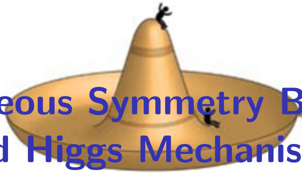


Spontaneous Symmetry Breaking and Higgs Mechanism

GK Doktorandenworkshop Althütte

Wolfgang G. Hollik

Institut für Theoretische Teilchenphysik | KIT Campus Süd

Karlsruhe Institute of Technology

December 11, 2012

“Prehistory of the Higgs boson”

[P. Higgs: C. R. Physique 8 (2007) 970]

"Prehistory of the Higgs boson"

[P. Higgs: C. R. Physique 8 (2007) 970]

1928: Heisenberg (Ferromagnetism, nearest-neighbour interaction)

"Prehistory of the Higgs boson"

[P. Higgs: C. R. Physique 8 (2007) 970]

1928: Heisenberg (Ferromagnetism, nearest-neighbour interaction)

1947: Bogoliubov (Superfluidity, Bose condensation with short-range repulsive two-body interaction)

“Prehistory of the Higgs boson”

[P. Higgs: C. R. Physique 8 (2007) 970]

1928: Heisenberg (Ferromagnetism, nearest-neighbour interaction)

1947: Bogoliubov (Superfluidity, Bose condensation with short-range repulsive two-body interaction)

1950: Ginzburg and Landau (Superconductivity, Bose condensation of charged quasi-particles)

“Prehistory of the Higgs boson”

[P. Higgs: C. R. Physique 8 (2007) 970]

1928: Heisenberg (Ferromagnetism, nearest-neighbour interaction)

1947: Bogoliubov (Superfluidity, Bose condensation with short-range repulsive two-body interaction)

1950: Ginzburg and Landau (Superconductivity, Bose condensation of charged quasi-particles)

1957: Bardeen, Cooper and Schrieffer identified the Bosons of Ginzburg–Landau (effective theory)

"Prehistory of the Higgs boson"

[P. Higgs: C. R. Physique 8 (2007) 970]

1928: Heisenberg (Ferromagnetism, nearest-neighbour interaction)

1947: Bogoliubov (Superfluidity, Bose condensation with short-range repulsive two-body interaction)

1950: Ginzburg and Landau (Superconductivity, Bose condensation of charged quasi-particles)

1957: Bardeen, Cooper and Schrieffer identified the Bosons of Ginzburg–Landau (effective theory)

1960: Nambu (relativistic models, scalar condensate breaks symmetry to generate masses for fermions)

"Prehistory of the Higgs boson"

[P. Higgs: C. R. Physique 8 (2007) 970]

- **1928:** Heisenberg (Ferromagnetism, nearest-neighbour interaction)
- **1947:** Bogoliubov (Superfluidity, Bose condensation with short-range repulsive two-body interaction)
- **1950:** Ginzburg and Landau (Superconductivity, Bose condensation of charged quasi-particles)
- **1957:** Bardeen, Cooper and Schrieffer identified the Bosons of Ginzburg–Landau (effective theory)
- **1960:** Nambu (relativistic models, scalar condensate breaks symmetry to generate masses for fermions)
- **1961:** Goldstone (spinless zero-mass excitations)

“Prehistory of the Higgs boson”

[P. Higgs: C. R. Physique 8 (2007) 970]

1928: Heisenberg (Ferromagnetism, nearest-neighbour interaction)

1947: Bogoliubov (Superfluidity, Bose condensation with short-range repulsive two-body interaction)

1950: Ginzburg and Landau (Superconductivity, Bose condensation of charged quasi-particles)

1957: Bardeen, Cooper and Schrieffer identified the Bosons of Ginzburg–Landau (effective theory)

1960: Nambu (relativistic models, scalar condensate breaks symmetry to generate masses for fermions)

1961: Goldstone (spinless zero-mass excitations)

1962: Goldstone, Salam and Weinberg (proved Goldstone’s theorem)

"Prehistory of the Higgs boson"

[P. Higgs: C. R. Physique 8 (2007) 970]

1928: Heisenberg (Ferromagnetism, nearest-neighbour interaction)

1947: Bogoliubov (Superfluidity, Bose condensation with short-range repulsive two-body interaction)

1950: Ginzburg and Landau (Superconductivity, Bose condensation of charged quasi-particles)

1957: Bardeen, Cooper and Schrieffer identified the Bosons of Ginzburg–Landau (effective theory)

1960: Nambu (relativistic models, scalar condensate breaks symmetry to generate masses for fermions)

1961: Goldstone (spinless zero-mass excitations)

1962: Goldstone, Salam and Weinberg (proved Goldstone's theorem)

1963: Anderson ("Higgs mechanism" in superconductors)

"Prehistory of the Higgs boson"

[P. Higgs: C. R. Physique 8 (2007) 970]

1928: Heisenberg (Ferromagnetism, nearest-neighbour interaction)

1947: Bogoliubov (Superfluidity, Bose condensation with short-range repulsive two-body interaction)

1950: Ginzburg and Landau (Superconductivity, Bose condensation of charged quasi-particles)

1957: Bardeen, Cooper and Schrieffer identified the Bosons of Ginzburg–Landau (effective theory)

1960: Nambu (relativistic models, scalar condensate breaks symmetry to generate masses for fermions)

1961: Goldstone (spinless zero-mass excitations)

1962: Goldstone, Salam and Weinberg (proved Goldstone's theorem)

1963: Anderson ("Higgs mechanism" in superconductors)

1964: Higgs (local gauge invariance fails axioms of Goldstone: evade Goldstone's theorem in gauge theories)

“Prehistory of the Higgs boson” (cont’d)

1964: Higgs (simples field-theoretic model (“Higgs model”), relativistic version of Ginzburg–Landau) (rejected by PL!)

“Prehistory of the Higgs boson” (cont’d)

1964: Higgs (simples field-theoretic model (“Higgs model”), relativistic version of Ginzburg–Landau) (rejected by PL!)

1964: Higgs 2nd version (+ general features of ssb gauge theories, massive scalar excitations remaining: Higgs bosons) [1]

“Prehistory of the Higgs boson” (cont’d)

1964: Higgs (simples field-theoretic model (“Higgs model”), relativistic version of Ginzburg–Landau) (rejected by PL!)

1964: Higgs 2nd version (+ general features of ssb gauge theories, massive scalar excitations remaining: Higgs bosons) [1]

1964: Englert and Brout (QFT instead of classical FT by Higgs) [2]

“Prehistory of the Higgs boson” (cont’d)

1964: Higgs (simples field-theoretic model (“Higgs model”), relativistic version of Ginzburg–Landau) (rejected by PL!)

1964: Higgs 2nd version (+ general features of ssb gauge theories, massive scalar excitations remaining: Higgs bosons) [1]

1964: Englert and Brout (QFT instead of classical FT by Higgs) [2]

1967: Weinberg (SSB of Glashow’s $\mathrm{SU}(2) \times \mathrm{U}(1)$)

"Prehistory of the Higgs boson" (cont'd)

1964: Higgs (simples field-theoretic model ("Higgs model"), relativistic version of Ginzburg–Landau) (rejected by PL!)

1964: Higgs 2nd version (+ general features of ssb gauge theories, massive scalar excitations remaining: Higgs bosons) [1]

1964: Englert and Brout (QFT instead of classical FT by Higgs) [2]

1967: Weinberg (SSB of Glashow's $\mathrm{SU}(2) \times \mathrm{U}(1)$)

1968: Salam (?) [in: Proceedings of the Eighth Nobel Symposium, p. 367]

“Prehistory of the Higgs boson” (cont’d)

1964: Higgs (simples field-theoretic model (“Higgs model”), relativistic version of Ginzburg–Landau) (rejected by PL!)

