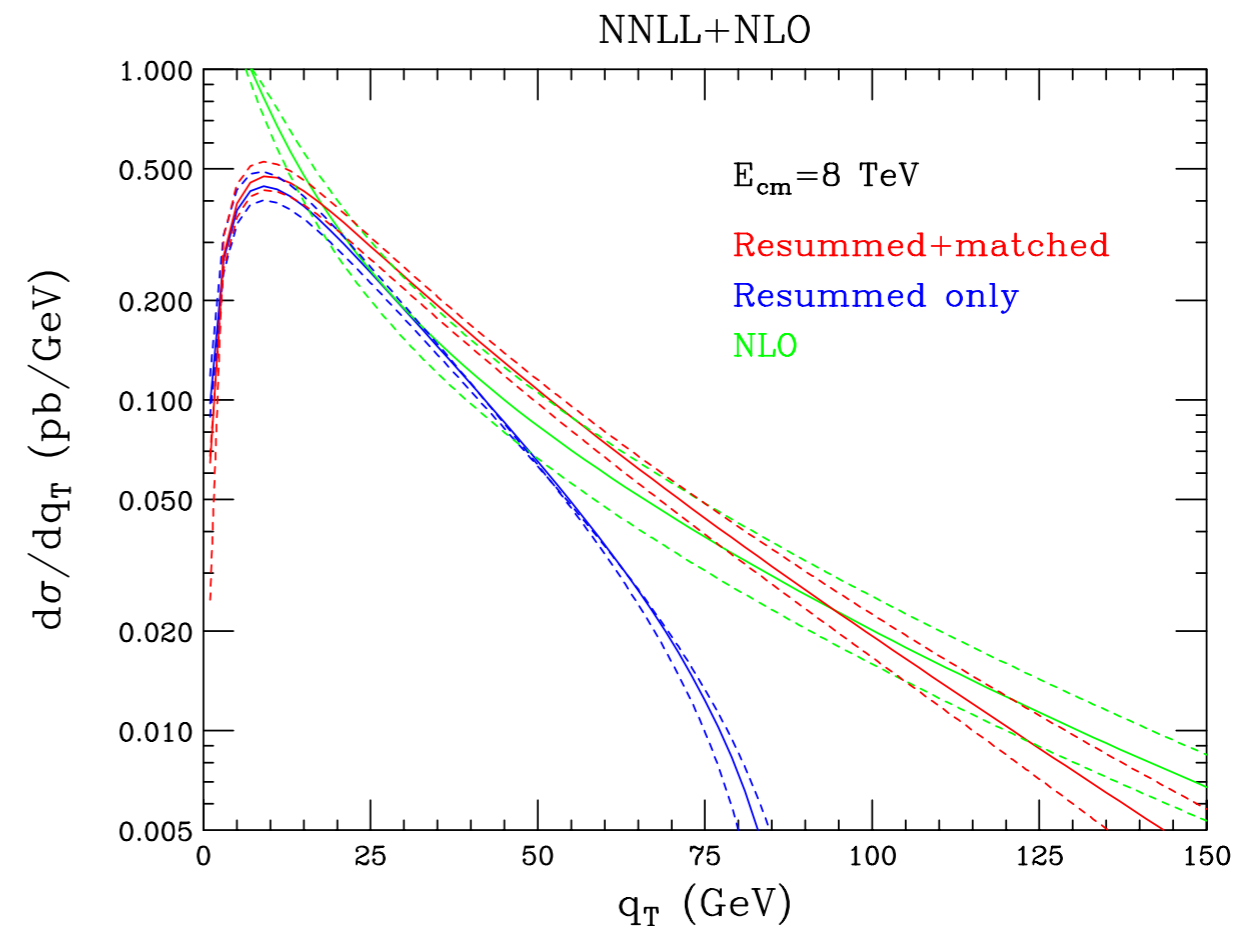
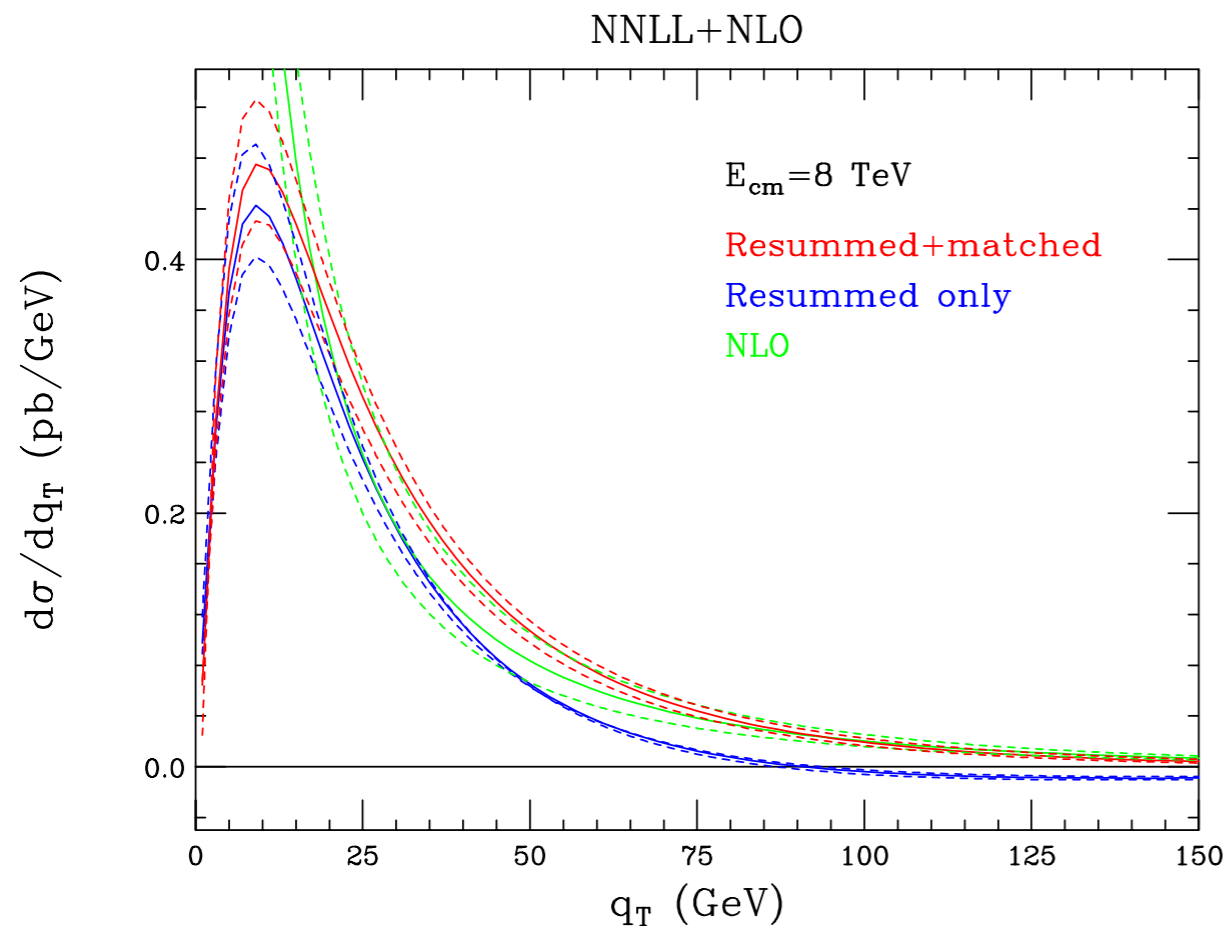
$$\frac{1}{\hat{\sigma}_{gg}} \frac{d^2 \hat{\sigma}_{gg}}{d\mathbf{q}_T^2} \sim \delta^2(\mathbf{q}_T) + \alpha_S \int d^2 \mathbf{p}_T \left[\frac{A_g}{\mathbf{p}_T^2} \ln \frac{m_H^2}{\mathbf{p}_T^2} + \frac{B_g}{\mathbf{p}_T^2} \right]_+ \delta^2(\mathbf{q}_T + \mathbf{p}_T) + \dots$$

