Topics in Quantum Field Theory and LHC phenomenology

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My goal in this talk

The goal of this talk is to introduce myself by telling you a few things about my research. I am a theoretical particle physicist with strong inclination to work on phenomenological problems. I also like things that can be done from first principles; this lead to my long-term interest in perturbative computations in Quantum Field Theory.

One of the things that I would like to illustrate with this talk is not only the ``depth'' of what I do, but also the ``breadth''. Because of that, I have chosen several topics that I want to discuss. They are:

1) Are there positronia contributions to electron anomalous magnetic moment ?

2) Is it possible to constrain the Higgs boson width at the LHC ?

3) Two-loop helicity amplitudes for off-shell production of vector bosons in hadron collisions;

4) Single top production at the LHC through NNLO QCD.

Are there positronia contributions to electron anomalous magnetic moment ?

In collaboration with A. Vainshtein and M. Voloshin

Electron anomalous magnetic moment

In any problem in Quantum Physics eigenstates of a Hamiltonian form a Hilbert space. In Quantum Field Theory, Hamiltonians are too complicated to solve for a Hilbert space exactly; as the result, we resort to perturbative methods.

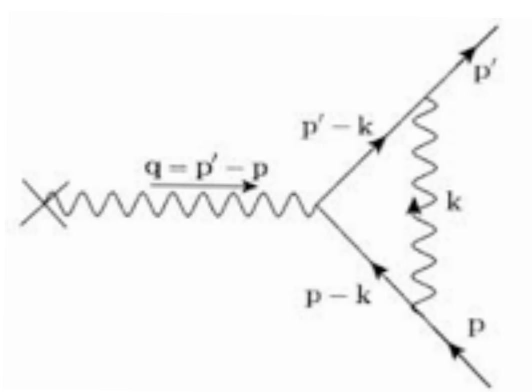
For example, in QED, the Hilbert space that we work with in perturbation theory consists of multi-particle states with free , non-interacting electrons, positrons and photons. However, it is clear that such an a Hilbert space is not be complete. Indeed, electron and positron can form bound states -- positronia-- which are clearly not part of the perturbative Hilbert space (in principle, these bound states decay, so strictly speaking they are not part of the Hilbert space, but we will ignore this subtlety).

The question that we would like to address is why no mistakes are made by neglecting contributions of bound states in the vast majority of perturbative computations ?

The discussion of this question is motivated by recent claims that positronia resonances do contribute to electron anomalous magnetic moment.

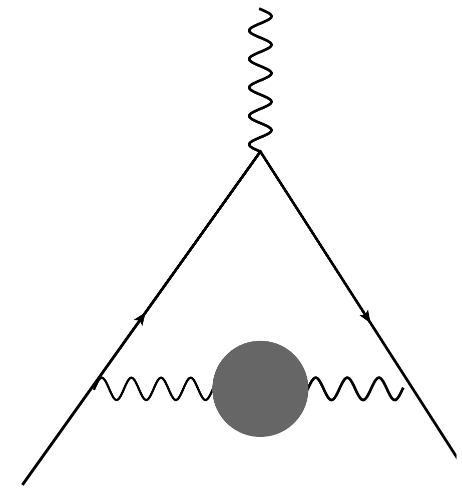
Electron anomalous magnetic moment

Electron anomalous magnetic moment is induced by quantum fluctuations of electromagnetic fields in the vacuum. The one-loop result is well-known.

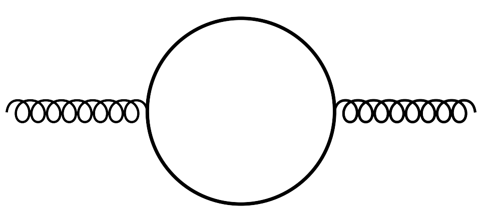


$$\vec{\mu} = g\mu_0\vec{s}, \qquad \mu_0 = \frac{e\hbar}{2mc}$$

$$a_\mu = \frac{\alpha}{2\pi} = \frac{g-2}{2}$$



Next, consider higher-order corrections to g-2, in particular those that are related to photon vacuum polarization contribution. To calculate those, write the vacuum polarization through a single form factor and use dispersion relations to compute it.

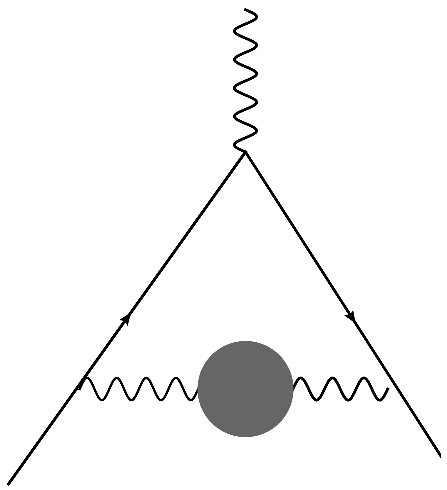


$$= (-g^{\mu\nu} + q^\mu q^\nu)\,\Pi(q^2), \qquad \Pi(q^2) = \frac{1}{\pi}\int \frac{\mathrm{d}s\ \mathrm{Im}\Pi(s)\ q^2}{(s-q^2-i0)s}$$

The vacuum polarization contribution to g-2 can then be computed as an integral over one-loop correction to the anomalous magnetic moment that originates from a massive vector boson exchange.

Electron anomalous magnetic moment

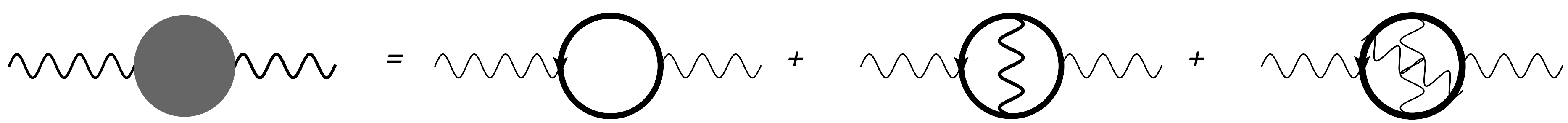
A simple computation gives



$$a_e = \frac{g-2}{2} = \frac{\alpha}{\pi^2} \int\limits_{s_0}^{\infty} \frac{\mathrm{d}s}{s}\, \mathrm{Im}\Pi(s)\; K(s).$$

$$K(s) = \int\limits_0^1 \frac{\mathrm{d}x\; x^2(1-x)}{x^2 + (1-x)s/m_e^2}.$$

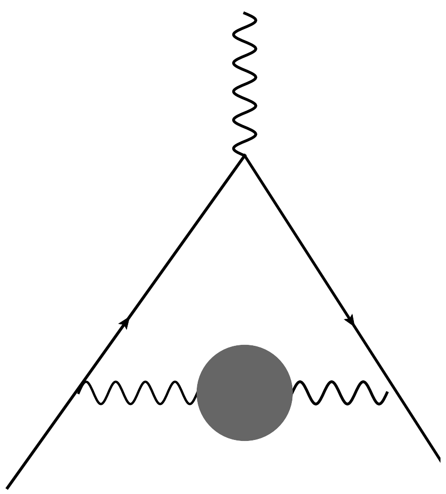
So, we find that K(s) is completely determined by one-loop QED and Im[Pi(s)] needs to be calculated. In principle, one can compute it in perturbation theory through conventional perturbative expansion



It is then easy to see that all these diagrams (and the majority of all perturbative diagrams) have imaginary parts that vanish for s < 4m^2. However, if we consider positronia contributions to Im[Pi(s)], we recognize that, because of binding energies, their contributions to Im[Pi(s)] originate from s < 4m^2. Therefore, it appears that when polarization operator is computed in perturbation theory, contribution of positronia bound states gets lost.

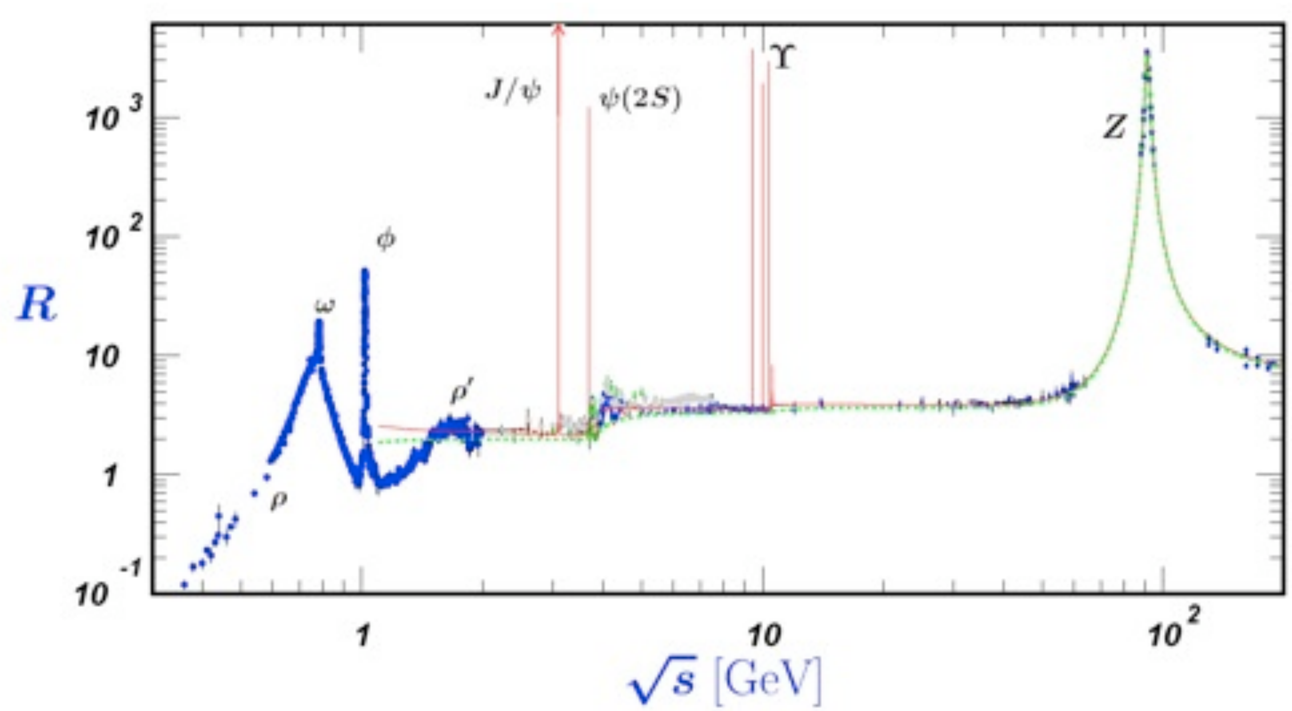
Electron anomalous magnetic moment

If we do not look ``inside" the positronium atom, and just go by its quantum numbers, we would say that positronium (ortho) is a vector meson, similar to rho, omega, phi etc. mesons. And there is a well-known example when we account for contributions of vector mesons to electron anomalous magnetic moment -- it is hadronic vacuum polarization contributions to g-2. Since there is not much difference between the rho-meson and the ortho-positronium, it is not clear why do we include the first and not the second in our calculations. Perhaps the positronium contribution is just too small to even talk about it?



$$\mathrm{Im}\Pi(s)|_{s=m_V^2} \sim \sigma_{e^+e^-\to V}(s) = \frac{12\pi^2\Gamma_{V\to e^+e^-}}{m_V}\delta(s-m_V^2)$$

$$a_e = \frac{g-2}{2} = \frac{\alpha}{\pi^2}\int_{s_0}^{\infty}\frac{\mathrm{d}s}{s}\,\mathrm{Im}\Pi(s)\,K(s).$$



Electron anomalous magnetic moment

Positronia contributions are definitely not large, but they appear in a well-defined order of perturbative expansion. To see this, we write a formula for the imaginary part of the photon vacuum polarization operator that contains both positronia poles and continuum contributions

$$a_e = \frac{g-2}{2} = \frac{\alpha}{\pi^2} \int_{s_0}^{\infty} \frac{\mathrm{d}s}{s}\, \mathrm{Im}\Pi(s)\; K(s).$$

$$|\Psi_n(0)|^2 = \frac{m_e^3 \alpha^3}{8\pi n^3}$$

$$\mathrm{Im}\Pi(s) = 16\pi^2 \alpha \sum \frac{|\Psi_n(0)|^2}{M_n} \delta(s - M_n^2) + \theta(s - 4m^2) C(s).$$

$$a_e^{\mathrm{Pos}} = \frac{\alpha^5}{4\pi} \zeta_3 \left(8 \ln 2 - \frac{11}{2} \right).$$

It follows that the contribution of positronia bound states to electron g-2 appears in fifth order in the perturbative expansion. Fifth order contribution to g-2 was recently calculated by Kinoshita et al. and the positronia contributions were not included in those calculations. A possible interpretation of this observation is that the fifth order results by Kinoshita et al. are incomplete and that the positronia contributions shown above need to be added there... Another possibility is that positronia contributions are (somehow) already included in perturbative g-2 computations, so that they do not need to be accounted for separately.

Which of the two interpretations is the correct one?

Electron anomalous magnetic moment

Our main reason for calling positronia contributions ``new'' is related to the fact that they appear as sub-threshold contributions to the imaginary part and sub-threshold contributions do not appear in any fixed order in perturbation theory. Hence, they are ``non-perturbative''.

However, postronia bound states is not the only manifestation of long-distance, non-perturbative nature of Coulomb interactions at small relative velocities; in fact, just above the threshold for electron-positron pair production the spectral density is distorted very strongly.

$$\Pi(s) = \frac{2\pi\alpha}{m_e^2} G(0,0,E) + \mathrm{const}, \qquad G(\vec{x},\vec{y},E) = \langle \vec{x} | (H_c - E)^{-1} | \vec{y} \rangle$$

$$C(s) = \frac{2\pi\alpha}{m_e^2} \mathrm{Im} G(0,0,E) = \frac{\pi\alpha^2}{2} \frac{1}{1 - e^{-\pi\alpha/\beta}}.$$

In perturbation theory, the expansion of Pi(s) in the fine structure constant even above the threshold is not convergent if the relative velocity of electron and positron is sufficiently small; this means that in addition to positronia, there is another non-perturbative contribution of the continuum:

$$C(s)|_{\mathrm{PT}} = \frac{\pi\alpha^2}{2} \left(\frac{\beta}{\pi\alpha} + \frac{1}{2} + \frac{\pi\alpha}{12\beta} - \frac{1}{720} \left(\frac{\pi\alpha}{\beta} \right)^3 + \ldots \right).$$

Both, positronium and continuum contributions need to be computed together to understand if non-pertubative QED physics affects predictions for g-2.

Electron anomalous magnetic moment

Let us define ``non-perturbative" more accurately. A possible definition is based on the analytic properties of the result w.r.t. fine structure constant. Perturbation theory by construction is based on the Taylor expansion in the fine structure constant. But non-perturbative results do not have to be Taylor-expandable at $\alpha = 0$. Indeed, bound states (positronia) exist only for attractive interaction $\alpha > 0$; while for repulsive interactions there should be no bound states in the spectrum at the first place.

To make this property explicit, we re-write positronium contribution in the following way

$$a_e^{\rm Pos} = \frac{\alpha^5 + |\alpha|^5}{8\pi}\zeta_3\left(8\ln 2 - \frac{11}{2}\right).$$

The non-analyticity in the fine structure constant is apparent. The above result is peculiar; it shows that one-half of the positronium contribution is, in fact, perturbative while the other half is not.

Next, we need to consider continuum contribution to g-2. It can be cast into the following form

$$a_e^{\rm cont} = \frac{\alpha^2}{\pi}K(4m_e^2)I(\alpha,\beta_0), \quad I(\alpha,\beta_0) = \int\limits_0^{\beta_0} {\rm d}\beta\, \frac{\alpha\beta}{1-e^{-\pi\alpha/\beta}}.$$

The upper cut-off is introduced to ensure validity of non-relativistic approximation.