1964: Higgs 2nd version (+ general features of ssb gauge theories, massive scalar excitations remaining: Higgs bosons) [1]

1964: Englert and Brout (QFT instead of classical FT by Higgs) [2]

1967: Weinberg (SSB of Glashow’s $\mathrm{SU}(2) \times \mathrm{U}(1)$)

1968: Salam (?) [in: Proceedings of the Eighth Nobel Symposium, p. 367]

1971: t’Hooft (completed Veltman’s renormalization programme)

“Prehistory of the Higgs boson” (cont’d)

1964: Higgs (simples field-theoretic model (“Higgs model”), relativistic version of Ginzburg–Landau) (rejected by PL!)

1964: Higgs 2nd version (+ general features of ssb gauge theories, massive scalar excitations remaining: Higgs bosons) [1]

1964: Englert and Brout (QFT instead of classical FT by Higgs) [2]

1967: Weinberg (SSB of Glashow’s $\mathrm{SU}(2) \times \mathrm{U}(1)$)

1968: Salam (?) [in: Proceedings of the Eighth Nobel Symposium, p. 367]

1971: t’Hooft (completed Veltman’s renormalization programme)

1972: HEP Conference @ Fermilab (outbreak of theoretical activity)

"Prehistory of the Higgs boson" (cont'd)

1964: Higgs (simples field-theoretic model ("Higgs model"), relativistic version of Ginzburg–Landau) (rejected by PL!)

1964: Higgs 2nd version (+ general features of ssb gauge theories, massive scalar excitations remaining: Higgs bosons) [1]

1964: Englert and Brout (QFT instead of classical FT by Higgs) [2]

1967: Weinberg (SSB of Glashow's $\mathrm{SU}(2) \times \mathrm{U}(1)$)

1968: Salam (?) [in: Proceedings of the Eighth Nobel Symposium, p. 367]

1971: t'Hooft (completed Veltman's renormalization programme)

1972: HEP Conference @ Fermilab (outbreak of theoretical activity)

1973: discovery of weak neutral currents

“Prehistory of the Higgs boson” (cont’d)

1964: Higgs (simples field-theoretic model (“Higgs model”), relativistic version of Ginzburg–Landau) (rejected by PL!)

1964: Higgs 2nd version (+ general features of ssb gauge theories, massive scalar excitations remaining: Higgs bosons) [1]

1964: Englert and Brout (QFT instead of classical FT by Higgs) [2]

1967: Weinberg (SSB of Glashow’s $\mathrm{SU}(2) \times \mathrm{U}(1)$)

1968: Salam (?) [in: Proceedings of the Eighth Nobel Symposium, p. 367]

1971: t’Hooft (completed Veltman’s renormalization programme)

1972: HEP Conference @ Fermilab (outbreak of theoretical activity)

1973: discovery of weak neutral currents

“Prehistory of the Higgs boson” (cont’d)

1964: Higgs (simples field-theoretic model (“Higgs model”), relativistic version of Ginzburg–Landau) (rejected by PL!)

1964: Higgs 2nd version (+ general features of ssb gauge theories, massive scalar excitations remaining: Higgs bosons) [1]

1964: Englert and Brout (QFT instead of classical FT by Higgs) [2]

1967: Weinberg (SSB of Glashow’s $\mathrm{SU}(2) \times \mathrm{U}(1)$)

1968: Salam (?) [in: Proceedings of the Eighth Nobel Symposium, p. 367]

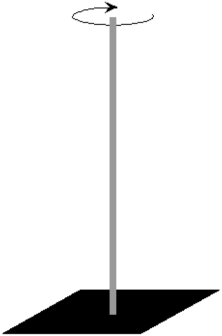
1971: t’Hooft (completed Veltman’s renormalization programme)

1972: HEP Conference @ Fermilab (outbreak of theoretical activity)

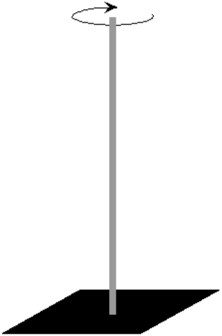
1973: discovery of weak neutral currents

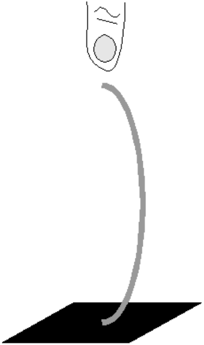
Anecdote besides: When Higgs met Nambu twenty years later, he revealed that he had been the referee of [1] and [2].

What does (spontaneous) symmetry breaking mean?

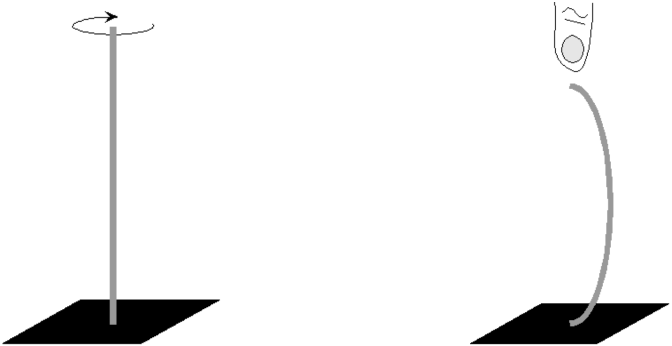


What does (spontaneous) symmetry breaking mean?





What does (spontaneous) symmetry breaking mean?



- a parameter assumes a critical value
- the symmetric configuration gets unstable
- the ground state ist degenerate

Degenerate vacua in quantum mechanics

Ferromagnet: rotational symmetric Hamiltonian

$$H = -J \sum_{i,j} \vec{S}_i \cdot \vec{S}_j$$

Degenerate vacua in quantum mechanics

Ferromagnet: rotational symmetric Hamiltonian

$$H = -J \sum_{i,j} \vec{S}_i \cdot \vec{S}_j$$

- Ground state rotationally invariant: $\vec{M} = \langle \vec{S}_i \rangle = 0$.
- Below critical temperature: non-zero magnetization $\vec{M} \neq 0$.
- New vacuum has $\mathrm{SO}(2)$ instead of $\mathrm{SO}(3)$ rotational symmetry.

Degenerate vacua in quantum mechanics

Ferromagnet: rotational symmetric Hamiltonian

$$H = -J \sum_{i,j} \vec{S}_i \cdot \vec{S}_j$$

- Ground state rotationally invariant: $\vec{M} = \langle \vec{S}_i \rangle = 0$.
- Below critical temperature: non-zero magnetization $\vec{M} \neq 0$.
- New vacuum has $\mathrm{SO}(2)$ instead of $\mathrm{SO}(3)$ rotational symmetry.

Degenerate vacua

- Instead of a single vacuum state, now: family of vacua related via rotations.
- System chooses the particular vacuum itself: symmetry is spontaneously broken by the choice of a vacuum.

Symmetric potential, non-symmetric ground state

Global symmetry

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - V(\phi, \phi^*)$$

impose *global* phase transformation: $\phi \to e^{i\theta}\phi$ ($\mathrm{U}(1)$ symmetry)

$$V(\phi, \phi^*) = V(|\phi|) = m^2 \phi\phi^* + \lambda\left(\phi\phi^*\right)^2$$

Symmetric potential, non-symmetric ground state

Global symmetry

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - V(\phi, \phi^*)$$

impose *global* phase transformation: $\phi \to e^{i\theta}\phi$ ($\mathrm{U}(1)$ symmetry)

$$V(\phi, \phi^*) = V(|\phi|) = m^2 \phi\phi^* + \lambda \left(\phi\phi^*\right)^2$$

Ground state: Minimizing the potential!

$$\frac{\partial V}{\partial \phi} = m^2 \phi^* + 2\lambda\phi^* \left(\phi\phi^*\right) \stackrel{!}{=} 0$$

Symmetric potential, non-symmetric ground state

Global symmetry

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - V(\phi, \phi^*)$$

impose *global* phase transformation: $\phi \to e^{i\theta}\phi$ ($\mathrm{U}(1)$ symmetry)

$$V(\phi, \phi^*) = V(|\phi|) = m^2 \phi\phi^* + \lambda\,(\phi\phi^*)^2$$

Ground state: Minimizing the potential!