$$\sim \int \frac{d^2 \mathbf{b}}{(2\pi)^2} e^{i\mathbf{b} \cdot \mathbf{q}_T} \left\{ 1 + \alpha_S \int d^2 \mathbf{p}_T \left[\frac{A_g}{\mathbf{p}_T^2} \ln \frac{m_H^2}{\mathbf{p}_T^2} + \frac{B_g}{\mathbf{p}_T^2} \right] (e^{i\mathbf{b} \cdot \mathbf{p}_T} - 1) + \dots \right\}$$

$$\sim \int \frac{d^2 \mathbf{b}}{(2\pi)^2} e^{i\mathbf{b} \cdot \mathbf{q}_T} \exp \left\{ \alpha_S \int d^2 \mathbf{p}_T \left[\frac{A_g}{\mathbf{p}_T^2} \ln \frac{m_H^2}{\mathbf{p}_T^2} + \frac{B_g}{\mathbf{p}_T^2} \right] (e^{i\mathbf{b} \cdot \mathbf{p}_T} - 1) \right\}$$

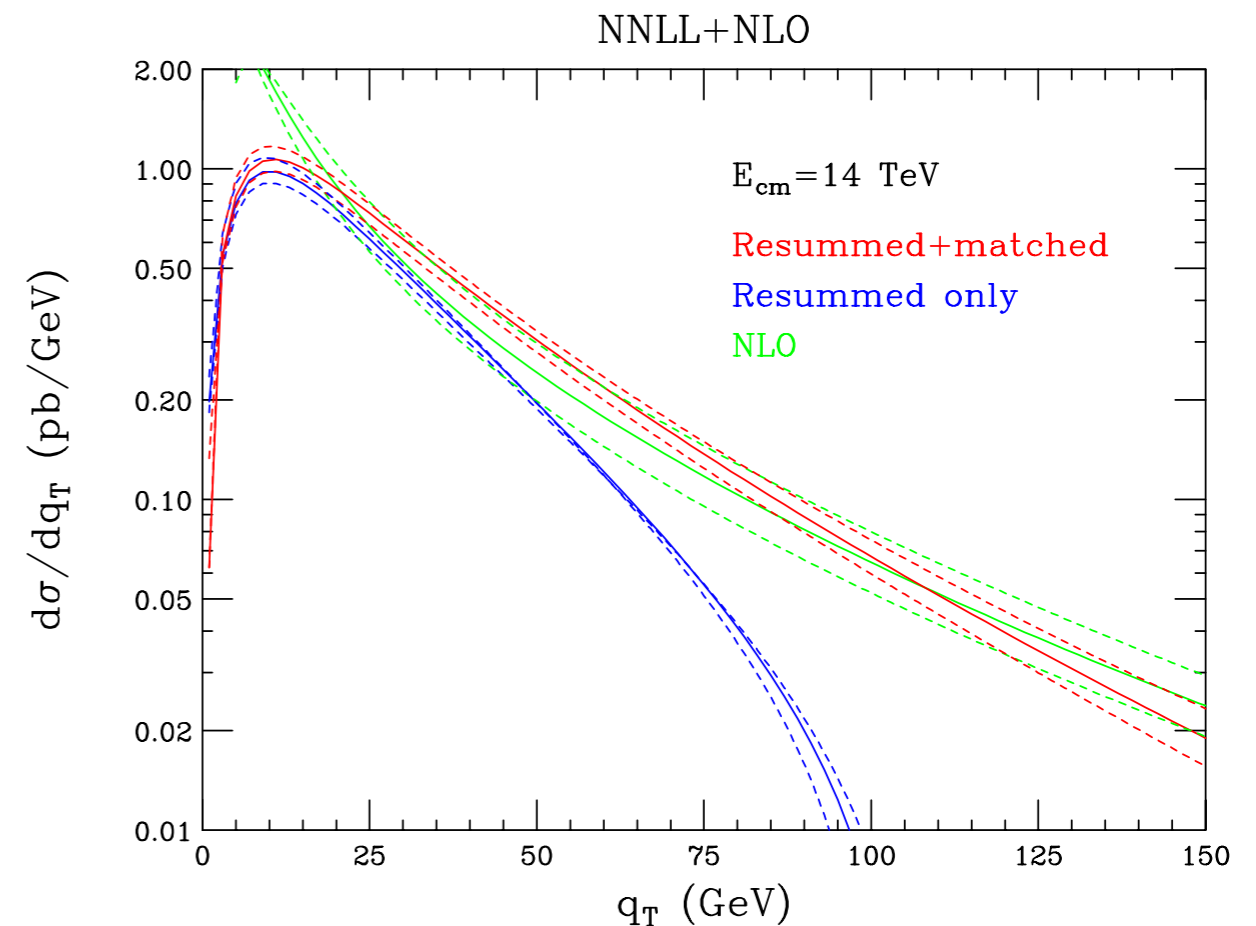
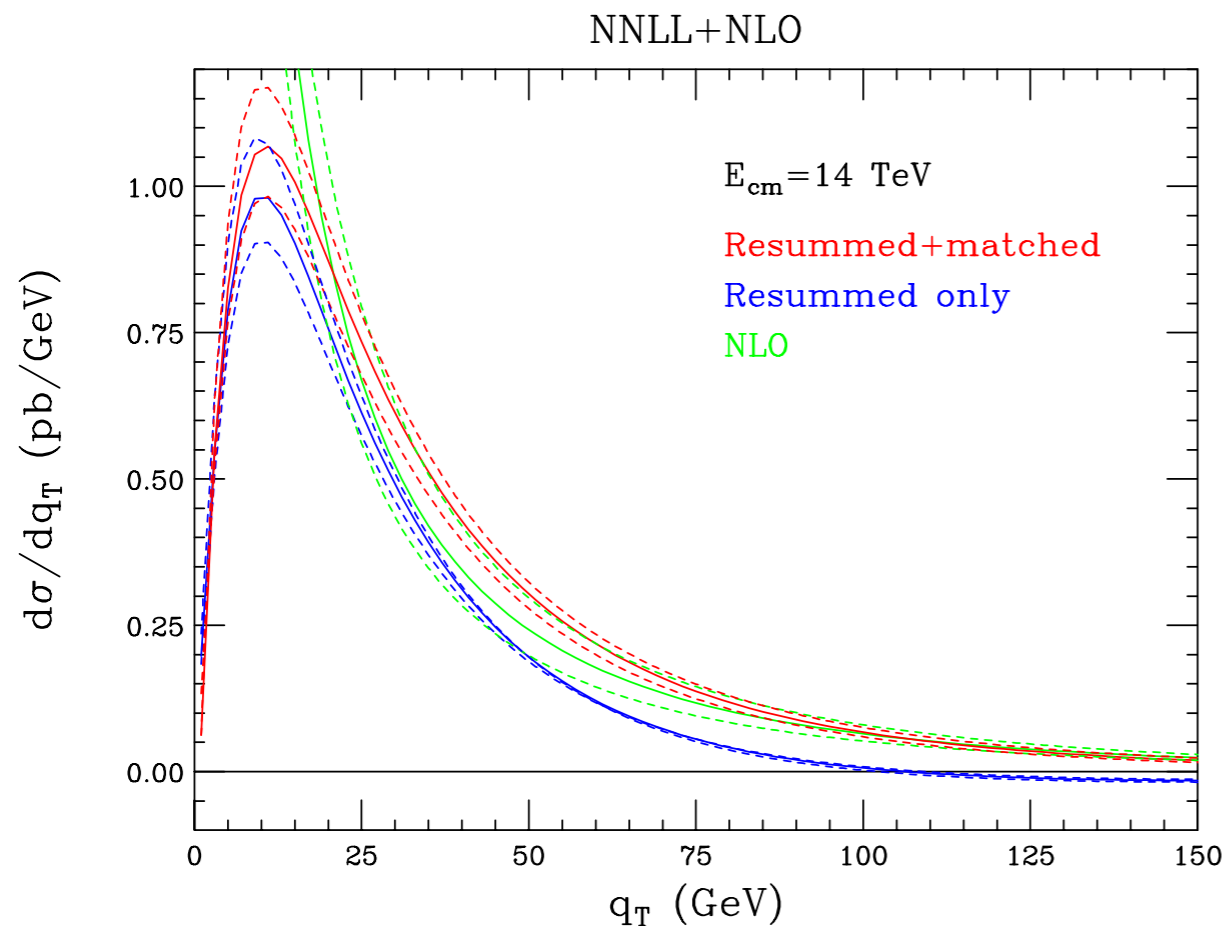
$$\frac{d\sigma}{dq_T} = \left[\frac{d\sigma}{dq_T} \right]_{\text{resum}} - \left[\frac{d\sigma}{dq_T} \right]_{\text{resum,NLO}} + \left[\frac{d\sigma}{dq_T} \right]_{\text{NLO}}$$