Electron anomalous magnetic moment

To calculate $I(\alpha,\beta)$, we split it into two parts; one part is responsible for the threshold behavior and the other part responsible for the ``large-beta" (perturbative) behavior

$$I(\alpha,\beta) = I_1(\alpha,\beta_0) + I_2(\alpha,\beta_0)$$

$$I_1 = \int_0^{\beta_0} \mathrm{d}\beta\beta\alpha \left(\frac{\beta}{\pi\alpha} + \frac{1}{2} + \frac{\pi\alpha}{12\beta}\right); \quad I_2 = \int_0^{\beta_0\to\infty} \mathrm{d}\beta\beta\alpha \left(\frac{1}{1-e^{-\pi\alpha/\beta}} - \frac{\beta}{\pi\alpha} - \frac{1}{2} - \frac{\pi\alpha}{12\beta}\right).$$

The first integral is obviously analytic; the second integral is independent of the velocity cut-off; its computation gives

$$I_2 = -\frac{|\alpha|^3\zeta_3}{8}$$

which implies that I2 is totally non-analytic. Threshold continuum contribution to g-2 reads

$$a_e^{\mathrm{cont,na}} = -\frac{|\alpha|^5}{8\pi}\zeta_3\left(8\ln 2 - \frac{11}{2}\right)$$

Combining positronia and threshold continuum contributions to g-2, we find the analytic result. Since the result is analytic in the fine structure constant, one can claim that it is not a new contribution in a sense that it is already contained in conventional perturbative computations

$$a_e^{\mathrm{Pos}} + a_e^{\mathrm{cont,na}} = -\frac{\alpha^5}{8\pi}\zeta_3\left(8\ln 2 - \frac{11}{2}\right)$$

Electron anomalous magnetic moment

There are different ways to prove this assertion. One option is reviewed below. The idea is to compute the contribution of the threshold region as a whole, without splitting it into positroniua and non-perturbative continuum contributions. This is done using the dispersion representation for the non-relativistic Green's function

$$a_e = \frac{2\alpha^2 K(4m^2)}{\pi m^3} \int \mathrm{d}E \; \mathrm{Im} G_E(0,0,E). \qquad G(0,0,E) = \frac{1}{\pi} \int\limits_{E_1}^{\infty} \frac{\mathrm{d}E' \mathrm{Im} G(0,0,E')}{E' - E - i0}, \qquad E_1 = -\frac{m_e \alpha^2}{4}.$$

$$\int \mathrm{d}E' \; G(0,0,E') = \lim_{E \to -\infty} \left(-\pi E G(0,0,E)\right).$$

$$\boxed{a_e = \frac{2\alpha^2 K(4m_e^2)}{\pi m_e^3} \times \lim_{E \to -\infty} \left[-\pi E G(0,0,E)\right].}$$

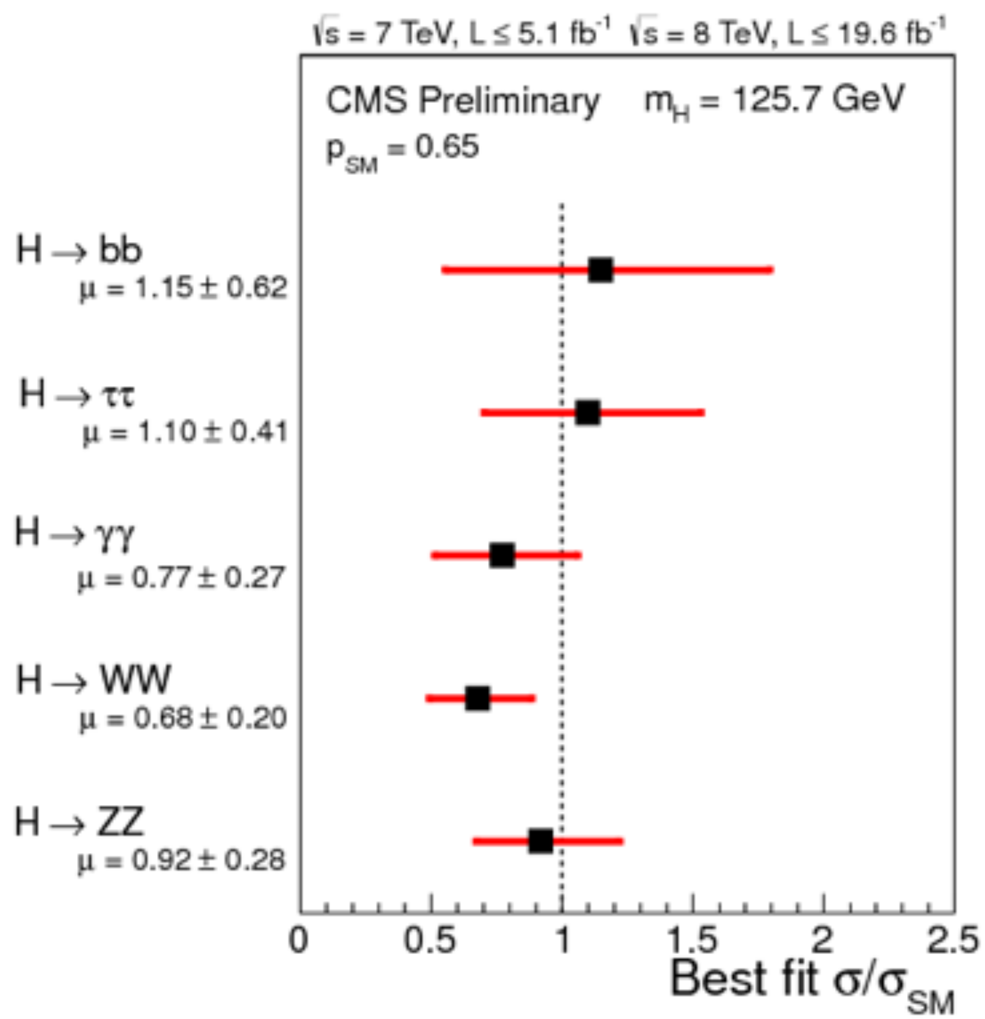
This formula is important since it shows that the anomalous magnetic moment is insensitive to (non-perturbative) threshold region, thanks to analyticity of the Green's function. Although fine details of the spectral density are highly non-trivial, the resulting integral is simple and ``perturbative" (analytic) in the fine structure constant. Hence, no new contributions due to bound states and other non-perturbative phenomena can appear in QED computations for electron g-2; perturbative calculations give complete result in that case.

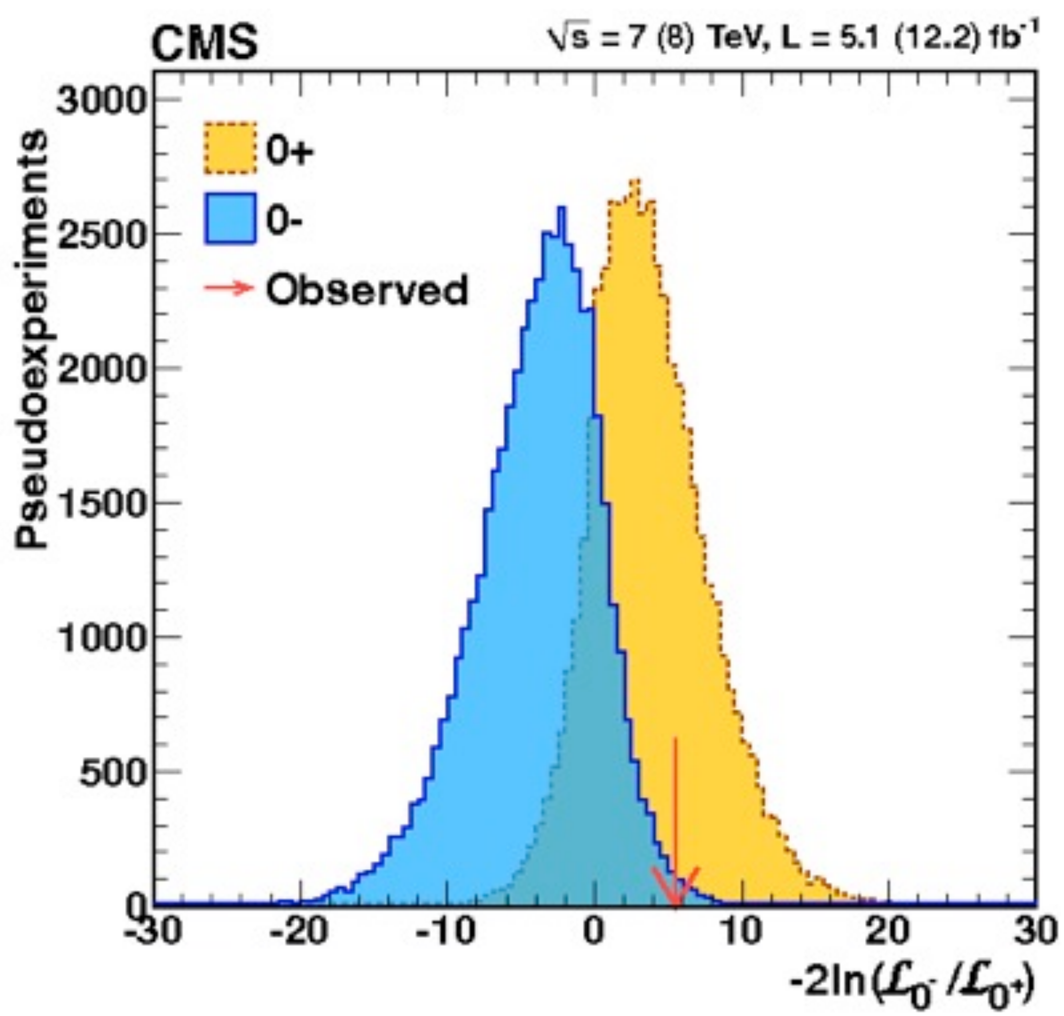
Is it possible to constrain the Higgs boson width at the LHC ?

In collaboration with F. Caola

Measuring the Higgs width at the LHC

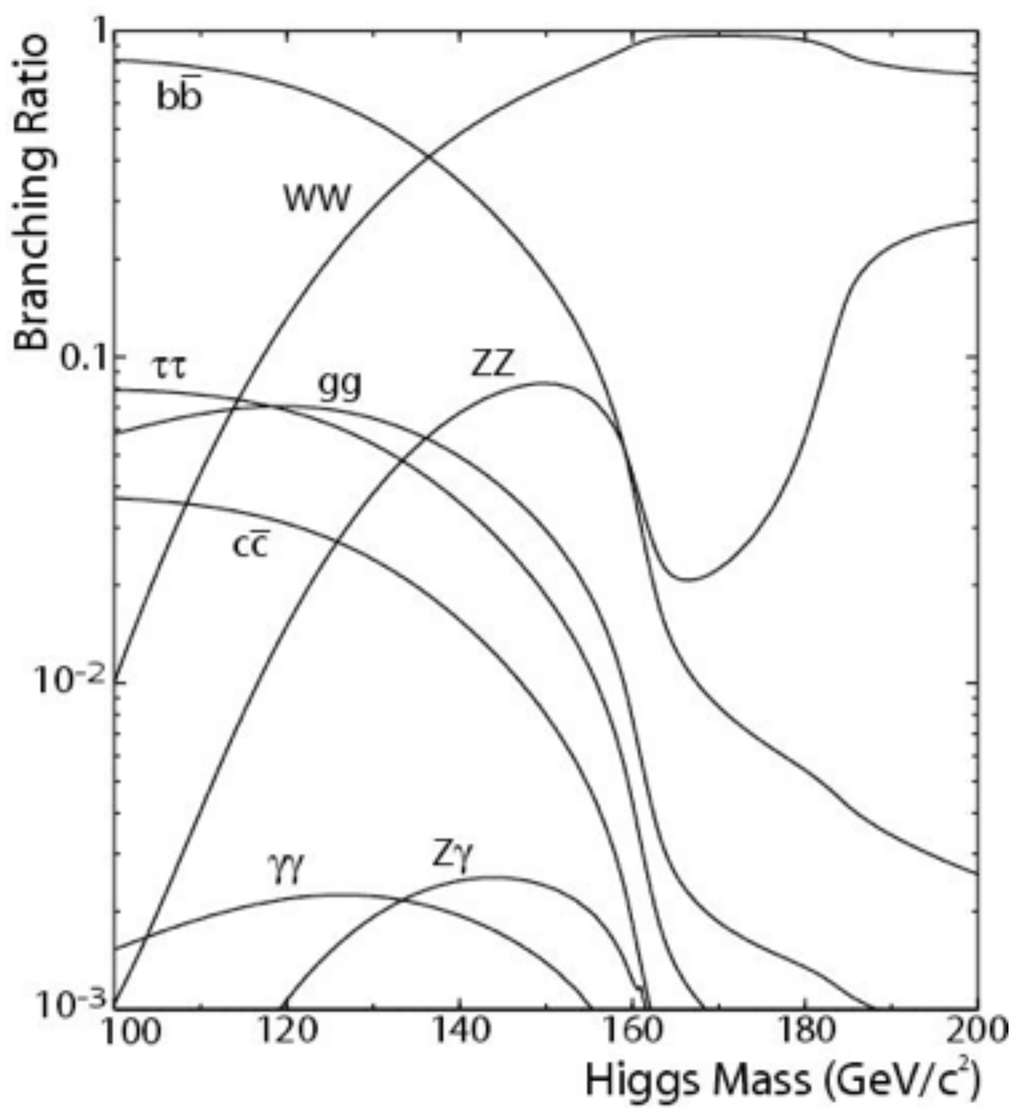
The new particle discovered at the LHC appears to be the Higgs boson of the Standard Model. Indeed, its production and decay rates, its spin and parity as well as its mass are all consistent with the Standard Model expectations. Further studies of these quantities with higher precision are definitely warranted but there are a few ``big items" in Higgs physics that, at the moment, we do not know much about. One such item is the Higgs boson width, the other one is the Higgs boson self-coupling and yet another one is the Higgs coupling to light fermions. In what follows, I will focus on one of these ``big items" -- the Higgs boson width.

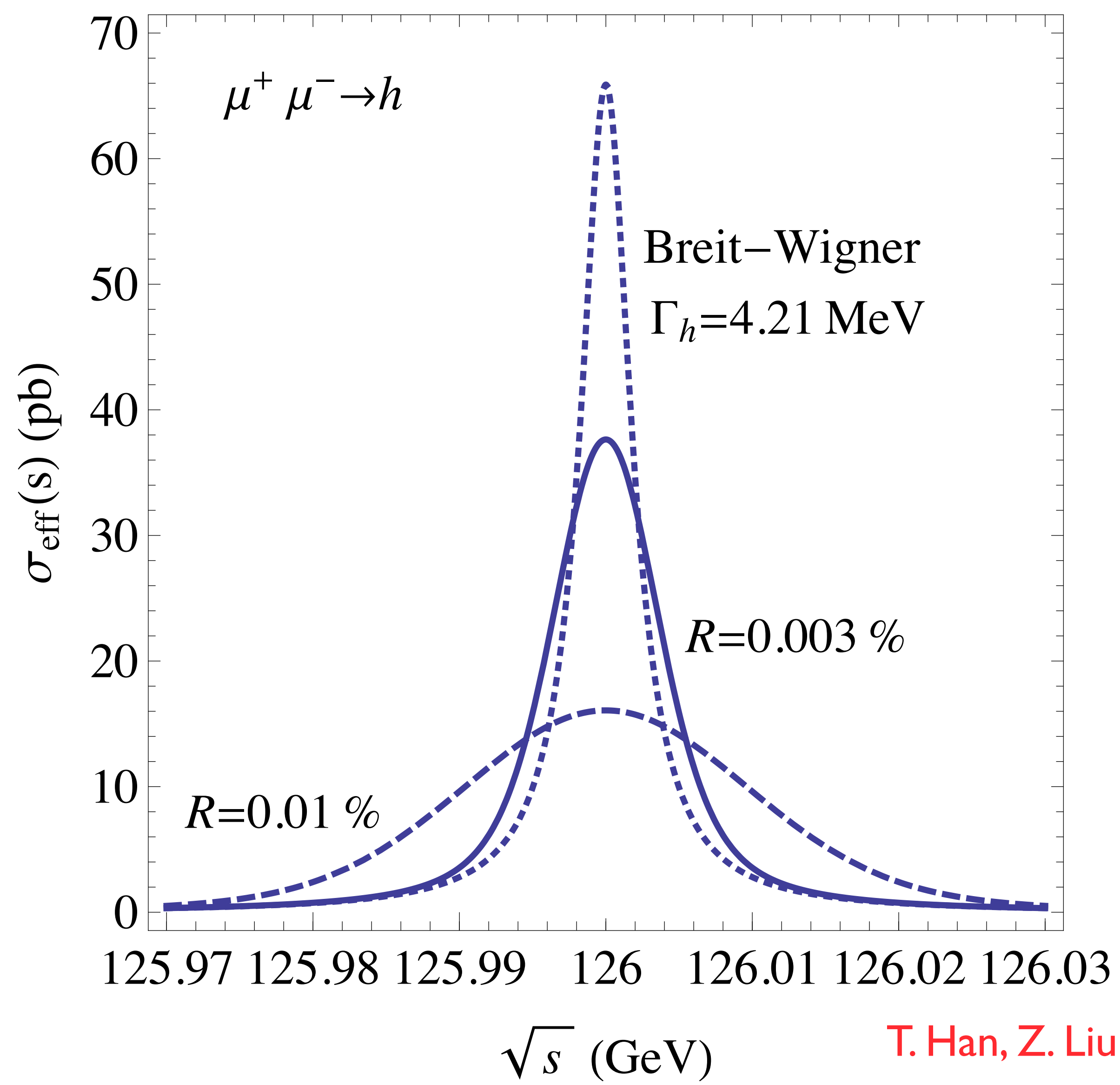




Measuring the Higgs width at the LHC

In the Standard Model, the width of 126 GeV Higgs boson is extremely small, it is just 4.2 MeV. It is almost impossible to measure it directly at any collider, with the exception of the muon one. At hadron and electron colliders, one can measure Higgs branching to invisible final state in the Bjorken process and then infer the total width from there. At an e+e- and muon collider, a measurement of the Higgs width with a few percent precision can probably be achieved.