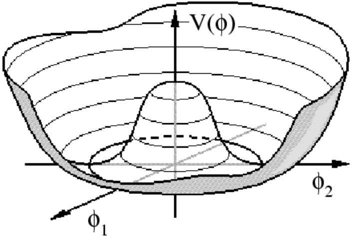
$$\frac{\partial V}{\partial \phi} = m^2 \phi^* + 2\lambda\phi^*\,(\phi\phi^*) \stackrel{!}{=} 0$$

- $m^2 > 0$: $\phi^* = 0 = \phi$
- $m^2 < 0$: local max $\phi = 0$, minima:

$$|\phi|^2 = -\frac{m^2}{2\lambda} = v^2 \quad \Leftrightarrow \quad |\langle 0|\phi|0\rangle|^2 = v^2$$

Mexican hat

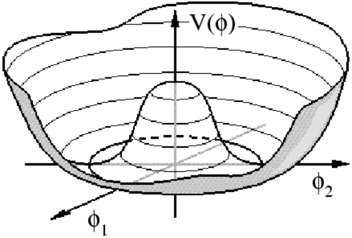
decomposing: $\phi = \phi_1 + i\phi_2$



Minima of V along circle $|\phi| = v$. If system chooses particular direction, e.g. $\phi_1 = v$ (meaning $\phi_2 = 0$), symmetry is *spontaneously broken*.

Mexican hat

decomposing: $\phi = \phi_1 + i\phi_2$



Minima of V along circle $|\phi| = v$. If system chooses particular direction, e.g. $\phi_1 = v$ (meaning $\phi_2 = 0$), symmetry is *spontaneously broken*.

Polar coordinates vs. real and imaginary parts

$$\phi(x) = \rho(x) e^{i\alpha(x)} = \phi_1(x) + i\phi_2(x),$$

expanding around the vacuum: $\phi(x) = v + \frac{1}{\sqrt{2}} \left(h(x) + i g(x) \right)$

Higgs and Goldstone particles

Plug the expansion $\phi(x) = v + \frac{1}{\sqrt{2}}\left(h(x) + ig(x)\right)$ into the potential $V(|\phi|) = m^2\phi\phi^* + \lambda\left(\phi\phi^*\right)^2$:

$$\mathcal{L} = \text{const.} + \frac{1}{2}\partial_\mu h \partial^\mu h + \frac{1}{2}\partial_\mu g \partial^\mu g - \frac{1}{2}\underbrace{\left(-2m^2\right)}_{m_h^2} h^2 + \mathcal{WW}.$$

- $h(x)$, $g(x)$ *real* scalar fields
- starting with one *complex* scalar $\phi(x)$ having mass m
- $m^2 < 0 \quad \hookrightarrow \quad m_h^2 > 0$: h acquires mass $m_h = \sqrt{-2m^2}$
- g is *massless* $\quad \hookrightarrow$ Goldstone boson

Higgs and Goldstone particles

Plug the expansion $\phi(x) = v + \frac{1}{\sqrt{2}}\left(h(x) + ig(x)\right)$ into the potential $V(|\phi|) = m^2\phi\phi^* + \lambda\left(\phi\phi^*\right)^2$:

$$\mathcal{L} = \text{const.} + \frac{1}{2}\partial_\mu h \partial^\mu h + \frac{1}{2}\partial_\mu g \partial^\mu g - \frac{1}{2}\underbrace{\left(-2m^2\right)}_{m_h^2} h^2 + \mathcal{WW}.$$

- $h(x)$, $g(x)$ *real* scalar fields
- starting with one *complex* scalar $\phi(x)$ having mass m
- $m^2 < 0 \quad \hookrightarrow \quad m_h^2 > 0$: h acquires mass $m_h = \sqrt{-2m^2}$
- g is *massless* $\quad \hookrightarrow$ Goldstone boson[a]

[a] Goldstone particles may be fermions as well: e.g. Goldstinos of SUSY breaking theories

Goldstone's theorem

Spontaneous breaking of a (continuous) symmetry
$\hookrightarrow$ *massless* particle: (Nambu-)Goldstone particle

The Goldstone Theorem

Goldstone's theorem

Spontaneous breaking of a (continuous) symmetry
$\hookrightarrow$ *massless* particle: (Nambu-)Goldstone particle

Manifold of vacua

- $\mathrm{U}(1)$ example: 1D vacuum

Goldstone's theorem

Spontaneous breaking of a (continuous) symmetry
$\hookrightarrow$ *massless* particle: (Nambu-)Goldstone particle

Manifold of vacua

- $\mathrm{U}(1)$ example: 1D vacuum
- dimensionality of vacua-manifold:
  $\#$ of generators that break the symmetry

The Goldstone Theorem

Goldstone's theorem

Spontaneous breaking of a (continuous) symmetry
$\hookrightarrow$ *massless* particle: (Nambu-)Goldstone particle

Manifold of vacua

- $\mathrm{U}(1)$ example: 1D vacuum
- dimensionality of vacua-manifold:
  $\#$ of generators that break the symmetry
- zero vacuum energy: $H|0\rangle = 0$

The Goldstone Theorem

Goldstone's theorem

Spontaneous breaking of a (continuous) symmetry
$\hookrightarrow$ *massless* particle: (Nambu-)Goldstone particle

Manifold of vacua

- $\mathrm{U}(1)$ example: 1D vacuum
- dimensionality of vacua-manifold:
  $\#$ of generators that break the symmetry
- zero vacuum energy: $H|0\rangle = 0$
- generator of symmetry trafo T^a: $[T^a, H] = 0$

$$H\left(T^a|0\rangle\right) = T^a H|0\rangle = 0$$

The Goldstone Theorem

Goldstone's theorem

Spontaneous breaking of a (continuous) symmetry
$\hookrightarrow$ *massless* particle: (Nambu-)Goldstone particle

Manifold of vacua

- $\mathrm{U}(1)$ example: 1D vacuum
- dimensionality of vacua-manifold:
  $\#$ of generators that break the symmetry
- zero vacuum energy: $H|0\rangle = 0$
- generator of symmetry trafo T^a: $[T^a, H] = 0$

$$H\left(T^a|0\rangle\right) = T^a H|0\rangle = 0$$

- if vacuum is not invariant under symmetry: $T^a|0\rangle \neq 0$,
  we have a new state with minimum energy, a new vacuum!