Higgs transverse momentum: 8 TeV



- Peak at $\sim 10 \text{ GeV}$: $\log(m_H^2/q_T^2) \sim 5.1$
- Resummation affects spectrum out to larger q_T

Higgs transverse momentum: 14 TeV



- Peak at $\sim 10 \text{ GeV}$: $\log(m_H^2/q_T^2) \sim 5.1$
- Resummation affects spectrum out to larger q_T

Resummation of Higgs q_T

$$d\sigma = \int dx_1 dx_2 f_a(x_1, \mu) f_b(x_2, \mu) d\hat{\sigma}_{ab}(x_1 x_2 s, \mu, \dots)$$

$$\frac{1}{\hat{\sigma}_{gg}} \frac{d^2 \hat{\sigma}_{gg}}{d\mathbf{q}_T^2} \sim \delta^2(\mathbf{q}_T) + \alpha_S \int d^2 \mathbf{p}_T \left[\frac{A_g}{\mathbf{p}_T^2} \ln \frac{m_H^2}{\mathbf{p}_T^2} + \frac{B_g}{\mathbf{p}_T^2} \right]_+ \delta^2(\mathbf{q}_T + \mathbf{p}_T) + \dots$$

$$\sim \int \frac{d^2 \mathbf{b}}{(2\pi)^2} e^{i\mathbf{b} \cdot \mathbf{q}_T} \left\{ 1 + \alpha_S \int d^2 \mathbf{p}_T \left[\frac{A_g}{\mathbf{p}_T^2} \ln \frac{m_H^2}{\mathbf{p}_T^2} + \frac{B_g}{\mathbf{p}_T^2} \right] (e^{i\mathbf{b} \cdot \mathbf{p}_T} - 1) + \dots \right\}$$

$$\sim \int \frac{d^2 \mathbf{b}}{(2\pi)^2} e^{i\mathbf{b} \cdot \mathbf{q}_T} \exp \left\{ \alpha_S \int d^2 \mathbf{p}_T \left[\frac{A_g}{\mathbf{p}_T^2} \ln \frac{m_H^2}{\mathbf{p}_T^2} + \frac{B_g}{\mathbf{p}_T^2} \right] (e^{i\mathbf{b} \cdot \mathbf{p}_T} - 1) \right\}$$

Resummation of Higgs E_T

$$d\sigma = \int dx_1 dx_2 f_a(x_1, \mu) f_b(x_2, \mu) d\hat{\sigma}_{ab}(x_1 x_2 s, \mu, \dots)$$

$$\frac{1}{\hat{\sigma}_{gg}} \frac{d^2 \hat{\sigma}_{gg}}{d\mathbf{q}_T^2} \sim \delta^2(\mathbf{q}_T) + \alpha_S \int d^2 \mathbf{p}_T \left[\frac{A_g}{\mathbf{p}_T^2} \ln \frac{m_H^2}{\mathbf{p}_T^2} + \frac{B_g}{\mathbf{p}_T^2} \right]_+ \delta^2(\mathbf{q}_T + \mathbf{p}_T) + \dots$$

$$\sim \int \frac{d^2 \mathbf{b}}{(2\pi)^2} e^{i\mathbf{b} \cdot \mathbf{q}_T} \left\{ 1 + \alpha_S \int d^2 \mathbf{p}_T \left[\frac{A_g}{\mathbf{p}_T^2} \ln \frac{m_H^2}{\mathbf{p}_T^2} + \frac{B_g}{\mathbf{p}_T^2} \right] (e^{i\mathbf{b} \cdot \mathbf{p}_T} - 1) + \dots \right\}$$

$$\sim \int \frac{d^2 \mathbf{b}}{(2\pi)^2} e^{i\mathbf{b} \cdot \mathbf{q}_T} \exp \left\{ \alpha_S \int d^2 \mathbf{p}_T \left[\frac{A_g}{\mathbf{p}_T^2} \ln \frac{m_H^2}{\mathbf{p}_T^2} + \frac{B_g}{\mathbf{p}_T^2} \right] (e^{i\mathbf{b} \cdot \mathbf{p}_T} - 1) \right\}$$

$$\frac{1}{\hat{\sigma}_{gg}} \frac{d\hat{\sigma}_{gg}}{dE_T} \sim \delta(E_T) + \alpha_S \int d^2 \mathbf{p}_T \left[\frac{A_g}{\mathbf{p}_T^2} \ln \frac{m_H^2}{\mathbf{p}_T^2} + \frac{B_g}{\mathbf{p}_T^2} \right]_+ \delta(E_T - |\mathbf{p}_T|) + \dots$$

$$\sim \int \frac{d\tau}{2\pi} e^{i\tau E_T} \exp \left\{ \alpha_S \int d^2 \mathbf{p}_T \left[\frac{A_g}{\mathbf{p}_T^2} \ln \frac{m_H^2}{\mathbf{p}_T^2} + \frac{B_g}{\mathbf{p}_T^2} \right] (e^{-i\tau |\mathbf{p}_T|} - 1) \right\}$$