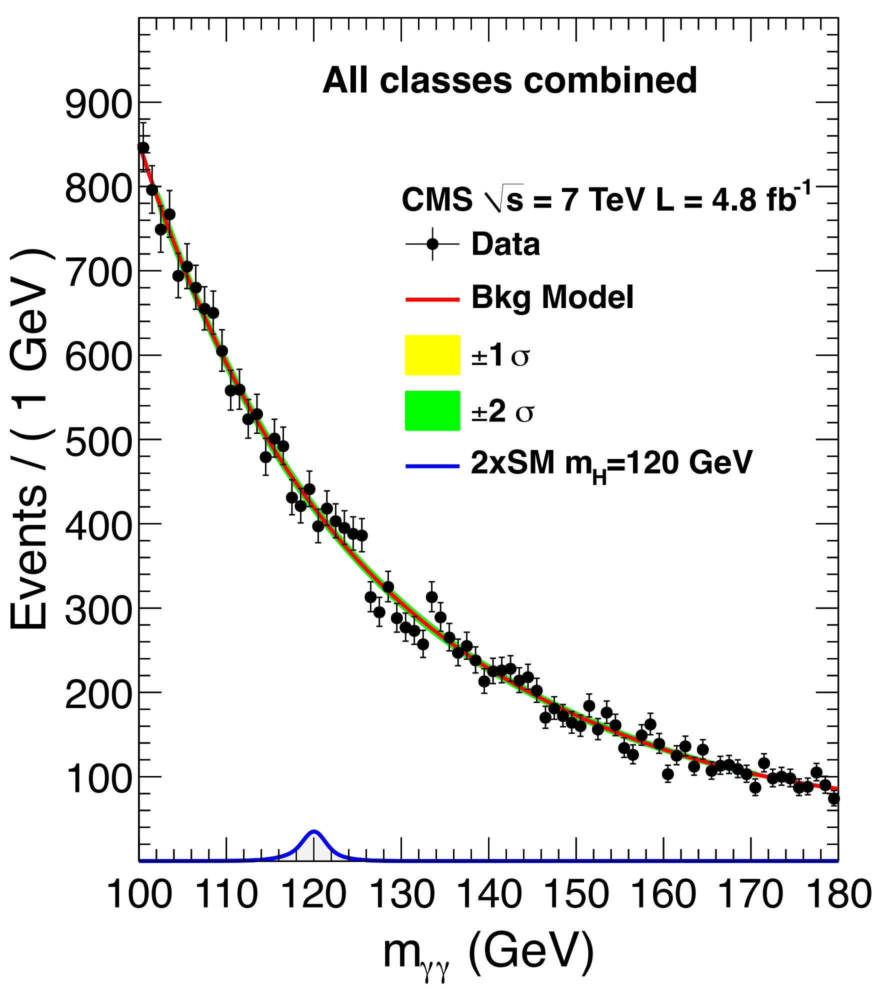


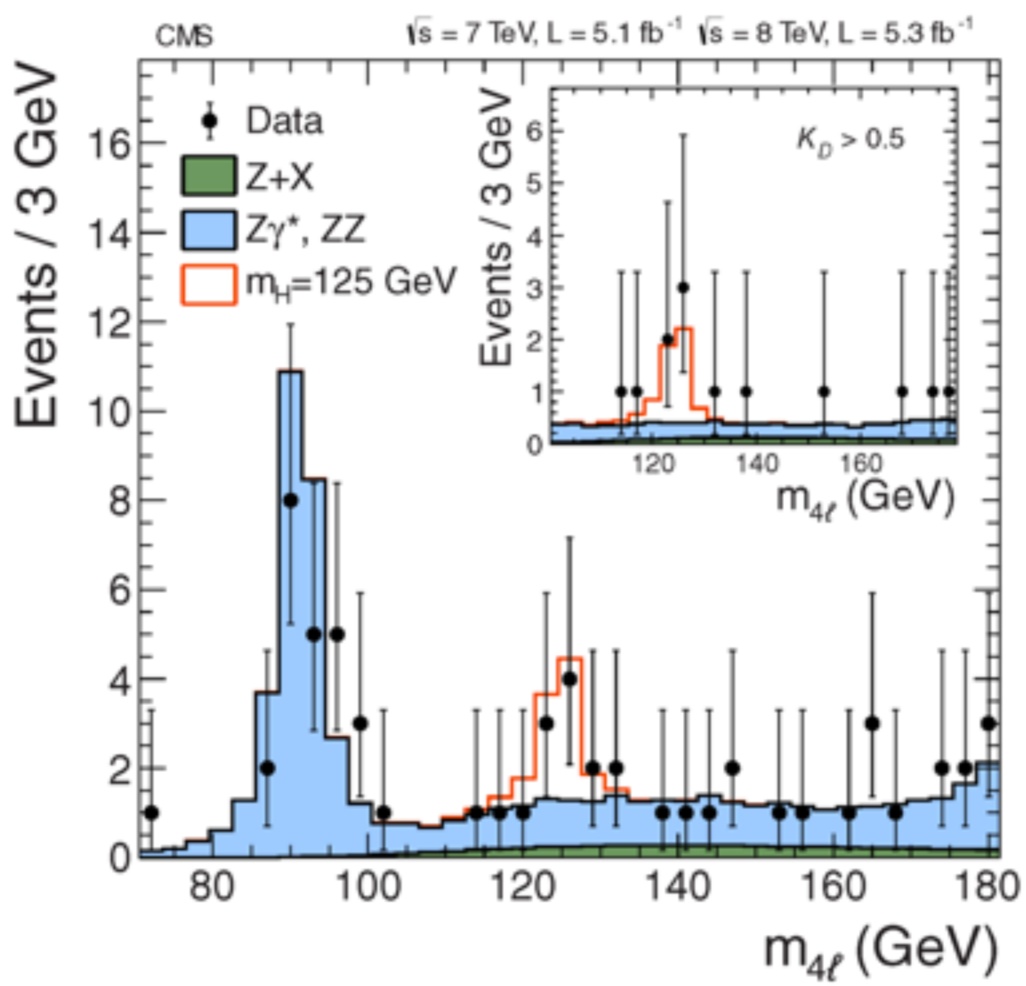
T. Han, Z. Liu

Measuring the Higgs width at the LHC

To measure the width directly, we typically study invariant mass distribution of the resonance decay products in the vicinity of a resonance and fit it to Breit-Wigner formula. Unfortunately, since the invariant mass of the Higgs decay products can be reconstructed with poor (for these purposes) resolution, the LHC is only sensitive to the Higgs width if it is in a few GeV range. The current direct limit on the Higgs width by the CMS collaboration is $\Gamma_H < 3.4$ GeV . The ultimate reach is estimated to be between 1 and 3 GeV.

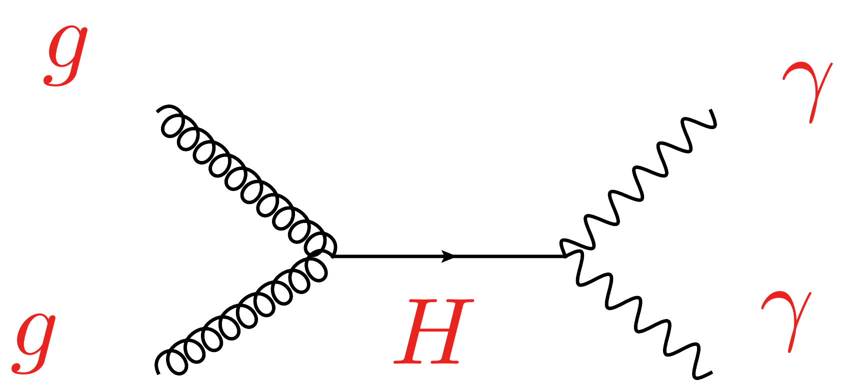
To get into an MeV range for the Higgs width measurement, we need to improve the sensitivity of our methods by a factor of a thousand ! Because of that, measuring the Higgs width at the LHC with any degree of precision was always considered an utopian endeavor.





Measuring the Higgs width at the LHC

The Higgs boson is a narrow resonance. It is mainly produced on-shell and this leads to a relation between production rates, Higgs couplings and the width.



$$\sigma_{i\to H\to f} \sim \frac{g_i^2 g_f^2}{\Gamma_H}$$

Unfortunately, such a relation makes it impossible to extract the couplings and the width separately from the measured on-shell cross-sections. Indeed, any on-shell cross-section is invariant under a simultaneous re-scaling of the Higgs couplings and the Higgs width

$$g \to \xi g, \quad \Gamma_H \to \xi^4 \Gamma_H \quad \Rightarrow \quad \sigma_H \to \sigma_H$$

Since the width of the Higgs boson is practically unconstrained, extraction of the Higgs couplings from production/decay rates suffers from significant ambiguity.

To resolve the ambiguity, we need to either measure the width of the Higgs boson or the Higgs couplings independently of each other.

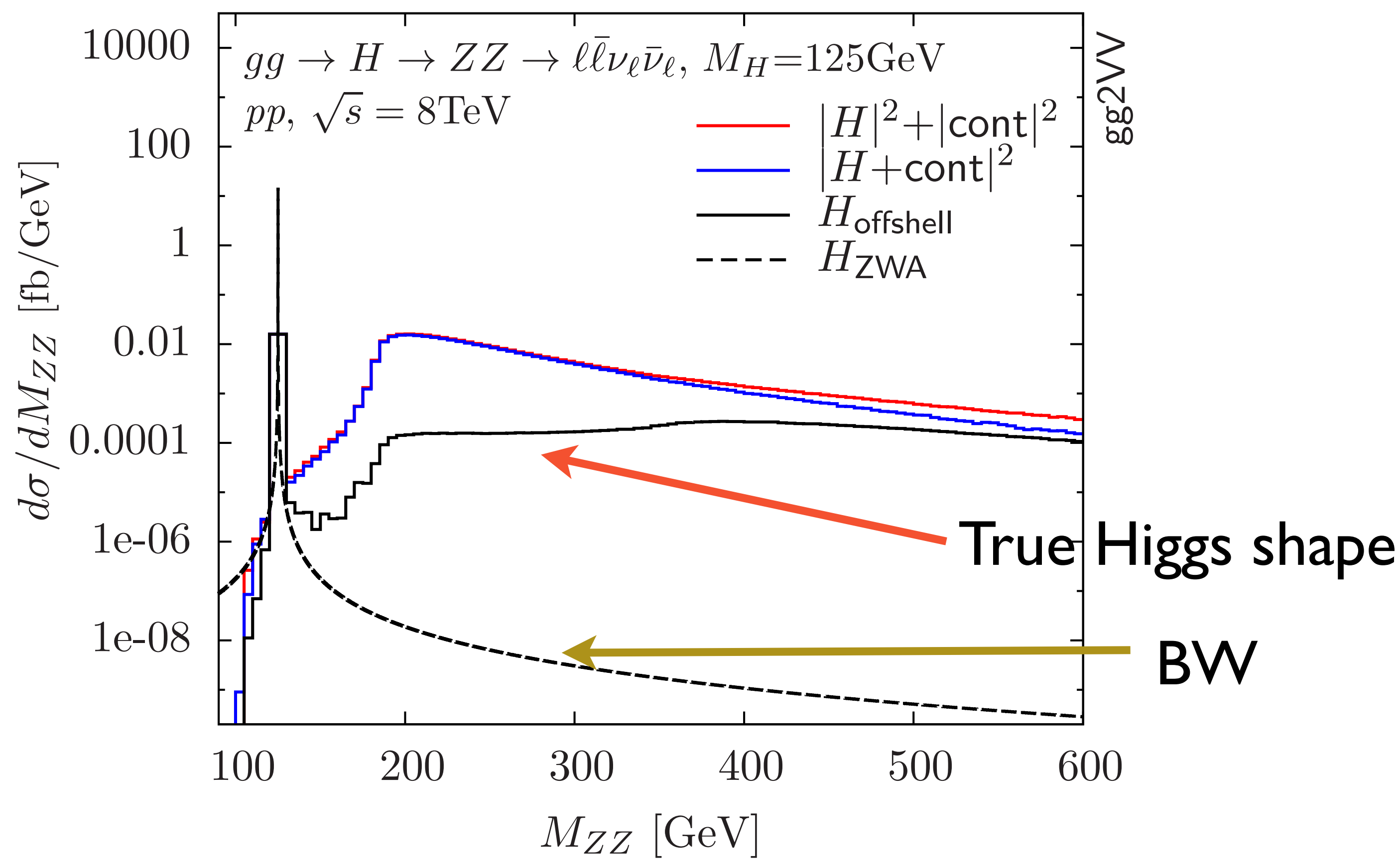
Measuring the Higgs width at the LHC

One can try to measure couplings of the Higgs boson when it is produced off-shell. The off-shell cross-section is proportional to couplings and is independent of the width, resolving the width/couplings ambiguity.

$$\sigma_{i \to H \to f} \sim \int \frac{\mathrm{d}s \quad g_i^2 g_f^2}{(s - m_h)^2 + m_h^2 \Gamma_h^2} \Big|_{s \gg m_h^2} \to \frac{g_i^2 g_f^2}{s}$$

The immediate problem with this idea is that off-shell contribution to Higgs boson production is expected to be extremely small.

However, Kauer and Passarino pointed out that a significant enhancement in the off-shell Higgs production rate exists, making the invariant mass distribution very different from the Breit-Wigner expectation.



Kauer, Passarino

Measuring the Higgs width at the LHC

One can use this enhancement in the off-shell Higgs production to resolve couplings/width degeneracy. The cleanest final state is ZZ (four leptons), so it is natural to look there.

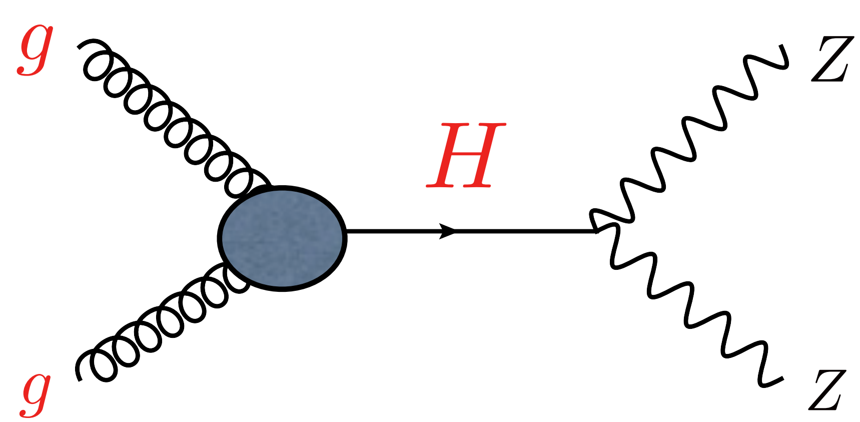
Caola, K.M.

The reason for significant off-shell rate is due to large cross-section for producing two longitudinally polarized Z bosons in decays of (strongly) off-shell Higgs.

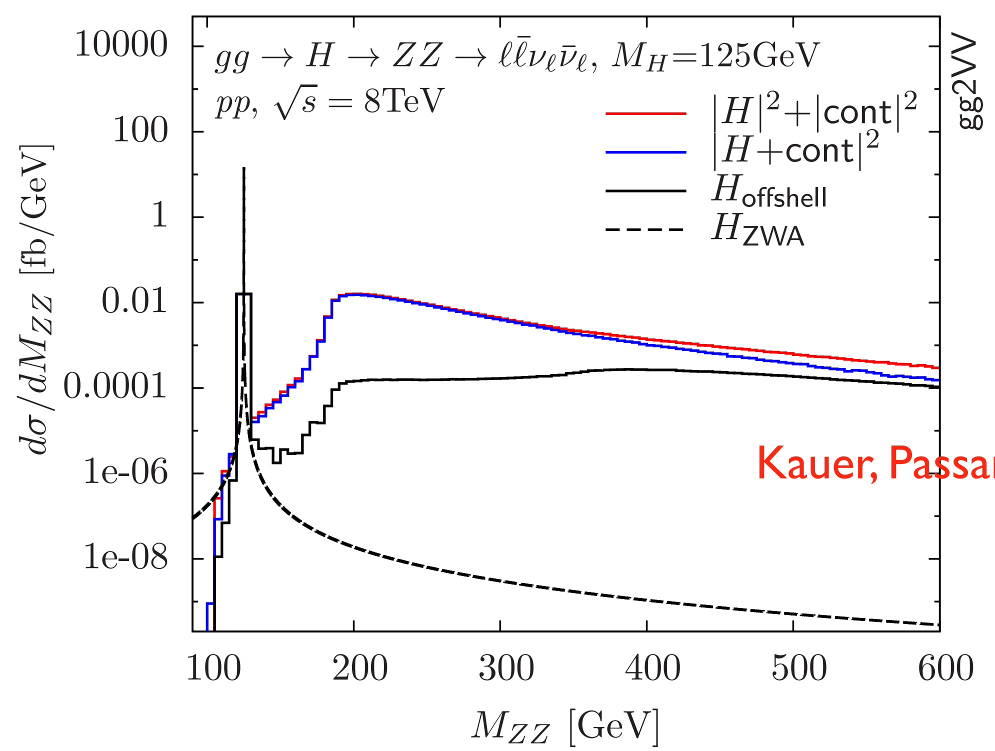
Kauer, Passarino

$$\mathcal{A}_{H^* \to Z_L Z_L} \sim \frac{s}{v} \qquad |\mathcal{A}_{gg \to H^* \to Z_L Z_L}|^2 \sim \frac{s^2}{(s-m_h^2)^2 + m_h^2 \Gamma_h^2} \Rightarrow \mathrm{const}, \quad s \gg m_h^2$$

For large invariant masses of the Z boson pair, the amplitude squared becomes independent of ZZ invariant mass, enhancing the off-shell production significantly. Off-shell cross-section is large; it is close to ten percent of the resonance cross-section.



$$\sigma_H(m_{ZZ} > 160 \text{ GeV}) \approx 0.1 \, \sigma_H$$



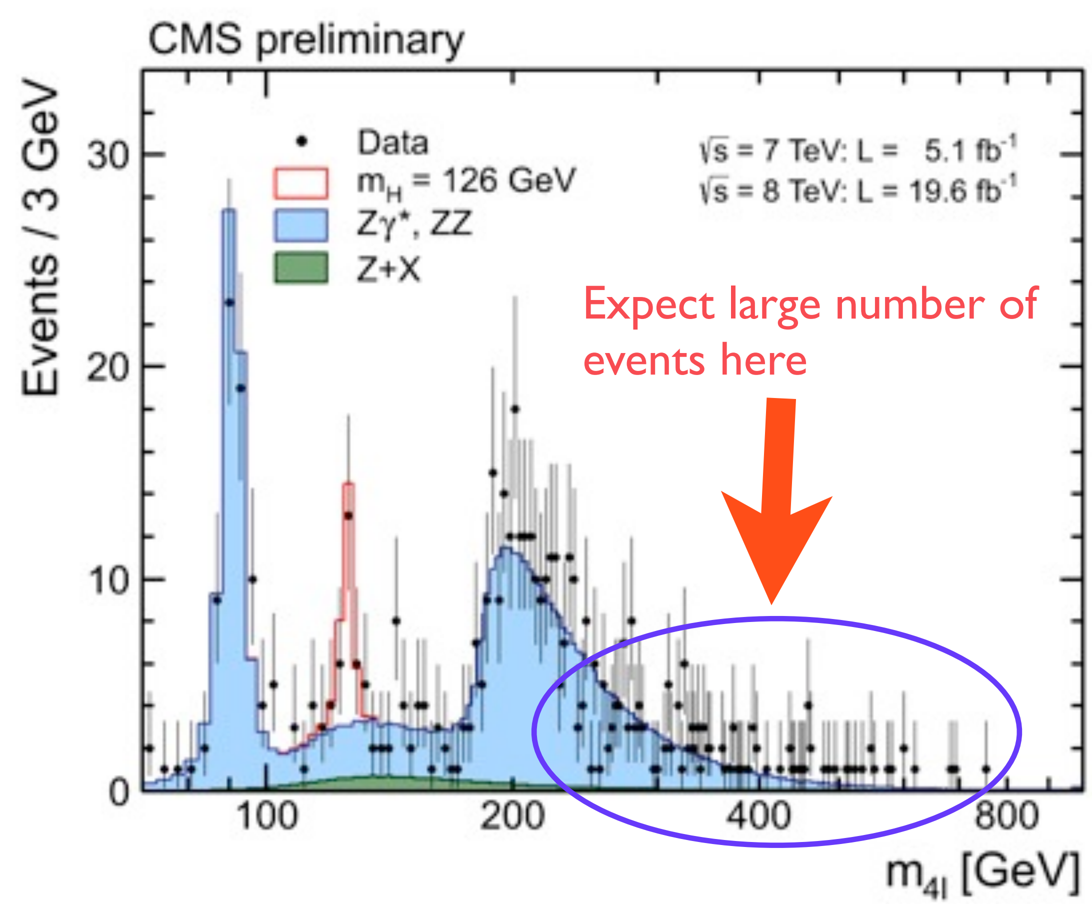
Kauer, Passarino

Higgs decay to ZZ

The off-shell production cross-section does not depend on the Higgs width but does depend on the Higgs couplings to initial state particles (gluons) and final state particles (Z bosons). This implies that if we change both the width of the Higgs and its couplings to other particles in such a way that the resonance cross-section does not change, the off-shell production cross-section changes proportionally to the Higgs width.

$$\sigma_H \sim \frac{g^2_{H\to gg} g^2_{H\to ZZ}}{\Gamma_H}; \quad \sigma_{\rm off} \sim g^2_{H\to gg} g^2_{H\to ZZ}. \qquad \sigma_H \sim \sigma_H^{\rm SM} \qquad \sigma_{\rm off} \sim \sigma_{\rm off}^{\rm SM} \frac{\Gamma_H}{\Gamma_H^{\rm SM}}$$

The current direct upper bound on the Higgs width is 3.4 GeV (CMS) which is 820 times larger than the Standard Model value. If the width were actually that large, Higgs couplings to gluons and ZZ should be different from their SM values to ensure agreement of the on-shell cross-section. However, once couplings are modified, one should expect a very large number of additional off-shell events that exceed by almost a factor of four a total number of ZZ events observed by the CMS!



$$N_{\rm off} \approx 0.1 \times N_{\rm peak} \times 820 \sim 1600 \gg N_{4l}^{\rm total}$$

Therefore, one can already put meaningful bounds on the Higgs width using current data on ZZ final states !