A non-Abelian example

Goldstone's theorem:

- one Goldstone particle for each generator which breaks the symmetry
- quantum numbers of those Goldstones are the same as the corresponding generators

A non-Abelian example

Goldstone's theorem:

- one Goldstone particle for each generator which breaks the symmetry
- quantum numbers of those Goldstones are the same as the corresponding generators

Abelian example:

$\mathrm{U}(1)$ symmetry: ϕ in 2-dimensional representation

A non-Abelian example

Group of spatial rotations: $\mathrm{SO}(3)$

- ϕ_i in fundamental (isovector) representation: $i = 1, 2, 3$

A non-Abelian example

Group of spatial rotations: $\mathrm{SO}(3)$

- ϕ_i in fundamental (isovector) representation: $i = 1, 2, 3$

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi_i\partial^\mu\phi_i - \frac{m^2}{2}\phi_i\phi_i - \lambda\left(\phi_i\phi_i\right)^2\,,$$

A non-Abelian example

Group of spatial rotations: $\mathrm{SO}(3)$

- ϕ_i in fundamental (isovector) representation: $i = 1, 2, 3$

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi_i \partial^\mu \phi_i - \frac{m^2}{2}\phi_i\phi_i - \lambda\left(\phi_i\phi_i\right)^2,$$

$$\mathcal{G} : \phi_i \to \left(e^{-i\alpha_k \omega^{(k)}}\right)_{ij} \phi_j = U_{ij}\phi_j.$$

A non-Abelian example

Group of spatial rotations: $\mathrm{SO}(3)$

- ϕ_i in fundamental (isovector) representation: $i = 1, 2, 3$

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi_i \partial^\mu \phi_i - \frac{m^2}{2}\phi_i\phi_i - \lambda\left(\phi_i\phi_i\right)^2,$$

$$\mathcal{G} : \phi_i \to \left(e^{-i\alpha_k \omega^{(k)}}\right)_{ij} \phi_j = U_{ij}\phi_j.$$

- Minimum of the potential with $m^2 < 0$:

$$|\phi_0| = \sqrt{\phi_1^2 + \phi_2^2 + \phi_3^2} = \left(\frac{-m^2}{4\lambda}\right)^{1/2} = v$$

- freedom to choose "physical" vacuum: $\vec{\phi}_0 = v\hat{e}_3$

Broken generators

Choosing vacuum as $\phi_0 = v\hat{e}_3$: *not* invariant under full group $\mathcal{G}$, but subgroup $\mathcal{H} \in \mathcal{G}$ (rotations around 3-axis)

$$\mathcal{H} : \vec{\phi}'_0 = exp^{i\alpha_3\omega^{(3)}}\vec{\phi}_0 = \vec{\phi}_0,$$

Broken generators

Choosing vacuum as $\phi_0 = v\hat{e}_3$: *not* invariant under full group $\mathcal{G}$, but subgroup $\mathcal{H} \in \mathcal{G}$ (rotations around 3-axis)

$$\mathcal{H} : \vec{\phi}'_0 = exp^{i\alpha_3\omega^{(3)}}\vec{\phi}_0 = \vec{\phi}_0,$$

but nevertheless: potential $V(\phi)$ *is* invariant under $\mathcal{G}$:

$$V(\phi') = V(\phi)$$

Broken generators

Choosing vacuum as $\phi_0 = v\hat{e}_3$: *not* invariant under full group $\mathcal{G}$, but subgroup $\mathcal{H} \in \mathcal{G}$ (rotations around 3-axis)

$$\mathcal{H} : \vec{\phi}_0' = exp^{i\alpha_3\omega^{(3)}}\vec{\phi}_0 = \vec{\phi}_0,$$

but nevertheless: potential $V(\phi)$ *is* invariant under $\mathcal{G}$:

$$V(\phi') = V(\phi)$$

How many Goldstone bosons?

ϕ_3 acquires vev: $\phi_3 = \chi + v$, $\langle\phi_1\rangle = 0$, $\langle\phi_2\rangle = 0$, $\langle\chi\rangle = 0$.

- quadratic term in the potential: only $\sim \chi^2$
  $$m_\chi^2 = 8v^2\lambda, \qquad m_{\phi_1} = m_{\phi_2} = 0.$$
- one generator $(\omega^{(3)})$ left: $\mathcal{H} = \mathrm{SO}(2) \cong \mathrm{U}(1)$
  $\hookrightarrow$ one massive field

Broken generators

Choosing vacuum as $\phi_0 = v\hat{e}_3$: *not* invariant under full group $\mathcal{G}$, but subgroup $\mathcal{H} \in \mathcal{G}$ (rotations around 3-axis)

$$\mathcal{H} : \vec{\phi}_0' = exp^{i\alpha_3 \omega^{(3)}} \vec{\phi}_0 = \vec{\phi}_0,$$

but nevertheless: potential $V(\phi)$ *is* invariant under $\mathcal{G}$:

$$V(\phi') = V(\phi)$$

How many Goldstone bosons?

ϕ_3 acquires vev: $\phi_3 = \chi + v$, $\langle \phi_1 \rangle = 0$, $\langle \phi_2 \rangle = 0$, $\langle \chi \rangle = 0$.

- quadratic term in the potential: only $\sim \chi^2$
  $$m_\chi^2 = 8v^2\lambda, \qquad m_{\phi_1} = m_{\phi_2} = 0.$$
- one generator $(\omega^{(3)})$ left: $\mathcal{H} = \mathrm{SO}(2) \cong \mathrm{U}(1)$
  $\hookrightarrow$ one massive field

\# of Goldstone particles: $n_G = \dim(\mathcal{G}/\mathcal{H}) = \dim \mathcal{G} - \dim \mathcal{H}$.

Abelian gauge symmetries

- up to now: *global* symmetries: $\phi \to e^{iq\theta}\phi$
- now: *local* (= gauge) symmetry: $\phi \to e^{iq\theta(x)}\phi$

$\mathrm{U}(1)$ gauge invariant Lagrangian:

$$\mathcal{L} = \left(D_\mu \phi\right)^* D^\mu \phi - V(|\phi|) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu},$$

gauge-covariant derivative: $D_\mu \phi = \left(\partial_\mu + iqA_\mu\right)\phi$,
field strength tensor: $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$,

$$V(|\phi|) = m^2 \phi^* \phi + \lambda \left(\phi^* \phi\right)^2,$$

minimum: $v = \sqrt{\frac{-m^2}{2\lambda}} \quad \hookrightarrow \quad \phi(x) = \left(v + \frac{1}{\sqrt{2}} h(x)\right) e^{i\alpha(x)}$

Abelian gauge symmetries

- up to now: *global* symmetries: $\phi \to e^{iq\theta}\phi$
- now: *local* (= gauge) symmetry: $\phi \to e^{iq\theta(x)}\phi$

$\mathrm{U}(1)$ gauge invariant Lagrangian:

$$\mathcal{L} = \left(D_\mu\phi\right)^* D^\mu\phi - V(|\phi|) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu},$$

gauge-covariant derivative: $D_\mu\phi = \left(\partial_\mu + iqA_\mu\right)\phi$,
field strength tensor: $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$,

$$V(|\phi|) = m^2\phi^*\phi + \lambda\left(\phi^*\phi\right)^2,$$

minimum: $v = \sqrt{\frac{-m^2}{2\lambda}} \quad \hookrightarrow \quad \phi(x) = \left(v + \frac{1}{\sqrt{2}}h(x)\right)e^{i\alpha(x)a}$

Abelian gauge symmetries

- up to now: *global* symmetries: $\phi \to e^{iq\theta}\phi$
- now: *local* (= gauge) symmetry: $\phi \to e^{iq\theta(x)}\phi$

U(1) gauge invariant Lagrangian:

$$\mathcal{L} = (D_\mu\phi)^* D^\mu\phi - V(|\phi|) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu},$$

gauge-covariant derivative: $D_\mu\phi = (\partial_\mu + iqA_\mu)\,\phi$,
field strength tensor: $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$,

$$V(|\phi|) = m^2\phi^*\phi + \lambda\,(\phi^*\phi)^2\,,$$

minimum: $v = \sqrt{\frac{-m^2}{2\lambda}} \quad \hookrightarrow \quad \phi(x) = \left(v + \frac{1}{\sqrt{2}}h(x)\right)e^{i\alpha(x)}$[a]

$$D_\mu\phi = \frac{1}{\sqrt{2}}\partial_\mu h(x) + iq(v + \frac{1}{\sqrt{2}}h(x))A_\mu.$$