Resummation of Higgs E_T

$$\frac{1}{\hat{\sigma}_{gg}} \frac{d\hat{\sigma}_{gg}}{dE_T} \sim \int_{-\infty}^{+\infty} \frac{d\tau}{2\pi} e^{i\tau E_T} \exp \left\{ \alpha_S \int d^2\mathbf{p}_T \left[\frac{A_g}{\mathbf{p}_T^2} \ln \frac{m_H^2}{\mathbf{p}_T^2} + \frac{B_g}{\mathbf{p}_T^2} \right] \left(e^{-i\tau|\mathbf{p}_T|} - 1 \right) \right\}$$

- Defined for $E_T \lesssim 0$

Resummation of Higgs E_T

$$\frac{1}{\hat{\sigma}_{gg}} \frac{d\hat{\sigma}_{gg}}{dE_T} \sim \int_{-\infty}^{+\infty} \frac{d\tau}{2\pi} e^{i\tau E_T} \exp \left\{ \alpha_S \int d^2\mathbf{p}_T \left[\frac{A_g}{\mathbf{p}_T^2} \ln \frac{m_H^2}{\mathbf{p}_T^2} + \frac{B_g}{\mathbf{p}_T^2} \right] \left(e^{-i\tau|\mathbf{p}_T|} - 1 \right) \right\}$$

- Defined for $E_T \leq 0$
- For $E_T < 0$, can close τ -contour in lower half-plane

Resummation of Higgs E_T

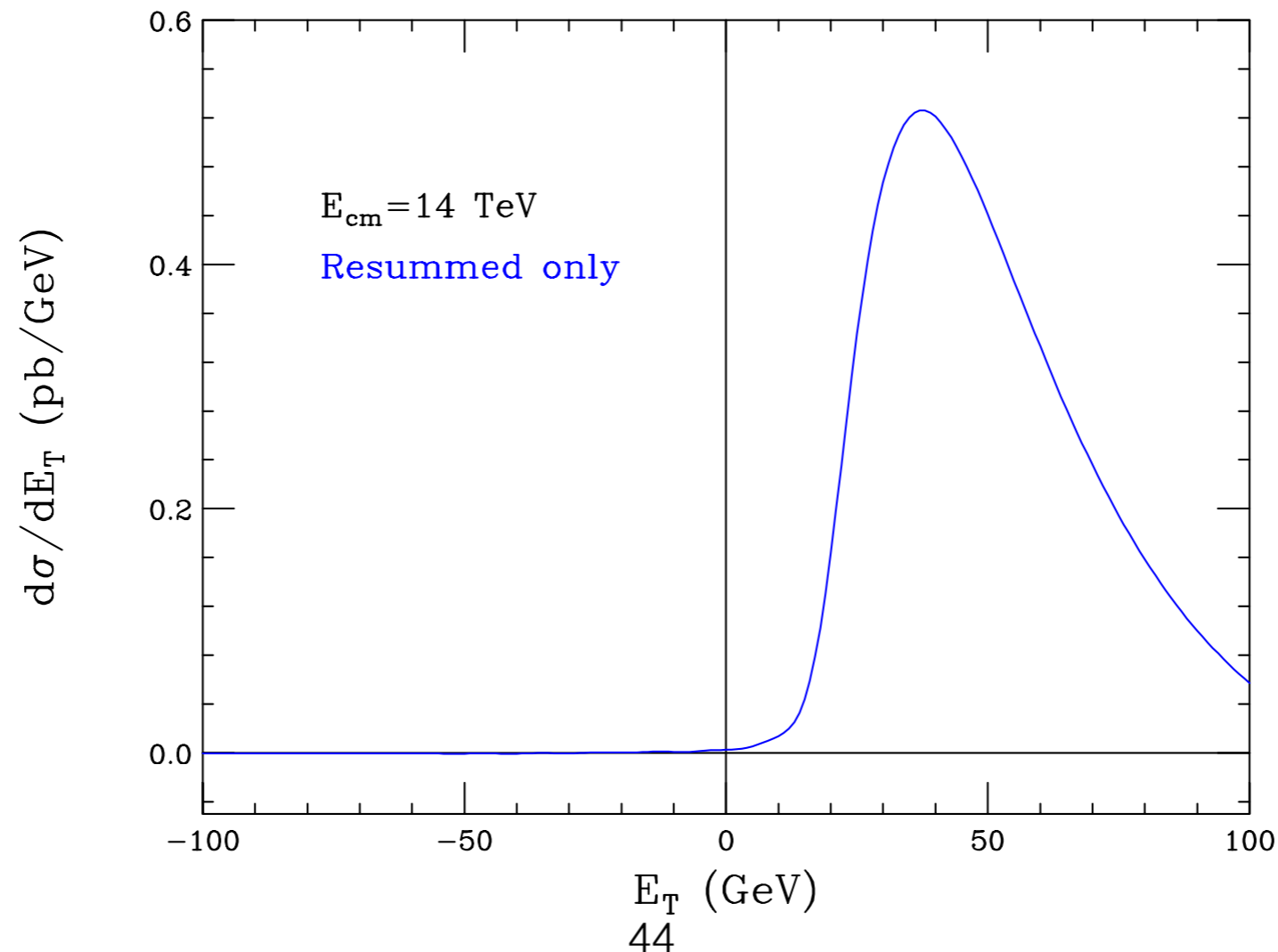
$$\frac{1}{\hat{\sigma}_{gg}} \frac{d\hat{\sigma}_{gg}}{dE_T} \sim \int_{-\infty}^{+\infty} \frac{d\tau}{2\pi} e^{i\tau E_T} \exp \left\{ \alpha_S \int d^2\mathbf{p}_T \left[\frac{A_g}{\mathbf{p}_T^2} \ln \frac{m_H^2}{\mathbf{p}_T^2} + \frac{B_g}{\mathbf{p}_T^2} \right] \left(e^{-i\tau|\mathbf{p}_T|} - 1 \right) \right\}$$