Caola, K.M.

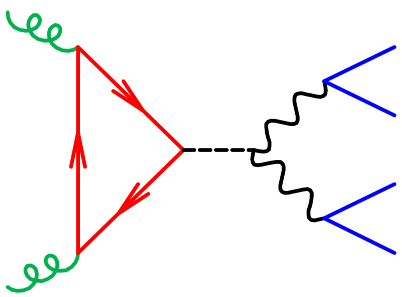
Complications: backgrounds and interferences

Since pairs of Z bosons can be produced in many different ways, ultimate constraints will require accurate predictions for production rates of 4-lepton final states in proton collisions, including backgrounds, and interferences of signal and backgrounds. There are three primary sources of Z-bosons that we should care about.

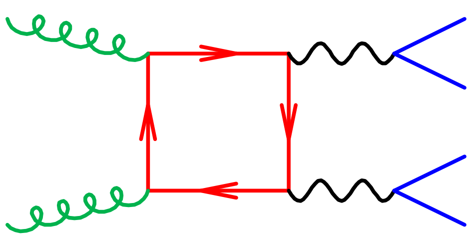
1) qq -> ZZ (NNLO QCD available since recently)

2) gg -> H ->ZZ (NNLO QCD) and
gg -> ZZ (LO, 1loop)

3) qg -> H q -> ZZq (NLO) and qg ->ZZq (NLO)

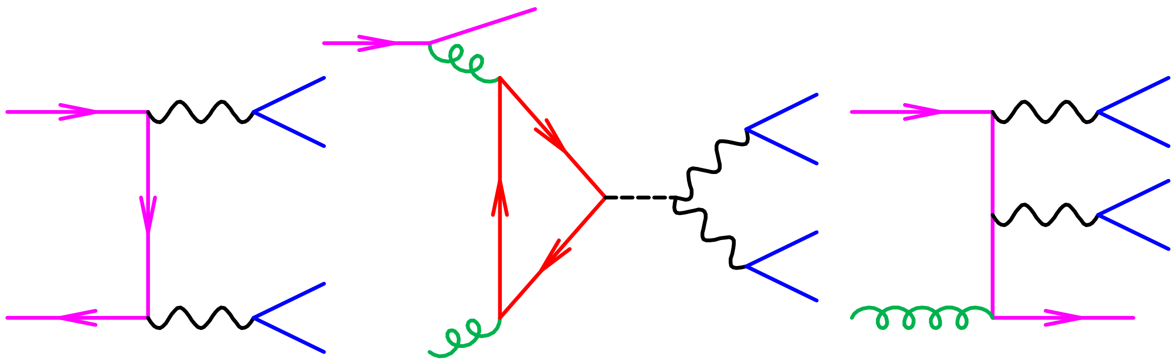


(a)



(b)

We should be also concerned about interference effects in the gg channel. Note that, although the interference in qg channels needs to be accounted for at the same order in strong and electroweak couplings, it is much smaller numerically.



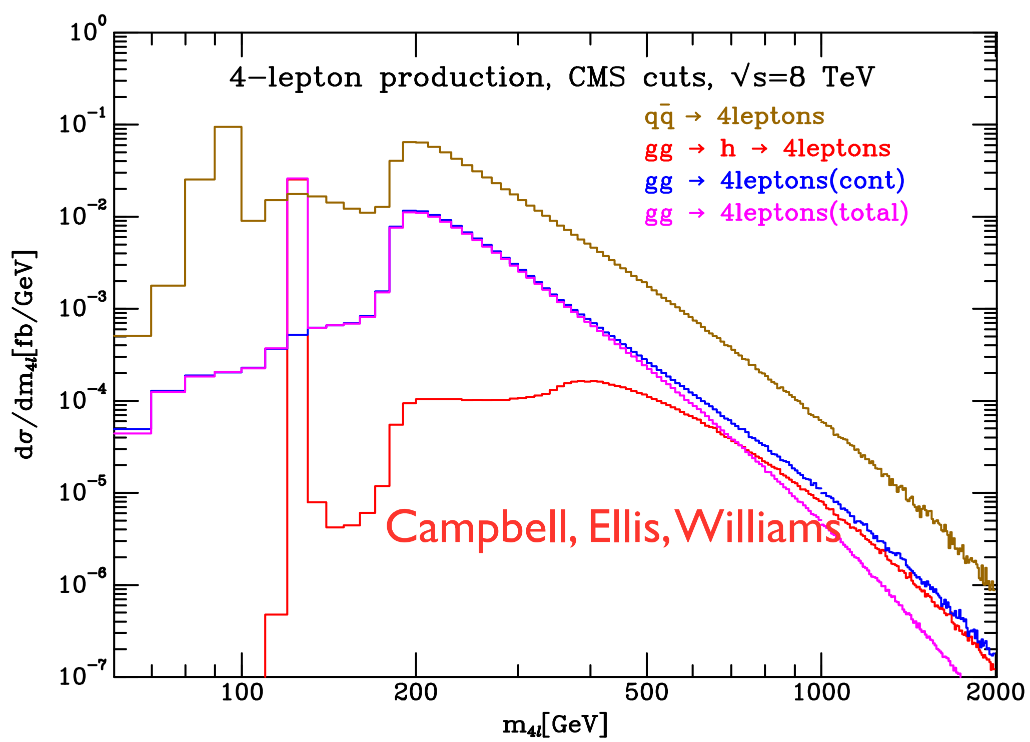
Campbell, Ellis, Williams

Magnitude of various contributions

How large is the total sample of events we have now and how large are different contributions to pp -> ZZ for realistic selection cuts?

Consider CMS 4-lepton events as an example. CMS observes 451 ZZ (4l) events in the invariant mass range between 100 and 800 GeV. In the Standard Model, these events can be decomposed into already indicated contributions; the largest one is qq -> ZZ, followed by gg->ZZ, followed by the resonance Higgs production. CMS expected to observe 432(30) ZZ events.

Off-shell production of the Higgs and its interference with gg -> ZZ production were not included in early CMS analysis because they are small in the Standard Model and because they do not affect properties of the Higgs resonance (off-shell effects, no impact on the Higgs properties extracted from peak cross-sections).



$$N_{qq\to ZZ} \approx N_{\rm tot}$$

$$N_{gg} \sim 10^{-1} \times N_{\rm tot}$$

$$N_H \sim 5 \times 10^{-2} \times N_{\rm tot}$$

$$N_{\rm off} \sim 10^{-2} N_{\rm tot}$$

$$N_{\rm int} \sim -2 \times 10^{-2} N_{\rm tot}$$

Constraining the width

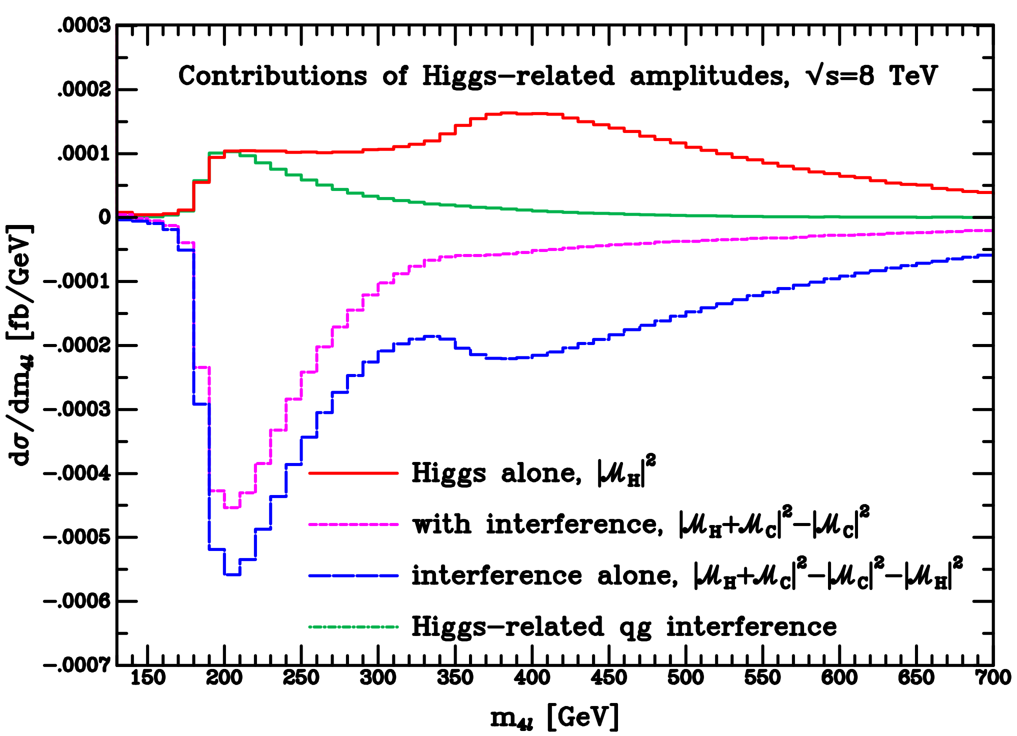
However, if we float the width of the Higgs boson, the number of expected events changes. To find the change, we note that the off-shell Higgs production cross-section scales as the width (or couplings raised to an appropriate power), and the interference scales as the square root of the width. Considering the ZZ invariant mass range from 100 GeV to 800 GeV, we find a new estimate for the number of events

$$N_{\rm exp} = 432 + 2.78\frac{\Gamma_H}{\Gamma_H^{\rm SM}} - 5.95\sqrt{\frac{\Gamma_H}{\Gamma_{\rm H}^{\rm SM}}} \pm 31$$

Requiring that observed (451) and expected number of events do not differ by more than two standard deviations, we derive an upper bound on the Higgs boson width

$$|N_{\rm nobs} - \bar{N}_{\rm exp}| < 62 \qquad \Gamma_H < 43\ \Gamma_H^{\rm SM} = 181\ {\rm MeV}\ (95\%{\rm C.L})$$

Caola, K.M.; Campbell, Ellis, Williams



Campbell, Ellis, Williams

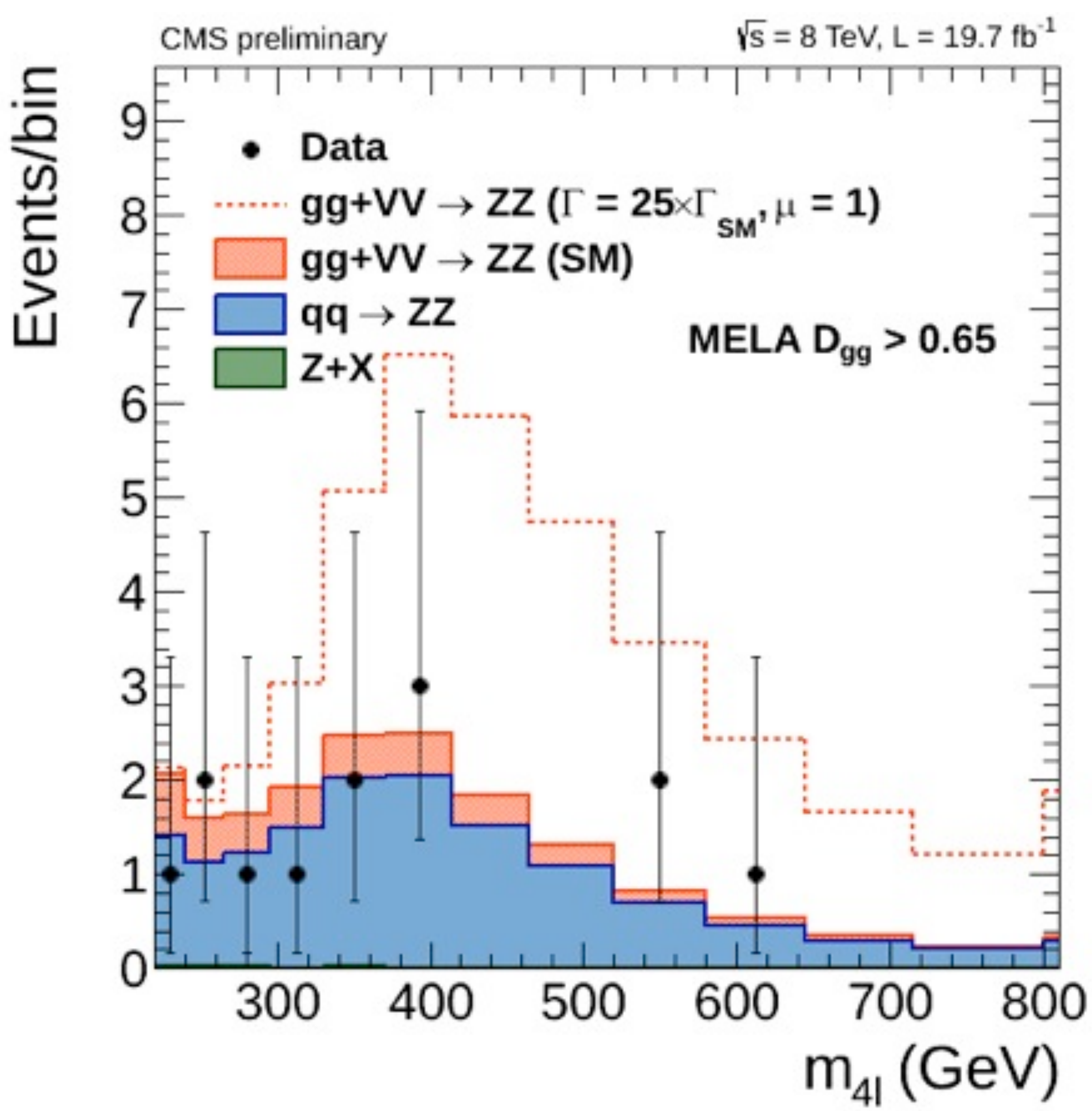
The analysis can be improved by focusing on the region of higher invariant masses. This is because the off-shell Higgs production is significant beyond 200 GeV while there is large negative interference below 200 GeV; removing contribution of that region, improves the constraint. For example, selecting events with 4-lepton invariant mass higher than 300 GeV, we find $\Gamma_H < 25.2\ \Gamma_H^{\rm SM} < 105\ {\rm MeV}(95\%{\rm C.L.})$

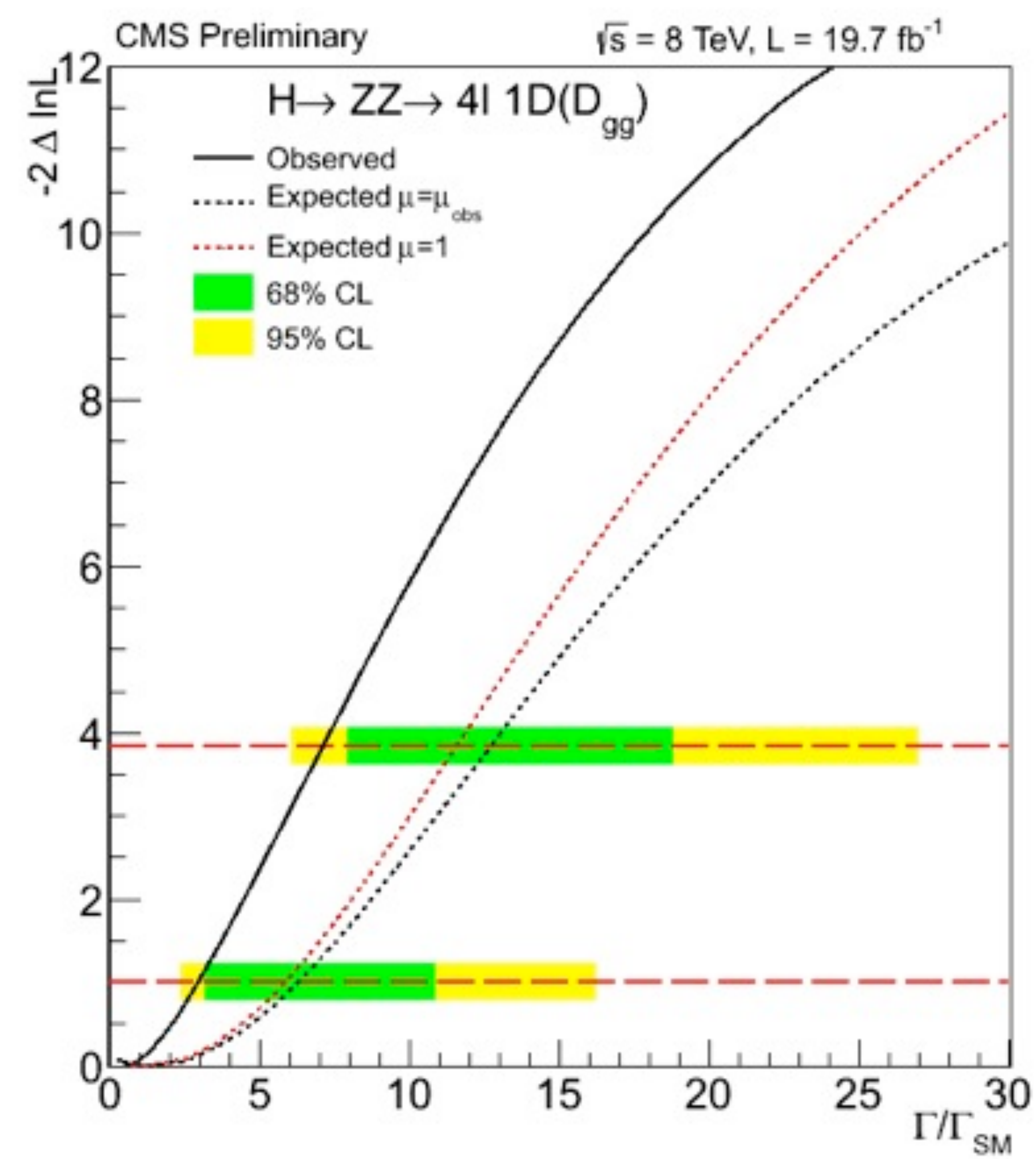
Caola, K.M.; Campbell, Ellis, Williams

Recent CMS measurement

CMS collaboration has presented results of the actual width measurement using off-shell ZZ production at the Moriond conferences, earlier this year. Very recently the preprint appeared (arXiv/1405.3455).