[a] phase $\alpha(x)$ can be removed by gauge transformation

Higgs mechanism: massive gauge bosons

Rewriting the Lagrangian:

$$\mathcal{L} = \frac{1}{2}\partial^\mu h(x)\partial_\mu h(x) - \frac{1}{2}\underbrace{2\lambda v^2}_{m_h^2} h(x)^2 - \lambda\left(\frac{v}{\sqrt{2}}h(x)^3 + \frac{1}{8}h(x)^4\right)$$

$$+ q^2\left(v + \frac{1}{\sqrt{2}}h(x)\right)^2 A_\mu A^\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

- term $\sim A_\mu A^\mu$: mass $m_A^2 = 2q^2v^2$

$$q^2v^2A_\mu A^\mu + \frac{2}{\sqrt{2}}q^2 v h(x) A_\mu A^\mu + \frac{q^2}{2}h(x)^2 A_\mu A^\mu$$

Higgs mechanism: massive gauge bosons

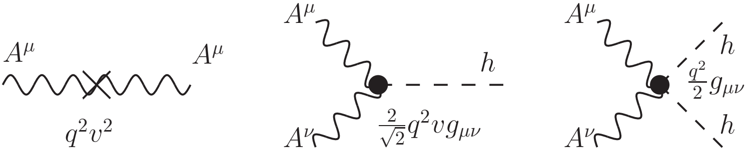
Rewriting the Lagrangian:

$$\mathcal{L} = \frac{1}{2}\partial^\mu h(x)\partial_\mu h(x) - \frac{1}{2}\underbrace{2\lambda v^2}_{m_h^2} h(x)^2 - \lambda\left(\frac{v}{\sqrt{2}}h(x)^3 + \frac{1}{8}h(x)^4\right)$$

$$+ q^2\left(v + \frac{1}{\sqrt{2}}h(x)\right)^2 A_\mu A^\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

- term $\sim A_\mu A^\mu$: mass $m_A^2 = 2q^2v^2$

$$q^2v^2 A_\mu A^\mu + \frac{2}{\sqrt{2}}q^2 v h(x) A_\mu A^\mu + \frac{q^2}{2}h(x)^2 A_\mu A^\mu$$



Massive Photons and Superconductivity

- Decomposing A^μ under spatial rotations $(\mathrm{SO}(3))$: $A^\mu \in \mathbf{0} \oplus \mathbf{1}$.
- 4 dof: -1 by Lorentz trafos -1 by gauge trafos: 2

Massive Photons and Superconductivity

- Decomposing A^μ under spatial rotations $(\mathrm{SO}(3))$: $A^\mu \in \mathbf{0} \oplus \mathbf{1}$.
- 4 dof: -1 by Lorentz trafos -1 by gauge trafos: 2

Degrees of freedom:

- massless vector: 2, complex scalar: 2
- massive vector: 3, one real scalar (Higgs boson): 1
- gauge symmetry broken, but $\partial_\mu A^\mu = 0$ still holds:
  - $\partial_\mu A^\mu \sim k_\mu \varepsilon^\mu(k)$
  - for $\varepsilon^\mu(k) \sim k^\mu$: $\partial_\mu A^\mu \sim k^2 = m_A^2 \neq 0$
  - rest frame: $k^\mu = (m_a, 0, 0, 0)$: $\varepsilon^\mu(k) = (1, 0, 0, 0)$ eliminated
  - $\mathbf{0}$ of A^μ eliminated: spin-0 part
- Goldstone boson?

Massive Photons and Superconductivity

- Decomposing A^μ under spatial rotations ($\mathrm{SO}(3)$): $A^\mu \in \mathbf{0} \oplus \mathbf{1}$.
- 4 dof: -1 by Lorentz trafos -1 by gauge trafos: 2

Degrees of freedom:

- massless vector: 2, complex scalar: 2
- massive vector: 3, one real scalar (Higgs boson): 1
- gauge symmetry broken, but $\partial_\mu A^\mu = 0$ still holds:
  - $\partial_\mu A^\mu \sim k_\mu \varepsilon^\mu(k)$
  - for $\varepsilon^\mu(k) \sim k^\mu$: $\partial_\mu A^\mu \sim k^2 = m_A^2 \neq 0$
  - rest frame: $k^\mu = (m_a, 0, 0, 0)$: $\varepsilon^\mu(k) = (1, 0, 0, 0)$ eliminated
  - $\mathbf{0}$ of A^μ eliminated: spin-0 part
- Goldstone boson? *eaten* by the gauge boson

Massive Photons and Superconductivity

- Decomposing A^μ under spatial rotations ($\mathrm{SO}(3)$): $A^\mu \in \mathbf{0} \oplus \mathbf{1}$.
- 4 dof: -1 by Lorentz trafos -1 by gauge trafos: 2

Degrees of freedom:

- massless vector: 2, complex scalar: 2
- massive vector: 3, one real scalar (Higgs boson): 1
- gauge symmetry broken, but $\partial_\mu A^\mu = 0$ still holds:
  - $\partial_\mu A^\mu \sim k_\mu \varepsilon^\mu(k)$
  - for $\varepsilon^\mu(k) \sim k^\mu$: $\partial_\mu A^\mu \sim k^2 = m_A^2 \neq 0$
  - rest frame: $k^\mu = (m_a, 0, 0, 0)$: $\varepsilon^\mu(k) = (1, 0, 0, 0)$ eliminated
  - $\mathbf{0}$ of A^μ eliminated: spin-0 part
- Goldstone boson? *eaten* by the gauge boson

Superconductivity

Realization of spontaneously broken $\mathrm{U}(1)$ in nature.
electric current: $\vec{j} = \sigma \vec{E}$, σ: conductivity, $\sigma \to \infty$: superconductor

Massive Photons and Superconductivity

- Decomposing A^μ under spatial rotations ($\mathrm{SO}(3)$): $A^\mu \in \mathbf{0} \oplus \mathbf{1}$.
- 4 dof: -1 by Lorentz trafos -1 by gauge trafos: 2

Degrees of freedom:

- massless vector: 2, complex scalar: 2
- massive vector: 3, one real scalar (Higgs boson): 1
- gauge symmetry broken, but $\partial_\mu A^\mu = 0$ still holds:
  - $\partial_\mu A^\mu \sim k_\mu \varepsilon^\mu(k)$
  - for $\varepsilon^\mu(k) \sim k^\mu$: $\partial_\mu A^\mu \sim k^2 = m_A^2 \neq 0$
  - rest frame: $k^\mu = (m_a, 0, 0, 0)$: $\varepsilon^\mu(k) = (1, 0, 0, 0)$ eliminated
  - $\mathbf{0}$ of A^μ eliminated: spin-0 part
- Goldstone boson? *eaten* by the gauge boson

Superconductivity

No electric field inside: $\dot{\vec{B}} = -\vec{\nabla} \times \vec{E} = 0 \quad \leftrightarrow \quad \vec{B}(t) = \vec{B}(0)$
if $\vec{B}(0) = 0$, magnetic field cannot penetrate inside the supercond.