- Defined for $E_T \leq 0$
- For $E_T < 0$, can close τ -contour in lower half-plane
- No singularities in lower half-plane

Resummation of Higgs E_T

$$\frac{1}{\hat{\sigma}_{gg}} \frac{d\hat{\sigma}_{gg}}{dE_T} \sim \int_{-\infty}^{+\infty} \frac{d\tau}{2\pi} e^{i\tau E_T} \exp \left\{ \alpha_S \int d^2\mathbf{p}_T \left[\frac{A_g}{\mathbf{p}_T^2} \ln \frac{m_H^2}{\mathbf{p}_T^2} + \frac{B_g}{\mathbf{p}_T^2} \right] \left(e^{-i\tau|\mathbf{p}_T|} - 1 \right) \right\}$$

- Defined for $E_T \leq 0$
- For $E_T < 0$, can close τ -contour in lower half-plane
- No singularities in lower half-plane



Resummation & matching of Higgs E_T

$$\left[\frac{d\sigma_H}{dQ^2 dE_T} \right]_{\text{res.}} = \frac{1}{2\pi} \sum_{a,b} \int_0^1 dx_1 \int_0^1 dx_2 \int_{-\infty}^{+\infty} d\tau e^{-i\tau E_T} f_{a/h_1}(x_1, \mu) f_{b/h_2}(x_2, \mu) W_{ab}^H(x_1 x_2 s; Q, \tau, \mu)$$

$$W_{ab}^H(s; Q, \tau, \mu) = \int_0^1 dz_1 \int_0^1 dz_2 C_{ga}(\alpha_S(\mu), z_1; \tau, \mu) C_{gb}(\alpha_S(\mu), z_2; \tau, \mu) \delta(Q^2 - z_1 z_2 s) \sigma_{gg}^H(Q, \alpha_S(Q)) S_g(Q, \tau)$$

$$S_g(Q, \tau) = \exp \left\{ -2 \int_0^Q \frac{dq}{q} \left[2A_g(\alpha_S(q)) \ln \frac{Q}{q} + B_g(\alpha_S(q)) \right] (1 - e^{iq\tau}) \right\}$$