Their bound on the Higgs width is $\Gamma < 5.4\,\Gamma_{\rm SM}$, i.e. even stronger than what earlier theoretical estimates suggested. This is a factor of 170 (!) improvement compared to the previous bound on the width. The corresponding limit on invisible branching rate is fifty percent.





General comments

1) CMS/ATLAS measurements prove that it is possible -- in practice -- to constrain the Higgs boson width using off-shell production of Z and W pairs.

1) It is important to get the logic of the measurement correctly: by going off-shell, we measure couplings. No width enters the off-shell physics. We infer the information about the width from the off-shell cross-section once couplings are known.

2) Even with all statistical tricks (likelihood etc.), at its core, this is a counting experiment that requires understanding of yields rather than shapes. Proper theoretical predictions for signal, background and interferences are therefore very important.

3) The main idea of the method is that excessive events at high-invariant mass of Z-boson pairs are interesting and may be related to Higgs physics. Interpretation of such excesses in terms of limits on the Higgs boson width is possible, as we have seen, but may require some care since it forces us to relate couplings measured at different invariant masses.

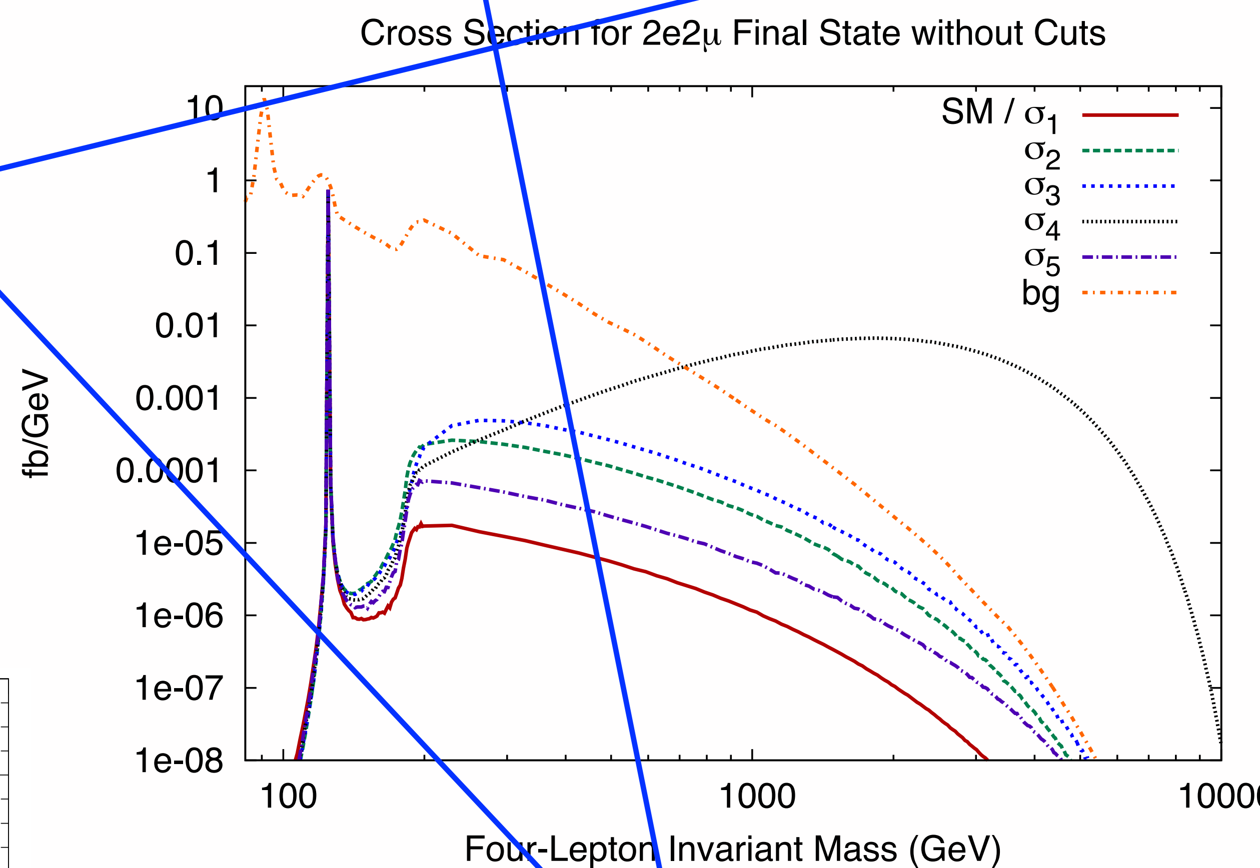
4) In general, a relation between on- and off-shell couplings may become less straightforward if the HZZ vertex contains anomalous couplings and the HGG vertex receives significant contributions from light degrees of freedom. Luckily, such effects can be constrained from various on-shell measurements, as I will discuss shortly.

Example: anomalous HZZ coupling

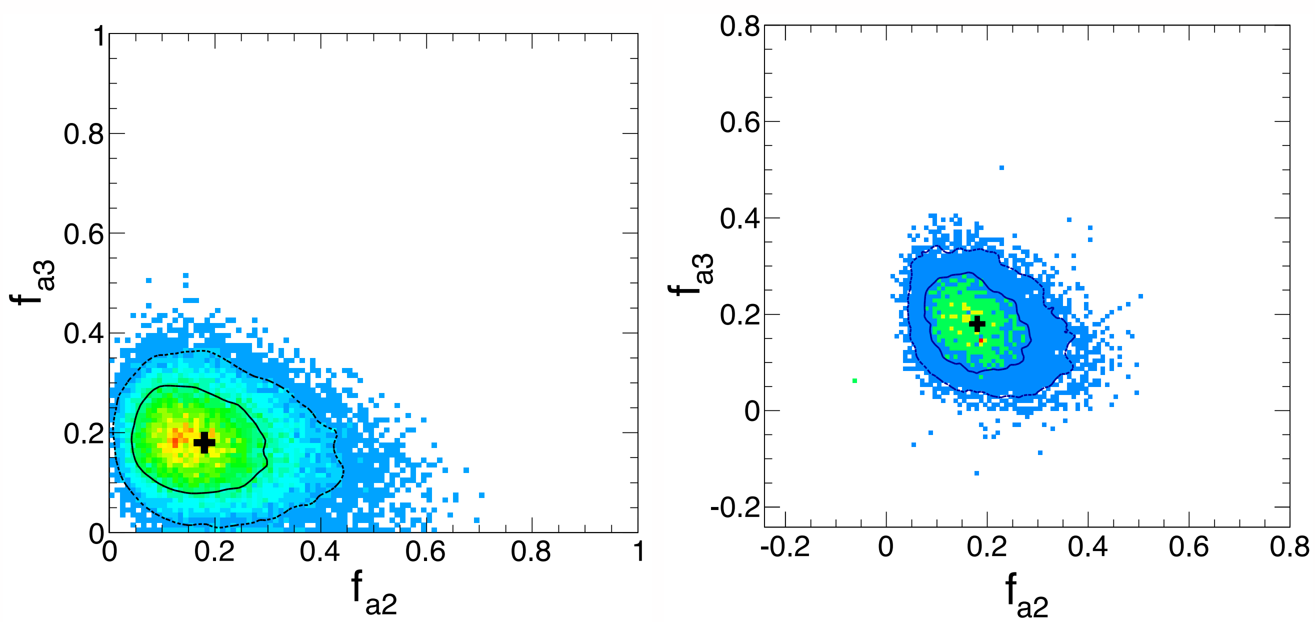
Basis of HZZ operators [Gainer, Lykken et al (2013)]

$$\mathcal{O}_1 = -\frac{M_Z^2}{v} H Z_\mu Z^\mu, \quad \mathcal{O}_2 = -\frac{1}{2v} H Z_{\mu\nu} Z^{\mu\nu} \quad \mathcal{O}_3 = -\frac{1}{2v} H Z_{\mu\nu} \tilde{Z}^{\mu\nu}, \quad \mathcal{O}_4 = \frac{2}{v} H Z_\mu \partial^2 Z^\mu$$

$$\mathcal{O}_6 = -\frac{M_Z^2}{M_H^2 v} Z_\mu Z^\mu \partial^2 H$$



Strong modification of the m_{4l} shape

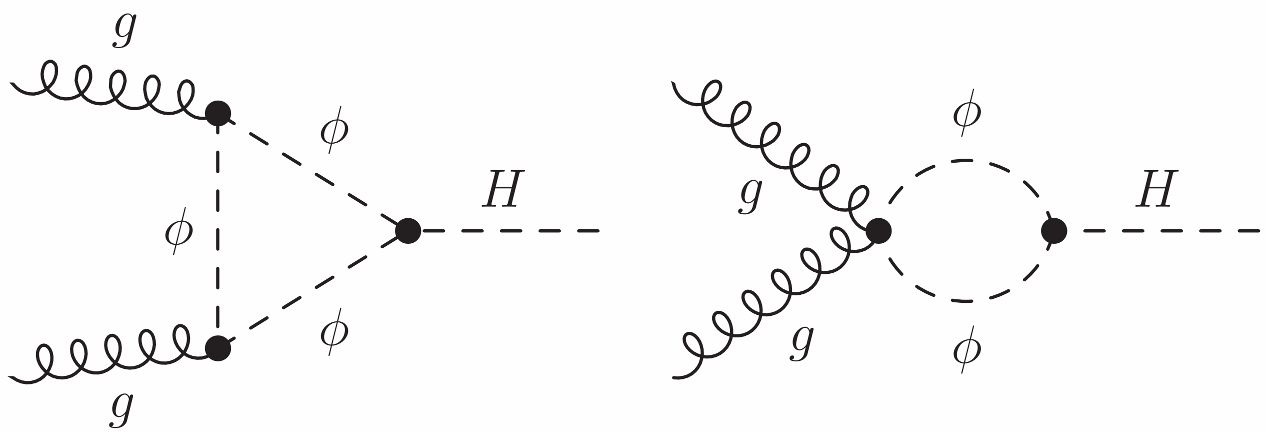


[Anderson et al. (2013)]

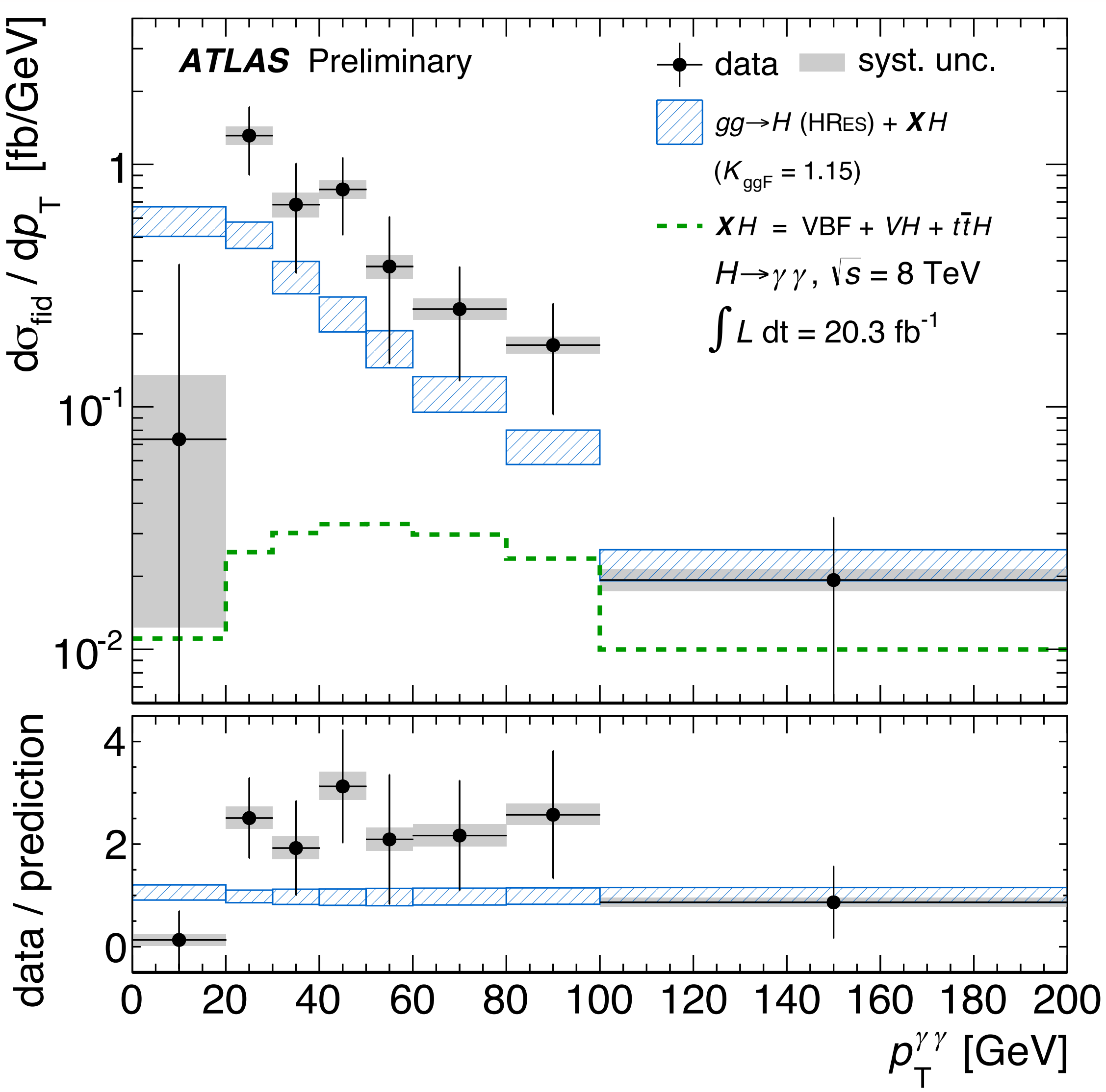
Modification of lepton angular distributions -> good control with 300 fb^{-1}

Example: light colored singlets in the loop

[Englert, Spannowsky (2014)]



Light d.o.f. can qualitatively change the analysis



m_ϕ	μ (h peak)	Γ_h/Γ_h^{SM}	$\overline{\sigma}/\overline{\sigma}^{SM}$ [$m(4\ell) \geq 330$ GeV][a]
70 GeV	$\simeq 1.0$	$\simeq 5$	-2%
170 GeV	$\simeq 1.0$	$\simeq 4.7$	$+80\%$
170 GeV	$\simeq 1.0$	$\simeq 1.7$	$+6\%$

[a]We impose the cut set used by CMS [18] without the MELA cut [35].

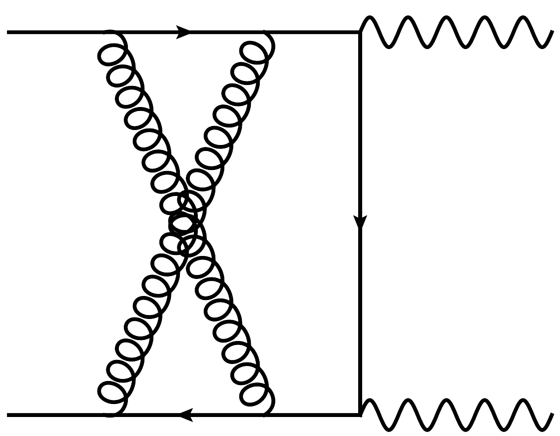
Can be detected by measuring the Higgs p_T distribution (need theoretical improvement, full m_t dependence)

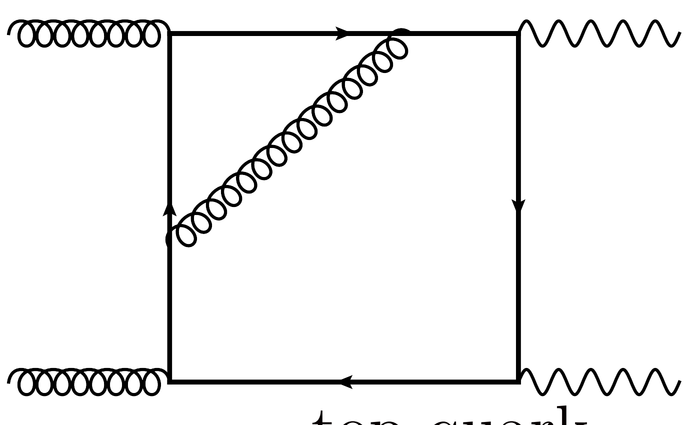
The ultimate reach of the width measurement

The ultimate reach of this method to constrain the width is determined by how well the number of ZZ events at high invariant mass can be predicted in the Standard Model. This requires NNLO QCD computations for qq->ZZ, two-loop NLO QCD computations for gg -> ZZ and the signal-background interference. Electroweak corrections may be also sizable, at high invariant mass.

Two loop computations are not easy. However, recently there appeared to be a breakthrough with two groups completing the necessary scalar integrals. These results where already used to construct the NNLO QCD predictions for ZZ production cross-section (Grazzini et al.). It was found that the corrections are at the level of 12 to 14 percent depending on the center-of-mass energy with the residual scale dependence at the level of three percent. Obtaining corrections to fiducial cross-sections is necessary.