Massive Photons and Superconductivity

- Decomposing A^μ under spatial rotations ($\mathrm{SO}(3)$): $A^\mu \in \mathbf{0} \oplus \mathbf{1}$.
- 4 dof: -1 by Lorentz trafos -1 by gauge trafos: 2

Degrees of freedom:

- massless vector: 2, complex scalar: 2
- massive vector: 3, one real scalar (Higgs boson): 1
- gauge symmetry broken, but $\partial_\mu A^\mu = 0$ still holds:
  - $\partial_\mu A^\mu \sim k_\mu \varepsilon^\mu(k)$
  - for $\varepsilon^\mu(k) \sim k^\mu$: $\partial_\mu A^\mu \sim k^2 = m_A^2 \neq 0$
  - rest frame: $k^\mu = (m_a, 0, 0, 0)$: $\varepsilon^\mu(k) = (1, 0, 0, 0)$ eliminated
  - $\mathbf{0}$ of A^μ eliminated: spin-0 part
- Goldstone boson? *eaten* by the gauge boson

Superconductivity

magnetic field drops exponentially: $B(x) = B(0)e^{-x/l}$
realized by massive photons: $m_A^2 = 2q^2v^2, \quad q = 2e \qquad l = m_A^{-1}$

Massive Photons and Superconductivity

- Decomposing A^μ under spatial rotations $(\mathrm{SO}(3))$: $A^\mu \in \mathbf{0} \oplus \mathbf{1}$.
- 4 dof: -1 by Lorentz trafos -1 by gauge trafos: 2

Degrees of freedom:

- massless vector: 2, complex scalar: 2
- massive vector: 3, one real scalar (Higgs boson): 1
- gauge symmetry broken, but $\partial_\mu A^\mu = 0$ still holds:
  - $\partial_\mu A^\mu \sim k_\mu \varepsilon^\mu(k)$
  - for $\varepsilon^\mu(k) \sim k^\mu$: $\partial_\mu A^\mu \sim k^2 = m_A^2 \neq 0$
  - rest frame: $k^\mu = (m_a, 0, 0, 0)$: $\varepsilon^\mu(k) = (1, 0, 0, 0)$ eliminated
  - $\mathbf{0}$ of A^μ eliminated: spin-0 part
- Goldstone boson? *eaten* by the gauge boson

Superconductivity

Interpretation: Higgs bosons $\rightarrow$ Cooper pairs, massive photons: electric and magnetic fields described by massive KG / Proca eq.

Discussion

- Writing down a mass term for the gauge field *by hand* ($\frac{1}{2}m_A^2 A_\mu A^\mu$) does *not* respect gauge symmetry: forbidden!

Discussion

- Writing down a mass term for the gauge field *by hand* ($\frac{1}{2}m_A^2 A_\mu A^\mu$) does *not* respect gauge symmetry: forbidden!
- Mass term by SSB: *soft* mass term.

Discussion

- Writing down a mass term for the gauge field *by hand* ($\frac{1}{2}m_A^2 A_\mu A^\mu$) does *not* respect gauge symmetry: forbidden!
- Mass term by SSB: *soft* mass term.
- Lagrangian *is* gauge invariant.

Discussion

- Writing down a mass term for the gauge field *by hand* ($\frac{1}{2}m_A^2 A_\mu A^\mu$) does *not* respect gauge symmetry: forbidden!
- Mass term by SSB: *soft* mass term.
- Lagrangian *is* gauge invariant.
- Symmetry breaking takes place at the level of the *vacuum*.

Discussion

- Writing down a mass term for the gauge field *by hand* ($\frac{1}{2}m_A^2 A_\mu A^\mu$) does *not* respect gauge symmetry: forbidden!
- Mass term by SSB: *soft* mass term.
- Lagrangian *is* gauge invariant.
- Symmetry breaking takes place at the level of the *vacuum*.
- Theory (= Lagrangian) respects the symmetry, but the ground state (= vacuum) does not!

Discussion

- Writing down a mass term for the gauge field *by hand* ($\frac{1}{2}m_A^2 A_\mu A^\mu$) does *not* respect gauge symmetry: forbidden!
- Mass term by SSB: *soft* mass term.
- Lagrangian *is* gauge invariant.
- Symmetry breaking takes place at the level of the *vacuum*.
- Theory (= Lagrangian) respects the symmetry, but the ground state (= vacuum) does not!
- @ high energies: $E \gg v$, v is small and can be neglected

Discussion

- Writing down a mass term for the gauge field *by hand* ($\frac{1}{2}m_A^2 A_\mu A^\mu$) does *not* respect gauge symmetry: forbidden!
- Mass term by SSB: *soft* mass term.
- Lagrangian *is* gauge invariant.
- Symmetry breaking takes place at the level of the *vacuum*.
- Theory (= Lagrangian) respects the symmetry, but the ground state (= vacuum) does not!
- @ high energies: $E \gg v$, v is small and can be neglected
- UV properties of theory are the same as for unbroken symmetry ($v = 0$)!

Discussion

- Writing down a mass term for the gauge field *by hand* ($\frac{1}{2}m_A^2 A_\mu A^\mu$) does *not* respect gauge symmetry: forbidden!
- Mass term by SSB: *soft* mass term.
- Lagrangian *is* gauge invariant.
- Symmetry breaking takes place at the level of the *vacuum*.
- Theory (= Lagrangian) respects the symmetry, but the ground state (= vacuum) does not!
- @ high energies: $E \gg v$, v is small and can be neglected
- UV properties of theory are the same as for unbroken symmetry ($v = 0$)!
- Broken gauge symmetry by hand is not renormalizable.

A Theory of Leptons

$$\mathcal{L} = i\bar{\psi}\partial\!\!\!/\psi - m\bar{\psi},$$

for massles fermions ($m = 0$): $\bar{\psi}\partial\!\!\!/\psi = \bar{\psi}_R\partial\!\!\!/\psi_R + \bar{\psi}_L\partial\!\!\!/\psi_L$,
where $\psi_{L,R} = P_{L,R}\psi$ and $P_L = \frac{1-\gamma_5}{2}$, $P_R = \frac{1+\gamma_5}{2}$.
Lepton Lagrangian (no righthanded components for neutrinos!):

$$\mathcal{L}_\ell = i\bar{\ell}_R\partial\!\!\!/\ell_R + i\bar{\ell}_L\partial\!\!\!/\ell_L + i\bar{\nu}_\ell\partial\!\!\!/\nu_\ell.$$

A Theory of Leptons

$$\mathcal{L} = i\bar{\psi}\partial\!\!\!/\psi - m\bar{\psi},$$

for massles fermions ($m = 0$): $\bar{\psi}\partial\!\!\!/\psi = \bar{\psi}_R\partial\!\!\!/\psi_R + \bar{\psi}_L\partial\!\!\!/\psi_L$,
where $\psi_{L,R} = P_{L,R}\psi$ and $P_L = \frac{1-\gamma_5}{2}$, $P_R = \frac{1+\gamma_5}{2}$.
Lepton Lagrangian (no righthanded components for neutrinos!):

$$\mathcal{L}_\ell = i\bar{\ell}_R\partial\!\!\!/\ell_R + i\bar{\ell}_L\partial\!\!\!/\ell_L + i\bar{\nu}_\ell\partial\!\!\!/\nu_\ell.$$

- internal symmetries?
- join together particles with the same space time properties:

$$L = \begin{pmatrix} \nu_\ell \\ \ell_L \end{pmatrix}, \qquad R = \ell_R$$

- $\mathcal{L}_\ell = i\bar{R}\partial\!\!\!/R + i\bar{L}\partial\!\!\!/L$

Symmetry transformations of the GWSM

$$\mathcal{L}_\ell = i\bar{R}\partial\!\!\!/ R + i\bar{L}\partial\!\!\!/ L,$$

$\mathcal{L}_\ell$ invariant under

$$L \to e^{-i\vec{\tau}\cdot\vec{\alpha}/2} L,$$
$$R \to R,$$

$\mathrm{SU}(2)$ transformations.