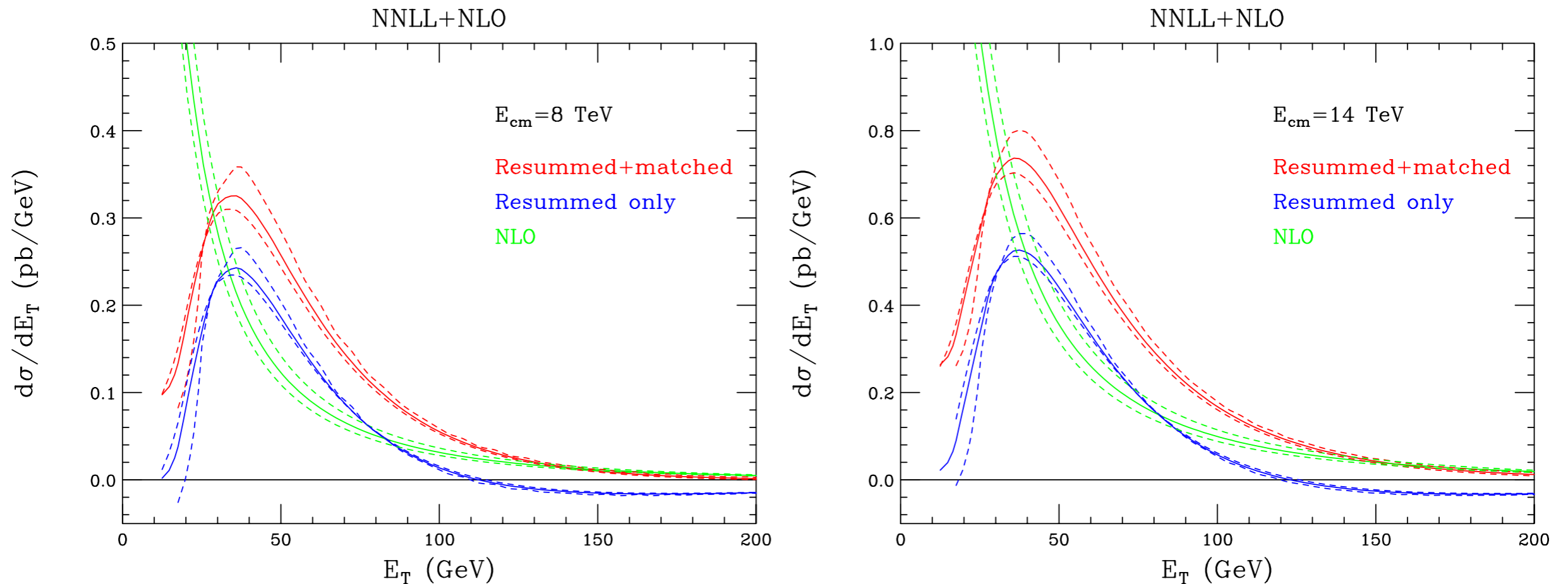
$$A_g(\alpha_S) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi} \right)^n A_g^{(n)},$$

$$B_g(\alpha_S) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi} \right)^n B_g^{(n)},$$

$$C_{ga}(\alpha_S, z) = \delta_{ga} \delta(1-z) + \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi} \right)^n C_{ga}^{(n)}(z)$$

$$\frac{d\sigma_H}{dE_T} = \left[\frac{d\sigma_H}{dE_T} \right]_{\text{resum}} - \left[\frac{d\sigma_H}{dE_T} \right]_{\text{resum,NLO}} + \left[\frac{d\sigma_H}{dE_T} \right]_{\text{NLO}}$$

Transverse energy distribution



- Peak at $\sim 35 \text{ GeV}$: $\log(m_H^2/E_T^2) \sim 2.6$
- Resummation affects spectrum out to much larger E_T
- Unlike q_T , the **Underlying Event** also contributes...

Summary

- QCD factorization allows precise predictions for LHC
 - ▶ Scale dependence is a (rough) guide to precision
- Higgs ggF cross section at 13 TeV is still very uncertain
 - ▶ My estimate: $\sigma_{\text{ggF}}(13 \text{ TeV}) = 53 \pm 11 \text{ pb}$
- Higgs transverse momentum resummed to NNLL+NLO
 - ▶ Peak $q_T \sim 10 \text{ GeV}$, independent of energy
- Radiated transverse energy resummed to (N)NLL+NLO
 - ▶ Peak $E_T \sim 35 \text{ GeV}$ in associated transverse energy
 - ▶ Contribution from Underlying Event to be considered ...