Further down the road are computations of NLO QCD corrections to gg -> ZZ and to the interference. When everything is completed, the quality of the Standard Model prediction for the off-shell ZZ production will be extremely high. A residual theoretical uncertainty for pp -> ZZ at the level of just a few percent can probably be reached within a year or two.





Higgs width measurements at the LHC

To recap the width story: one can obtain interesting information about Higgs boson properties -- in particular about the width -- from the off-shell production.

In the four-lepton channel, large off-shell effects are caused by the decay of an ``off-shell Higgs" to longitudinal Z bosons at large invariant masses. This leads to a plateau of Higgs-induced events. Measuring the number of events at this high-invariant mass region probes Higgs couplings to gluons and Z's and is independent of the Higgs width. The measured value of the Higgs on-shell production cross-section is then used to infer the value of the Higgs width.

Already with the current data, we can argue that the Higgs width can not exceed $\Gamma < 5.4\,\Gamma_{SM}$ O(15-20) times the SM value and significant improvements in this result are very likely. In fact, the very recent CMS measurement suggests an even stronger bound -- ; ATLAS has a slightly larger but comparable upper bound.

Further advances with the Higgs width measurements at the LHC using this methodology will require very precise theoretical predictions for ZZ production in proton collisions (the recent progress with multi-loop computations makes this well within reach) and detailed studies of on-shell couplings to Z's and gluons to constrain possible effects of higher-dimensional operators.

Two-loop scattering amplitude for the production of two vector bosons in quark antiquark collisions

In collaboration with F. Caola, J. Henn, A. Smirnov, V. Smirnov

Methods for NNLO QCD computations

There is more than one method for exclusive NNLO computations (i.e. cancelling IR/ collinear divergencies at the fully-differential level for) of hadron collider processes, at different stages of developments. I don't want to review these methods. Instead, I want to emphasize three points:

a) they are numerous ;

b) they are still recent and there is still a good deal of interesting LHC processes where they can be applied;

c) The field is very active (see the list below).

1) Czakon, Fiedler, Mitov: top quark pair production at NNLO;
2) J. Currie, T. Gehrmann, N.. Glover, A. Gehrmann - de Ridder, J. Pires: dijet production at NNLO;
3) G. Abeloff, A. Gehrmann - de Ridder, P. Maierhofer, S. Pozzorini: top quark pair production at NNLO;
4) R. Boughezal, F. Caola, K. Melnikov, F. Petriello and M. Schulze: H+jet at NNLO;
5) M. Bruscherseifer, F. Caola, K. Melnikov : top decay at NNLO;
6) M. Bruscherseifer, F. Caola, K. Melnikov: t-channel single top production at NNLO (large N);
7) F. Cascioli, T. Gehrmann, M. Grazzini, et al. : ZZ production at NNLO ;
8) C. Anastasiou, A. Lazopoulos, F. Herzog, R.Mueller, Higgs production in bottom fusion;

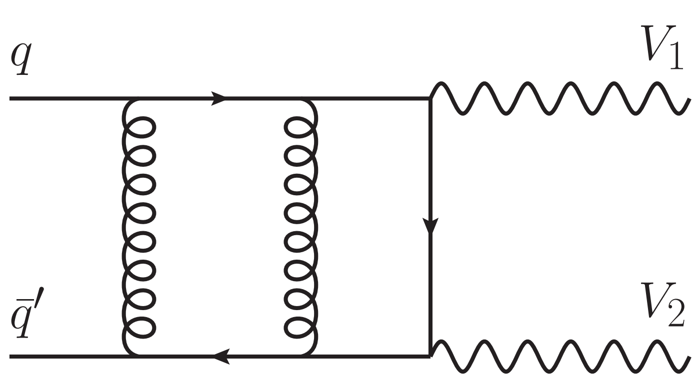
In what follows, I would like to discuss one technical aspect of NNLO computations -- calculation of two-loop scattering amplitudes.

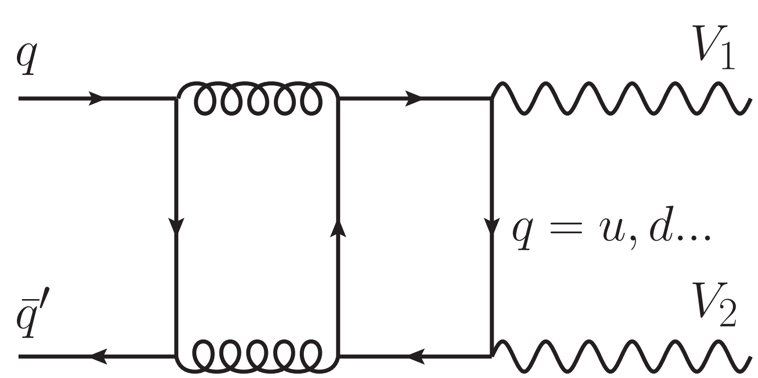
Two-loop virtual corrections

Two-loop virtual corrections is a necessary ingredient of any NNLO computation.
As with any computation of virtual corrections, the complexity increases with larger number of external particles and with larger number of kinematic invariants (masses included).

The technology that is currently used for these computations involves three steps:
a) diagrammatic analysis;
b) reduction to master integrals using integration-by-parts identities;
c) computation of master integrals.

To give you an idea of how these computations are done, I will consider production of an arbitrary pair of vector bosons (either on- or off-shell) in the collisions of a quark and an antiquark (neglecting all contributions with single intermediate vector bosons)





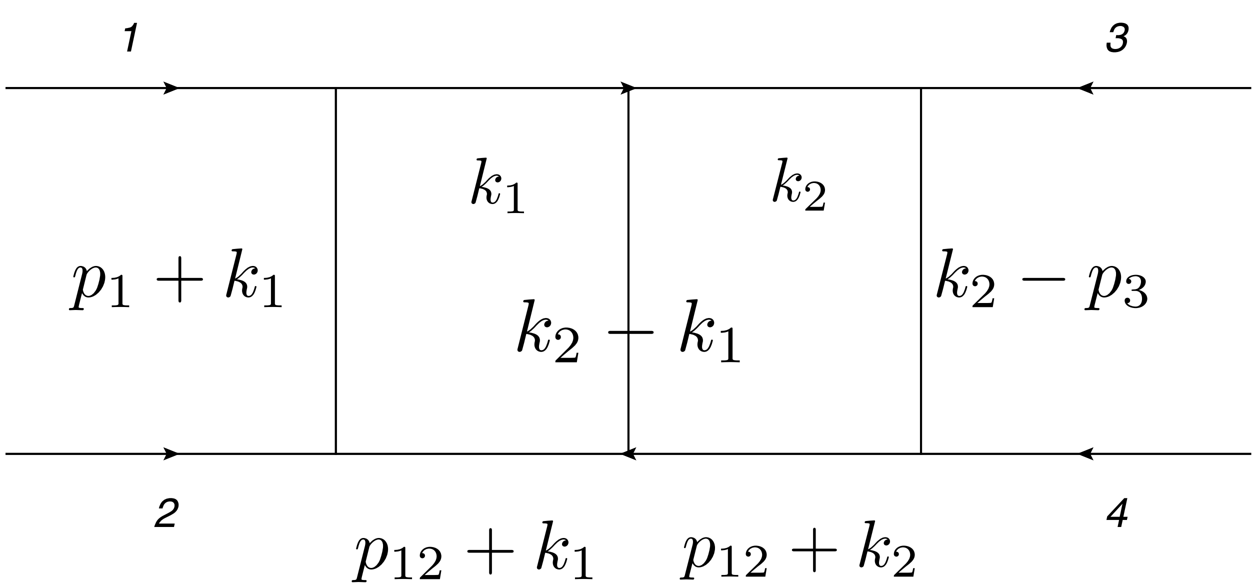
$$\mathcal{M}(\lambda_q, \lambda_5, \lambda_7) = i\left(\frac{g_W}{\sqrt{2}}\right)^4 \delta_{i_1 i_2} D_3(p_3) D_4(p_4) C^{\lambda_7}_{l,V_2} C^{\lambda_5}_{l,V_1} \epsilon_3^{\mu}(\lambda_5)\epsilon_4^{\nu}(\lambda_7)$$

$$\times \left[C^{\lambda_q}_{\bar{q}',V_2} C^{\lambda_q}_{q,V_1} \mathcal{A}^{(d)}_{\mu\nu}(p_1^{\lambda_q}, p_3, p_4, p_2^{-\lambda_q}) + C^{\lambda_q}_{\bar{q}',V_1} C^{\lambda_q}_{q,V_2} \mathcal{A}^{(d)}_{\nu\mu}(p_1^{\lambda_q}, p_4, p_3, p_2^{-\lambda_q}) + C^{n_g}_{V_1 V_2} \mathcal{A}^{n_g}_{\mu\nu}(p_1^{\lambda_q}, p_2^{-\lambda_q}; p_3, p_4) \right]$$

Two-loop virtual corrections

The problem with two-loop computations is that no algebraic framework exists for expressing tensor integrals through Lorentz scalar integrals. This is in variance with the Passarino-Veltman procedure at one loop.

At two-loops a similar task is accomplished by the integration-by-parts technique. However, this technique can only be applied if Feynman diagrams are written in a ``closed form" w.r.t  scalar products of loop momenta and external momenta of ``primary particles". This implies that polarization vectors and momenta of decay products of primary particles must be eliminated. This isn't easy and requires understanding of the Lorenz decomposition of the amplitude into form factors and construction of appropriate projection operators.



$$\otimes (k_2 + p_1)^2 \; (k_1 - p_3)^2$$

Two-loop virtual corrections

This procedure is straightforward in principle but it becomes increasingly cumbersome for larger multiplicities. A particular problematic issue is to understand how to properly treat gamma5 in closed fermion loops.

$$\mathcal{M} = \mathcal{A}_{\mu\nu}(p_1,p_2,p_3,p_4)\epsilon_3^{\mu}\epsilon_4^{\mu}$$

$$W^+(p_3) \to \nu(p_5) + e^+(p_6), \quad \epsilon_3^{\mu} = \langle 5|\gamma^{\mu}|6]$$

$$W^-(p_4) \to e^-(p_7) + \bar{\nu}(p_8), \quad \epsilon_4^{\nu} = \langle 7|\gamma^{\mu}|8]$$

$$A_{\mu\nu} = \bar{v}_{p_2}\hat{p}_{\perp}u_{p_1}A_{\mu\nu}^{(1,a)} + \bar{v}_{p_2}\gamma^{\mu}u_{p_1}A_{\nu}^{(2,a)} + \bar{v}_{p_2}\gamma^{\nu}u_{p_1}A_{\mu}^{(3,a)} + \bar{v}_{p_2}\gamma^{[\mu}\hat{p}_{\perp}\gamma^{\nu]}u_{p_1}A^{(4,a)}$$

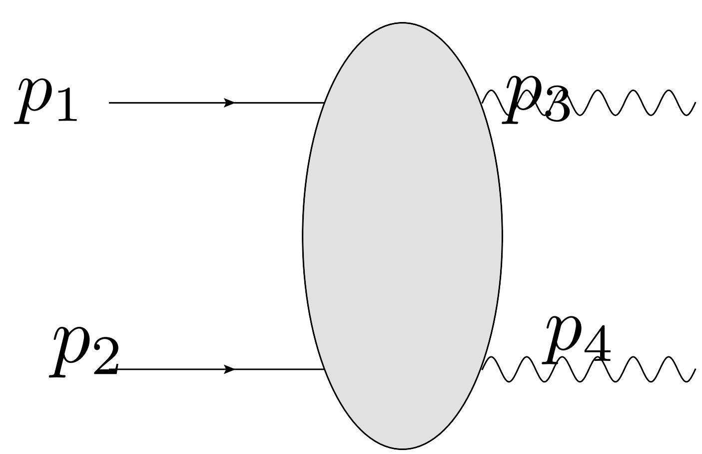
$$p_{3,4} = \alpha_{3,4}p_1 + \beta_{3,4}p_2 \pm p_{\perp} \qquad p_3^2 = m_3^2 \qquad p_4^2 = m_4^2$$

$$A_{\mu\nu}^{(1,a)} = T_1 g_{\mu\nu} + T_2 p_{1\mu}p_{1\nu} + T_3 p_{1\mu}p_{2\nu} + T_4 p_{1,\mu}p_{\perp\nu} + T_5 p_{2\mu}p_{1\nu} + \ldots\ldots T_{10}p_{\perp\mu}p_{\perp\nu}$$

$$A_{\mu}^{(2a)} = T_{11}p_{1\mu} + T_{12}p_{2\mu} + T_{13}p_{\perp\mu}$$

$$A_{\mu}^{(3a)} = T_{14}p_{1\mu} + T_{15}p_{2\mu} + T_{16}p_{\perp\mu}$$

$$A_4 = T_{17}$$



The above decomposition is way too general; it does not use constraints that follow from vector current conservation. The T form-factors are independent of momenta and polarizations of vector boson decay products; they are expressed through Feynman integrals of the type shown on the previous transparency.

Two-loop virtual corrections

We can express the amplitude (left-left-left, as an example) in a compact form using spinor-helicity notations. We find that physical helicity amplitude depends on nine form factors only.

$$\mathcal{M}^{(a)} = -F_1\langle 57\rangle[86]\langle 2\hat{3}1] + F_2\langle 15\rangle\langle 17\rangle[16][18]\langle 2\hat{3}1] + F_3\langle 15\rangle\langle 27\rangle[16][28]\langle 2\hat{3}1]$$
$$+F_5\langle 17\rangle\langle 25\rangle[18][26]\langle 2\hat{3}1] + F_6\langle 25\rangle\langle 27\rangle[26][28]\langle 2\hat{3}1] + F_{14}\langle 15\rangle\langle 27\rangle[16][18]$$
$$+F_{11}\langle 25\rangle\langle 17\rangle[16][18] + F_{12}\langle 25\rangle\langle 27\rangle[16][28] + F_{15}\langle 25\rangle\langle 27\rangle[26][18]$$

$$F_1 = -2T_1, \quad F_2 = T_2 - \alpha_3\alpha_4 T_{10} - \alpha_3 T_8 + \alpha_4 T_4,$$

$$F_3 = T_3 - \frac{4T_{17}}{s} - \alpha_3\beta_4 T_{10} - \alpha - 3T_9 + \beta_4 T_4$$

$$\ldots\ldots\ldots$$

$$F_{15} = 2T_{15} - 2\beta_3 T_{16}$$

. To compute these form factors, we construct projection operators.

$$\mathcal{A}_{\mu\nu} = \bar{v}_{p_2}\hat{\Gamma}_{\mu\nu}u_{p_1}. \qquad \sum \mathcal{A}_{\mu\nu} \times \bar{u}_{p_1}\hat{O}v_{p_2} = \mathrm{Tr}\left[\hat{p}_2\Gamma_{\mu\nu}\hat{p}_1\hat{O}\right]$$

Two-loop virtual corrections

$$\mathcal{A}_{\mu\nu} = \bar{v}_{p_2}\hat{\Gamma}_{\mu\nu}u_{p_1}. \qquad \sum \mathcal{A}_{\mu\nu} \times \bar{u}_{p_1}\hat{O}v_{p_2} = \mathrm{Tr}\left[\hat{p}_2\Gamma_{\mu\nu}\hat{p}_1\hat{O}\right]$$

$$G_1 = -\frac{\mathrm{Tr}\left[\hat{p}_2\Gamma_{\mu\nu}\hat{p}_1\hat{p}_\perp\right]}{4p_\perp^2(p_1\cdot p_2)^3} \times p_1^\mu p_1^\nu, \qquad G_1 = T_6.$$

$$G_2 = -\frac{\mathrm{Tr}\left[\hat{p}_2\Gamma_{\mu\nu}\hat{p}_1\hat{p}_\perp\right]}{4p_\perp^2(p_1\cdot p_2)^3} \times p_2^\mu p_2^\nu, \qquad G_2 = T_2.$$

. . . .

The projection operators are constructed using the trial-and-error method; at the end, it turns out that our goal can be achieved with relatively simple projection operators.

$$G_{17} = -\frac{\mathrm{Tr}\left[\hat{p}_2\Gamma_{\mu\nu}\hat{p}_1\left(\gamma^\nu\hat{p}_\perp\gamma^\mu - \mu \leftrightarrow \nu\right)\right]}{8p_\perp^2(p_1p_2)}, \qquad G_{17} = -(2d^2 - 14d + 20)T_{17} + (p_1p_2)T_5 - (p_1p_2)T_3.$$

$$T_1 = \frac{G_{10} - G_9 - G_4 - G_3}{d-3}, \quad T_2 = G_2, \quad \ldots\ldots.., \quad T_{17} = -\frac{G_4 - G_3 + G_{17}}{2(d-3)(d-4)}.$$

Finally, we combine equations for T's to obtain the physical form factors F; these form factors are expressed in terms of two-loop four-point integrals of the type shown earlier.

The integrals satisfy many linear equations that originate from integration-by-parts identities that allow one to map all the integrals that are needed to a small set of integrals called ``master integrals''.

Two-loop virtual corrections

Procedures for the reduction of two-loop integrals to master integrals are non-trivial. Large number of interesting ideas that facilitate this process appeared in the past O(10) years. In addition, significant effort went into an automation of integration-by-parts (IBP) procedure; now public programs (AIR,FIRE, REDUZE) and their more powerful private versions exist. The complexity of the IBP process increases with the number of kinematic invariants and masses that are present in the problem. At the moment, 2->2 processes can be dealt with; anything beyond that has never been seriously tried.