- connection weak isospin I_W and electric charge Q:

$$L : Q = I_W^3 - \frac{1}{2}; \qquad R : Q = I_W^3 - 1.$$

- gauging this $\mathrm{SU}(2)$: three massless gauge fields!
- further symmetry of $\mathcal{L}_\ell$:

$$\mathrm{U}(1) : R \to e^{i\beta} R$$

- what about L?: $L \to e^{iq\beta} L$

Weak Hypercharge

$$R \to e^{iy_R \beta/2} R$$
$$L \to e^{iy_L \beta/2} L,$$

with the “weak hypercharge” $y_{L,R}$: Y_W being generator of $U(1)$.

Weak Hypercharge

$$R \to e^{iy_R\beta/2}R$$
$$L \to e^{iy_L\beta/2}L,$$

with the "weak hypercharge" $y_{L,R}$: Y_W being generator of $U(1)$.

(quasi-)Gell-Mann–Nishijima relation:

$$Q = T_W^3 + \frac{Y_W}{2}$$

Weak Hypercharge

$$R \to e^{iy_R\beta/2} R$$
$$L \to e^{iy_L\beta/2} L,$$

with the "weak hypercharge" $y_{L,R}$: Y_W being generator of $U(1)$.

(quasi-)Gell-Mann–Nishijima relation:

$$Q = T_W^3 + \frac{Y_W}{2}$$

$$L \quad \text{has} \quad Y_W = -1,$$
$$R \quad \text{has} \quad Y_W = -2.$$

Weak Hypercharge

$$R \to e^{iy_R\beta/2} R$$

$$L \to e^{iy_L\beta/2} L,$$

with the "weak hypercharge" $y_{L,R}$: Y_W being generator of $U(1)$.

(quasi-)Gell-Mann–Nishijima relation:

$$Q = T_W^3 + \frac{Y_W}{2}$$

$$L \quad \text{has} \quad Y_W = -1,$$

$$R \quad \text{has} \quad Y_W = -2.$$

Symmetry of the Lagrangian:

$$\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$$

Weak Hypercharge

$$R \to e^{iy_R \beta/2} R$$
$$L \to e^{iy_L \beta/2} L,$$

with the "weak hypercharge" $y_{L,R}$: Y_W being generator of $U(1)$.

(quasi-)Gell-Mann–Nishijima relation:

$$Q = T_W^3 + \frac{Y_W}{2}$$

$$L \quad \text{has} \quad Y_W = -1,$$
$$R \quad \text{has} \quad Y_W = -2.$$

Symmetry of the Lagrangian:

$$\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$$

Covariant Derivative:

$$D_\mu = \partial_\mu - igT^a A_\mu^a - ig' Y_W B_\mu$$

Lagrangian of the Electroweak Standard Model:

$$\begin{aligned}
\mathcal{L}_{\text{EW}} &= i\bar{R}\not{D}R + i\bar{L}\not{D}L - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} - \frac{1}{4}F^a_{\mu\nu}F^{a,\mu\nu}, \\
D_\mu &= \partial_\mu - igT^aA^a_\mu - ig'Y_WB_\mu, \\
G_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu, \\
F^a_{\mu\nu} &= \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc}A^b_\mu A^c_\nu, \\
[T^a, T^b] &= f^{abc}T^c.
\end{aligned}$$

Electroweak Symmetry Breaking (EWSB)

Lagrangian of the Electroweak Standard Model:

$$\begin{aligned}
\mathcal{L}_{\mathsf{EW}} &= i\bar{R}\not{D}R + i\bar{L}\not{D}L - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} - \frac{1}{4}F^a_{\mu\nu}F^{a,\mu\nu}, \\
D_\mu &= \partial_\mu - igT^a A^a_\mu - ig'Y_W B_\mu, \\
G_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu, \\
F^a_{\mu\nu} &= \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu, \\
[T^a, T^b] &= f^{abc} T^c.
\end{aligned}$$

How to break $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$?

Lagrangian of the Electroweak Standard Model:

$$\begin{aligned}
\mathcal{L}_{\text{EW}} &= i\bar{R}\not{D}R + i\bar{L}\not{D}L - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} - \frac{1}{4}F^a_{\mu\nu}F^{a,\mu\nu}, \\
D_\mu &= \partial_\mu - igT^a A^a_\mu - ig'Y_W B_\mu, \\
G_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu, \\
F^a_{\mu\nu} &= \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu, \\
[T^a, T^b] &= f^{abc} T^c.
\end{aligned}$$

How to break $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$?
Introduce complex scalar isospinor ("the Higgs field"):

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix},$$

with quantum numbers $I_W = \frac{1}{2}$ and $Y_W = 1$.

Electroweak Symmetry Breaking (EWSB)

Lagrangian of the Electroweak Standard Model:

$$\begin{aligned}
\mathcal{L}_{\mathsf{EW}} &= i\bar{R}\not{D}R + i\bar{L}\not{D}L - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} - \frac{1}{4}F^a_{\mu\nu}F^{a,\mu\nu}, \\
D_\mu &= \partial_\mu - igT^aA^a_\mu - ig'Y_W B_\mu, \\
G_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu, \\
F^a_{\mu\nu} &= \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc}A^b_\mu A^c_\nu, \\
[T^a, T^b] &= f^{abc}T^c.
\end{aligned}$$

How to break $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$?
Introduce complex scalar isospinor ("the Higgs field"):

$$\langle\Phi\rangle = \begin{pmatrix} 0 \\ v \end{pmatrix},$$

due to $\mathrm{SU}(2) \otimes \mathrm{U}(1)$-invariant quartic potential: $v^2 = -\frac{m^2}{2\lambda}$.

Electroweak Gauge Bosons

Due to $\mathrm{SU}(2) \otimes \mathrm{U}(1)$ symmetry, we can choose

$$\Phi(x) = \begin{pmatrix} 0 \\ v + \frac{1}{\sqrt{2}} h(x) \end{pmatrix},$$

in the “unitary gauge”.

Electroweak Gauge Bosons

Due to $\mathrm{SU}(2) \otimes \mathrm{U}(1)$ symmetry, we can choose

$$\Phi(x) = \begin{pmatrix} 0 \\ v + \frac{1}{\sqrt{2}} h(x) \end{pmatrix},$$

in the "unitary gauge".

$(D_\mu \phi)^\dagger D^\mu \phi \quad \hookrightarrow$ quadratic terms for gauge fields:

$$\frac{1}{4} v^2 \left(g A^3_\mu - g' B_\mu \right) \left(g A^{3\,\mu} - g' B^\mu \right) + \frac{1}{2} g^2 v^2 A^+_\mu A^{-\,\mu},$$

where the generators $T^a = \sigma^a / 2$ were used and $Y_W = 1/2$ set.
$A^\pm_\mu = \frac{1}{\sqrt{2}} (A^1_\mu \pm i A^2_\mu)$.

Electroweak Gauge Bosons

Due to $\mathrm{SU}(2) \otimes \mathrm{U}(1)$ symmetry, we can choose

$$\Phi(x) = \begin{pmatrix} 0 \\ v + \frac{1}{\sqrt{2}} h(x) \end{pmatrix},$$

in the "unitary gauge".

$\left(D_\mu \phi\right)^\dagger D^\mu \phi \quad \hookrightarrow$ quadratic terms for gauge fields:

$$\frac{1}{4} v^2 \left(g A^3_\mu - g' B_\mu\right) \left(g A^{3\,\mu} - g' B^\mu\right) + \frac{1}{2} g^2 v^2 A^+_\mu A^{-\,\mu},$$

where the generators $T^a = \sigma^a/2$ were used and $Y_W = 1/2$ set.
$A^\pm_\mu = \frac{1}{\sqrt{2}}(A^1_\mu \pm i A^2_\mu)$.

mass terms for

- $Z^0_\mu \sim g A^3_\mu - g' B_\mu$,
- $W^\pm_\mu = A^\pm_\mu = \frac{1}{\sqrt{2}}(A^1_\mu \pm i A^2_\mu)$.