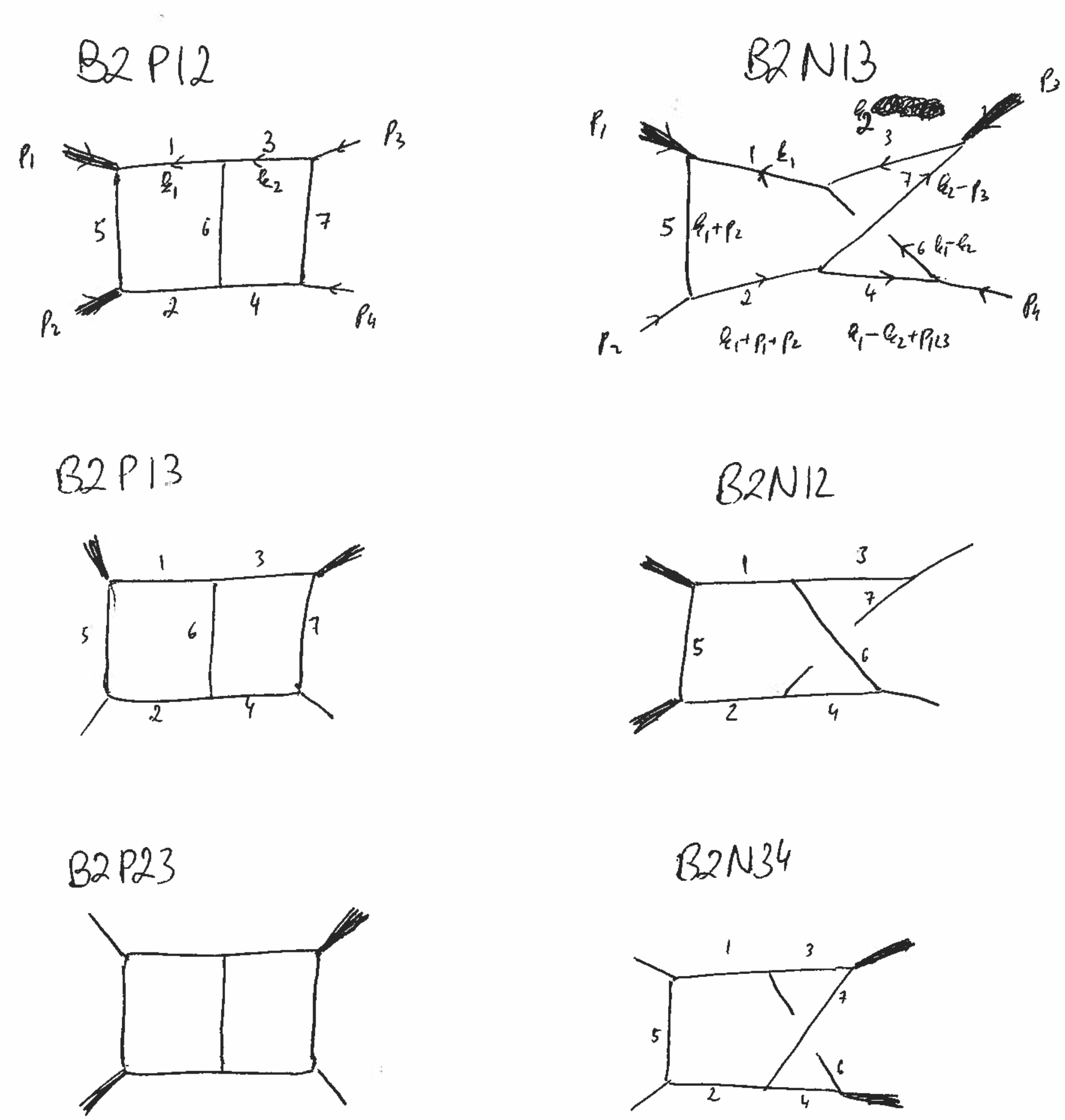
Calculations of master integrals is a much less straightforward procedure; it was traditionally done on a case-by-case basis. An interesting recent development is related to Henn's conjecture that postulates that it is always possible to choose a set of master integrals in such a way that they satisfy differential equations of the following form

$$\partial_x \vec{f} = \epsilon \hat{A}_x(x,y,z\ldots)\vec{f}$$

The important point is that on the right-hand side, the dimensional regularization parameter appears explicitly, and only as a multiplicative pre-factor. It is then possible to solve these equations iteratively order-by-order in (d-4).

While differential equations were used to find master integrals for a long time starting from papers by Kotikov and Remiddi in the early 1990s, the idea by Henn streamlines and simplifies such computations significantly. This already lead to very impressive advances (e.g. master integrals for Bhabha, V1 V2 production) that may have interesting consequences for phenomenology.

Two-loop virtual corrections



For the case of double vector boson production, we can identify six different two-loop topologies; the differential equations can be ``rationalized'' with the following (typical) change of variables

$$\frac{s}{m_3^2} = (1+x)(1+xy),$$

$$\frac{t}{m_3^2} = -xz, \quad \frac{m_4^2}{m_3^2} = x^2 y$$

$$\sqrt{(s-m_3^2-m_4^2)^2 - 4m_3^2 m_4^2} = m_3^2 x(1-y)$$

$$G(a_n, a_{n-1}, \ldots, a_1, t) = \int_0^t \frac{\mathrm{d}t_n}{t_n - a_n} G(a_{n-1}, \ldots a_1, t_n)$$

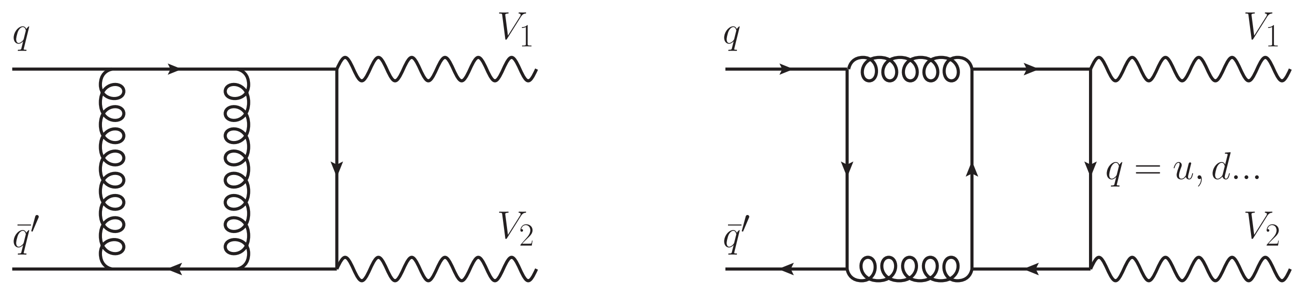
$$d\vec{f} = \epsilon(dA) \times f, \quad A = \sum A_i \log \alpha_i$$

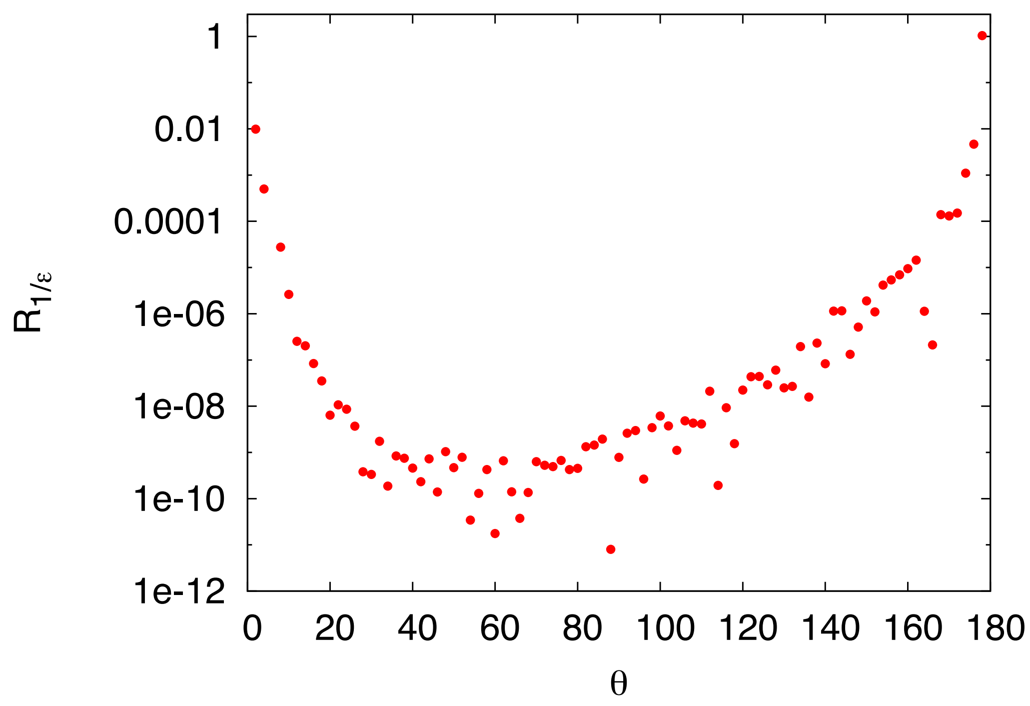
$$\alpha = \{x, y, z, 1+x, 1-y, 1+xy, z-y, 1+y(1+x)-z,$$
$$xy+z, 1+x(1+y-z), 1+xz,$$
$$1+y-z, z+x(z-y)+xyz, z-y+yz+xyz\}$$

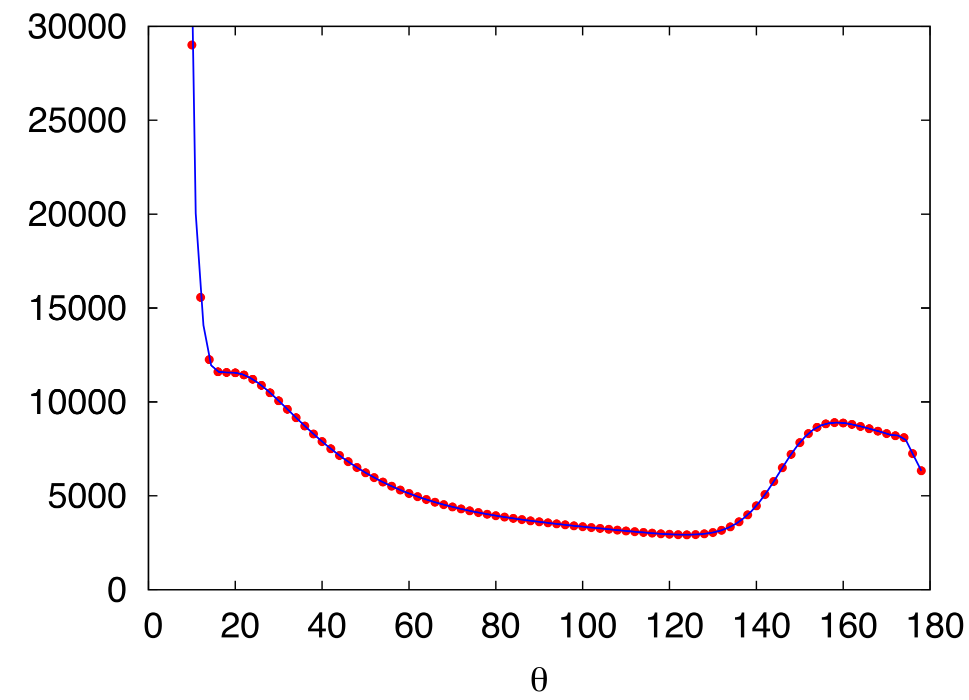
Important issues: finding a suitable basis; choice of ``rational variables''; boundary conditions for solutions of differential equations, analytic continuation.
Numerical evaluation of Goncharov's polylogarithms and their mapping on conventional polylogarithms.

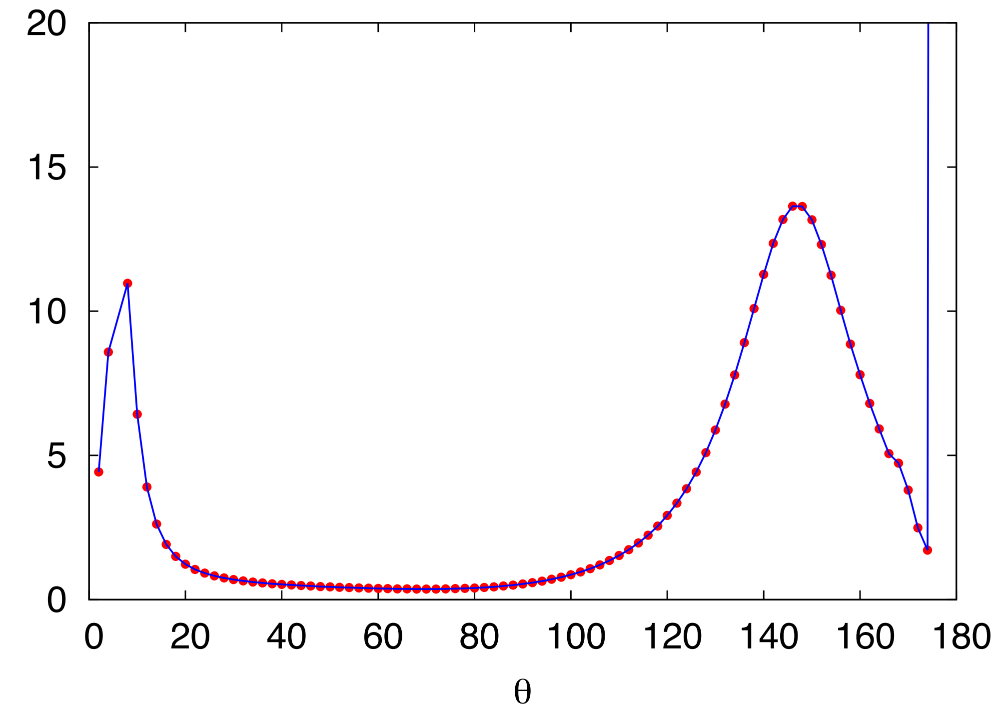
Two-loop amplitude for 4-lepton production

Analytic expression for two-loop virtual amplitude is too large to be shown. Here we display some numerical results for the kinematics relevant for WW* background to Higgs boson searches.









Comparison of an analytic prediction for 1/ep poles with the results of explicit computation. Two-loop helicity amplitudes squared as a function of vector boson scattering angle for different types of contribution.

Summary on two-loop virtual corrections

To summarize the situation with the two-loop virtual corrections for collider physics processes , let me say that

1) they are needed since they are always part of any NNLO computation;

2) they can be computed in many ways (direct Feynman parameter integration, numerics, Mellin-Barnes, differential equations) but their computation is always difficult;

3) recent advances seem to streamline computations of master integrals so that one can expect significant progress in computing two-loop virtual corrections to various 2 -> 2 processes;

4) larger number of kinematic invariants (multi-leg, masses etc.) makes such computations increasingly complicated and, at the moment, we do not know if two-loop computations for 2->2 amplitudes with larger number of kinematic invariants or 2->3 processes are feasible within this framework;

5) There are interesting attempts to understand if two-loop computations can be done using unitarity techniques that turned out to be so powerful at one-loop. While there was an impressive progress in this field related to classification of integrand residuals based on algebraic geometry concepts, there are still many outstanding issues. Currently , the main problem there seems to be the lack of understanding of how to avoid the use of integration-by-parts.

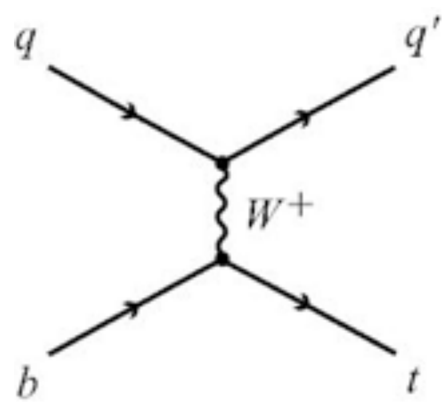
t-channel single top production in NNLO QCD

M. Brucherseifer, F. Caola, K. Melnikov

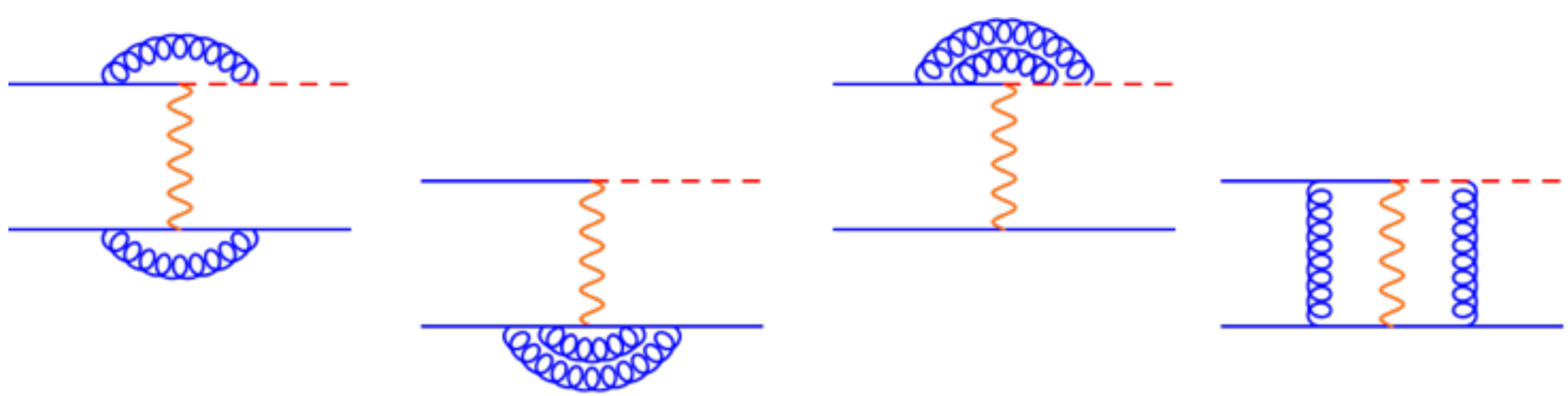
t-channel single-top production at NNLO

As I already mentioned, technology for NNLO QCD computations (cancellation of infra-red and collinear divergencies in exclusive observables) sufficiently matured in the past few years, to allow for sophisticated computations to be carried through. To show you an example, I would like to discuss the t-channel contribution to single-top production at NNLO QCD.

This process occurs due to an exchange of a W-boson in the t-channel. As the result, there is no color transfer from light-quark line to heavy-quark line at LO and NLO. It appears for the first time at NNLO where it is color-suppressed. We will neglect these contributions in our NNLO computation.



The relevant two-loop amplitudes are shown below; they involve one-loop corrections applied to heavy- and light-quark lines separately and the two-loop corrections to either heavy- or light-quark lines. The last diagram is the color-suppressed interference effect and we do not consider it (color suppression).



Ingredients for single-top NNLO computation

1) Two-loop form factors for heavy- (tWb) and light-quark (qWq') weak transitions are needed and they are known.

Bonciani, Ferroglia; Bell; Astarian, Greub and Pecjak;
Beneke, Huber and Li; Huber

2) Amplitudes for 0-> tbW(ll')gg and 0->tbW(ll')qq and 0->qq'W(ll)gg etc. Such amplitudes are either available or can be computed in a straightforward way;

R.K.Ellis and J. Campbell

3) Collinear limits of all amplitudes (known in a general, universal form);

4) Soft limits for tree-level amplitudes (known; eikonal factors are slightly more difficult for massive particles).

5) Soft limits for one-loop scattering amplitude that include top quarks are less well-known; they require the soft-current at one loop for the massive fermion.

Bierenbaum, Czakon and Mitov

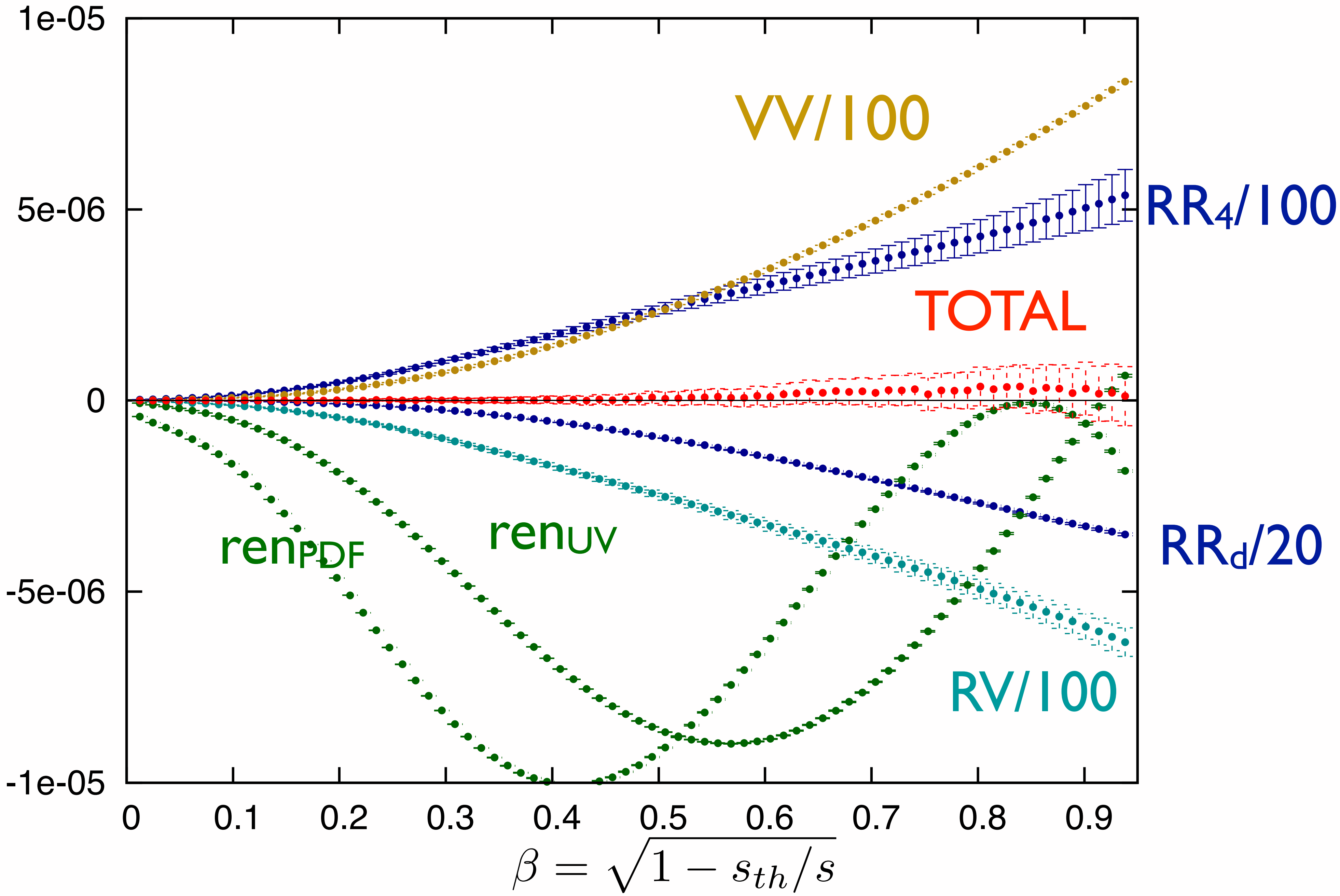
6) One-loop amplitudes for bW -> t g are known in a compact form and can be borrowed from e.g. MCFM;

J. Campbell and F. Tramontano

With these ingredients at place, one needs to perform phase-space partitioning (simple for heavy-quark line since no final state singularities), calculate the relevant limits, remove remaining singularities by performing renormalization (PDFs including). All of this has to be done for a multitude of partonic channels (quark-quark, quark-gluon etc.) -- a bit of a logistic nightmare.

t-channel single-top production at NNLO

Since all calculations are done numerically, cancellation of infra-red and collinear divergencies in the final result are also not exact. In fact, the degree of cancellation provides a useful check on the correctness of the implementation of various contributions.



$1/\epsilon$ poles, summing individual contributions

t-channel single top production at NNLO

We obtain the following results for the single-top cross-sections at leading, next-to-leading and next-to-next-to-leading order in perturbative QCD at 8 TeV LHC.

8 TeV LHC, MSTW2008, m_t = 173.2 GeV

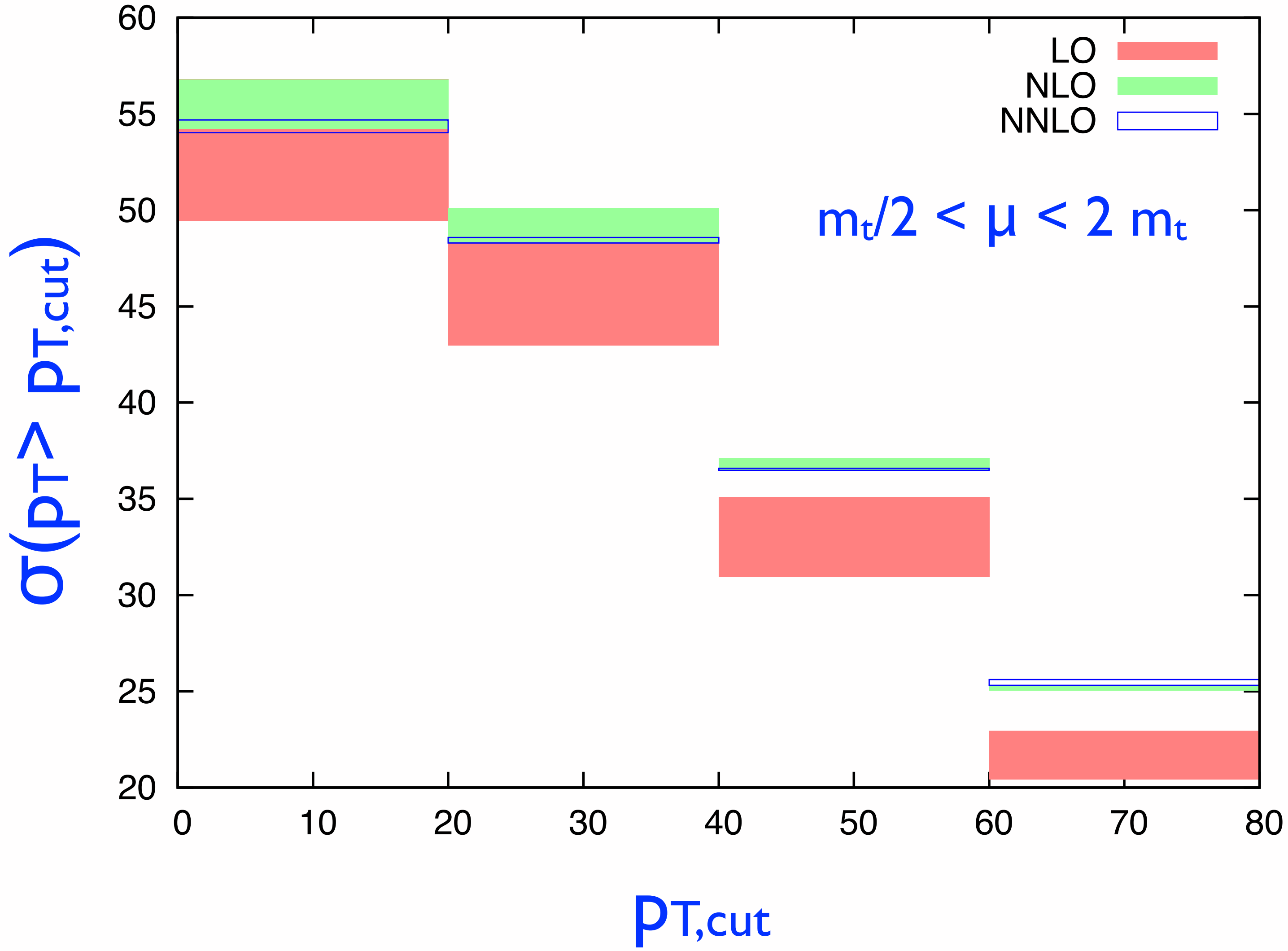
$$\sigma_{\rm LO} = 53.8^{+3.0}_{-4.3} \ \ {\rm pb} \qquad\qquad \sigma_{\rm NLO} = 55.1^{+1.6}_{-0.9} \ \ {\rm pb}$$

$$\sigma_{\rm NNLO} = 54.2^{+0.5}_{-0.2} \ {\rm pb}$$

- $\mu_R=\mu_F= \{m_t/2, m_t, 2\ m_t\}$
- next-to-leading order corrections at central scale are very small, much smaller than their natural O(10%) size; this is a consequence of significant cancellations between different channels.
- Delicate interplay/cancellations between different channels -> important to consistently compute corrections to all of them;
- The NNLO result is very close to the NLO result (-1.6%), reduced μ dependence -> good theoretical control

t-channel single top production at NNLO

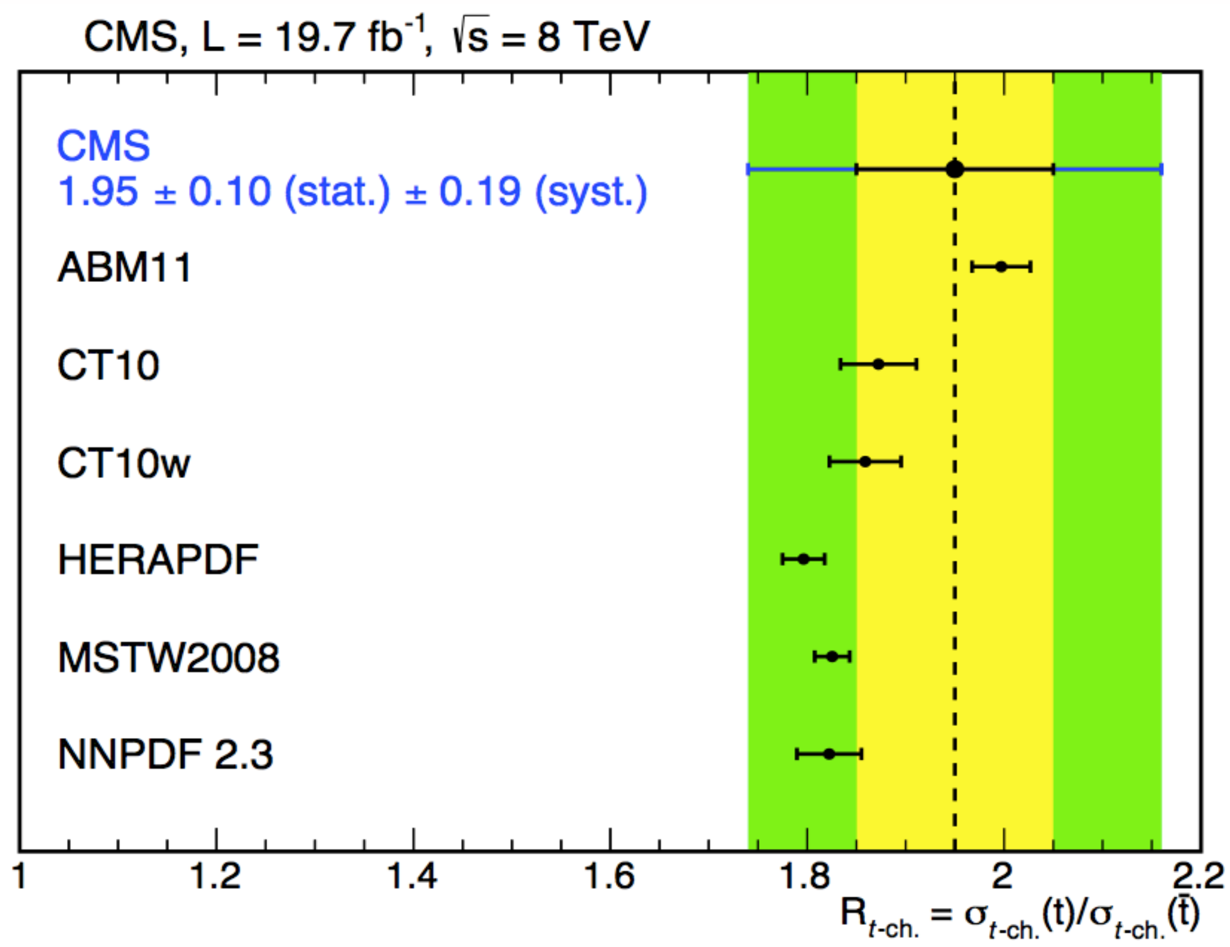
$p_\perp$	σ_{LO}, pb	σ_{NLO}, pb	δ_{NLO}	σ_{NNLO}, pb	δ_{NNLO}
0 GeV	$53.8^{+3.0}_{-4.3}$	$55.1^{+1.6}_{-0.9}$	$+2.4\%$	$54.2^{+0.5}_{-0.2}$	-1.6%
20 GeV	$46.6^{+2.5}_{-3.7}$	$48.9^{+1.2}_{-0.5}$	$+4.9\%$	$48.3^{+0.3}_{-0.02}$	-1.2%
40 GeV	$33.4^{+1.7}_{-2.5}$	$36.5^{+0.6}_{-0.03}$	$+9.3\%$	$36.5^{+0.1}_{+0.1}$	-0.1%
60 GeV	$22.0^{+1.0}_{-1.5}$	$25.0^{+0.2}_{+0.3}$	$+13.6\%$	$25.4^{-0.1}_{+0.2}$	$+1.6\%$



- Unnaturally small corrections disappear in kinematic quantities beyond the total cross-section. For example, we may consider cross-sections with a cut on a minimal value of top quark pt.
- NLO QCD corrections in this case become O(10%) at higher values of pt.
- On the contrary, the NNLO corrections are O(1%) for all values of pt;
- Scale dependence typically improves;

t-channel single top production at NNLO

8 TeV LHC, MSTW2008, m_t = 173.2 GeV



An interesting observable is the ratio of single top quark and antiquark production cross-sections.

$$\sigma_{t,\mathrm{LO}}/\sigma_{\bar{t},\mathrm{LO}} = 1.85$$

$$\sigma_{t,\mathrm{NLO}}/\sigma_{\bar{t},\mathrm{NLO}} = 1.83$$

$$\sigma_{t,\mathrm{NNLO}}/\sigma_{\bar{t},\mathrm{NNLO}} = 1.83$$

The results for the ratio appear to be very stable against inclusion of higher-order QCD corrections, at least for the choice of PDFs indicated above. Note strong PDF dependence -- should eventually give a useful constraint on quark/anti-quark PDF ratios. Note that scale variation errors at LO and NLO are not good indicators of higher orders, as it is often the case with ratios.

NNLO QCD computations

The story of NNLO QCD computations for hadron colliders is interesting. Many pieces that need to be known for any such computation have been known for a very long time but it was not understood how to put the various pieces together in a consistent way. Several methods for NNLO QCD computations, that appeared in the past few years, solve this problem. A method that I am mostly familiar with is based on the FKS phase-space partitioning and (imporved) sector decomposition; It seems to be robust and applicable to various 2 -> 2 processes and, perhaps, beyond.

All existing NNLO QCD methods are based on proper ingredients -- scattering amplitudes, universal soft and collinear limits etc. -- and therefore probably scale in an optimal way with increased number of particles.

The technology for computing two-loop integrals -- essential ingredients for these computations -- is also rapidly developing and may surpass the 2->2 threshold. An interesting new development here is an attempt to extend unitarity methods to two loops but it is too early to say how successful these extensions are going to be.

What to expect from NNLO in the coming years ?

Progress with NNLO computations in the past two years was very impressive. There is no doubt that the new NNLO technology will keep being applied to increasingly broad classes of processes. In many ways, NNLO is replacing NLO as a theoretical frontier for applying perturbative QFT to hadron collider phenomenology. There is currently a NNLO wish-list created as part of the Snowmass community planning exercise in US that happened last year. Below is a summary of some processes from the wish-list and some comments of what they are useful for and what needs to be done:

1) H+ j	Higgs transverse momentum distribution; Higgs decays to observable final states and loops with massive particles
2) H+V	Couplings; H -> bb @NNLO
3) HH	Higgs self-coupling; NLO with exact top mass dependence, virtual corrections;
4) tt+jet	Jet bins for forward-backwat top asymmetry;
5) single top	tWb couplings; top decay through NNO
6) dijets	PDF fits, contact four-quark operators
7) tri-jets	strong coupling constant; reductions, master integrals
8) V+j	PDFs (gluon), backgrounds
9) V1V2	anomalous couplings, backgrounds (Higgs) ; fiducial volume cross-sections
10) gg -> V1 V2	background to Higgs, signal-background interference; loops with massive particles
11) jets in DIS	strong coupling; PDFs etc.

Massive particles in loops; production and decays; 2 -> 3 processes; interface with parton showers.

Many interesting phenomenological applications should be expected in the comming years.

Instead of conclusions

An appropriate conclusion for the talk like that is to list my research interests.

1) Collider (LHC) physics;

2) Higgs physics (predictions for rates and shapes, (e.g. H+j), anomalous couplings, Higgs pair production, Higgs width measurements at the LHC);

3) Top quark physics (single top production, top quark decays, spin correlations, top quark mass);

4) Physics of electroweak gauge bosons (NNLO QCD predictions for VV*,V+jets, background to Higgs production etc.);

5) Low-energy tests of the Standard Model (g-2, muon decay etc.);

6) Technology for higher-order perturbative computations in general and for exclusive NNLO QCD computations in particular;