Summary of EWSM

Weak mixing angle:

$$\tan\theta_W = \frac{g'}{g},$$

$\sin^2\theta_W = 0.2312.$

Summary of EWSM

Weak mixing angle:

$$\tan\theta_W = \frac{g'}{g},$$

$\sin^2\theta_W = 0.2312.$

$$A^0_\mu = \cos\theta_W B_\mu + \sin\theta_W A^3_\mu \qquad \text{photon,}$$
$$Z^0_\mu = -\sin\theta_W B_\mu + \cos\theta_W A^3_\mu \qquad Z\text{-boson}$$

Summary of EWSM

Weak mixing angle:

$$\tan\theta_W = \frac{g'}{g},$$

$\sin^2\theta_W = 0.2312.$

$$A^0_\mu = \cos\theta_W B_\mu + \sin\theta_W A^3_\mu \qquad \text{photon},$$
$$Z^0_\mu = -\sin\theta_W B_\mu + \cos\theta_W A^3_\mu \qquad Z\text{-boson}$$

Masses:

$$m_Z = \frac{v}{\sqrt{2}}\sqrt{g^2 + g'^2},$$
$$m_W = \frac{v}{\sqrt{2}} g,$$
$$\frac{m_W}{m_Z} = \cos\theta_W.$$

Photon remains massless! Coupling: $e = g\sin\theta_W$.

Masses for Fermions

No tree-level mass allowed!

There is no way to combine left and righthanded fields in the SM representations (!) in a gauge invariant way:

- lefthanded fermions: $\mathbf{2}$ of $\mathrm{SU}(2)_L$
- righthanded fermions: $\mathbf{1}$ of $\mathrm{SU}(2)_L$

$$\mathcal{L}_{\text{mass}} \sim \bar{\Psi}\Psi = \bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L,$$

with

$$L = \begin{pmatrix} \nu_\ell \\ \ell_L \end{pmatrix}, \qquad R = \ell_R$$

$$\hookrightarrow \bar{L}R = \begin{pmatrix} \bar{\nu}_\ell & \bar{\ell}_L \end{pmatrix} \cdot \ell_R$$

undefined in the sense of inner tensor product:

no $\mathrm{SU}(2)_L$ invariant Lagrangian

(open/uncontracted $\mathrm{SU}(2)$ index)

Yukawa Interactions!

Way out:

- Construct gauge invariant mass terms via Higgs mechanism

Yukawa Interactions!

Way out:

- Construct gauge invariant mass terms via Higgs mechanism

- tree-level fermion mass: $\bar{\psi}_L \, m \, \psi_R$

Yukawa Interactions!

Way out:

- Construct gauge invariant mass terms via Higgs mechanism

- tree-level fermion mass: $\bar{\psi}_L \, m \, \psi_R$
- mass term carries $\mathrm{SU}(2)_L$ quantum number! $\hookrightarrow$ $\bar{\mathbf{2}} \cdot \mathbf{2} \; \mathbf{1} = \mathbf{1}$

Yukawa Interactions!

Way out:

- Construct gauge invariant mass terms via Higgs mechanism

- tree-level fermion mass: $\bar{\psi}_L \, m \, \psi_R$
- mass term carries $\mathrm{SU}(2)_L$ quantum number! $\hookrightarrow \; \bar{\mathbf{2}} \cdot \mathbf{2} \; \mathbf{1} = \mathbf{1}$
- Higgs field is doublet of $\mathrm{SU}(2)_L$

Yukawa Interactions!

Way out:

- Construct gauge invariant mass terms via Higgs mechanism

- tree-level fermion mass: $\bar{\psi}_L \, m \, \psi_R$
- mass term carries $\mathrm{SU}(2)_L$ quantum number! $\hookrightarrow \; \bar{\mathbf{2}} \cdot \mathbf{2} \; \mathbf{1} = \mathbf{1}$
- Higgs field is doublet of $\mathrm{SU}(2)_L$

Yukawa Interactions!

Way out:

- Construct gauge invariant mass terms via Higgs mechanism

- tree-level fermion mass: $\bar{\psi}_L \, m \, \psi_R$
- mass term carries $\mathrm{SU}(2)_L$ quantum number! $\hookrightarrow \; \bar{\mathbf{2}} \cdot \mathbf{2} \; \mathbf{1} = \mathbf{1}$
- Higgs field is doublet of $\mathrm{SU}(2)_L$

Yukawa couplings to leptons

$$\begin{aligned} \mathcal{L}_{\text{Yukawa}} &= Y_\ell \bar{L} \cdot \Phi \; R + \text{ h. c.} \\ &= Y_\ell \left(\bar{\nu}_\ell \;\; \bar{\ell}_L \right) \cdot \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \ell_R + \text{ h. c.} \\ \mathcal{L}^{\text{SSB}}_{\text{Yukawa}} &= Y_\ell \left(\bar{\nu}_\ell \;\; \bar{\ell}_L \right) \cdot \begin{pmatrix} 0 \\ v \end{pmatrix} \ell_R + \text{ h. c.} \\ &= Y_\ell \, v \, \bar{\ell}_L \ell_R + \text{ h. c.} \qquad \hookrightarrow \; m_\ell = v \, Y_\ell \end{aligned}$$

How the Flavour Comes into the Game...

What happens, if we add additional fermions to the SM?
"Families": adding groups of fermions with the same quantum numbers (spin, gauge charges, ...) but different masses

How the Flavour Comes into the Game...

What happens, if we add additional fermions to the SM?
"Families": adding groups of fermions with the same quantum numbers (spin, gauge charges, ...) but different masses

Flavour related to Yukawa sector

How the Flavour Comes into the Game...

What happens, if we add additional fermions to the SM?
"Families": adding groups of fermions with the same quantum numbers (spin, gauge charges, ...) but different masses

Flavour related to Yukawa sector

- kinetic terms: flavour blind $\bar{\psi}_i \partial\!\!\!/ \psi_i$

How the Flavour Comes into the Game...

What happens, if we add additional fermions to the SM?
"Families": adding groups of fermions with the same quantum numbers (spin, gauge charges, ...) but different masses

Flavour related to Yukawa sector

- kinetic terms: flavour blind $\bar{\psi}_i \partial\!\!\!/ \psi_i$
- gauge interaction: flavour blind $\bar{\psi}_i D\!\!\!\!/ \psi_i$

How the Flavour Comes into the Game...

What happens, if we add additional fermions to the SM?
"Families": adding groups of fermions with the same quantum numbers (spin, gauge charges, ...) but different masses

Flavour related to Yukawa sector

- kinetic terms: flavour blind $\bar{\psi}_i \partial\!\!\!/ \psi_i$
- gauge interaction: flavour blind $\bar{\psi}_i D\!\!\!\!/ \psi_i$
- Yukawa interactions: responsible for masses, couplings differ for each family $y_{ij} \bar{\psi}_{L,i} \Phi \psi_{R,j}$

How the Flavour Comes into the Game...

What happens, if we add additional fermions to the SM?
"Families": adding groups of fermions with the same quantum numbers (spin, gauge charges, ...) but different masses

Flavour related to Yukawa sector

- kinetic terms: flavour blind $\bar{\psi}_i \partial\!\!\!/ \psi_i$
- gauge interaction: flavour blind $\bar{\psi}_i D\!\!\!\!/ \psi_i$
- Yukawa interactions: responsible for masses, couplings differ for each family $y_{ij} \bar{\psi}_{L,i} \Phi \psi_{R,j}$

How the Flavour Comes into the Game...

What happens, if we add additional fermions to the SM?
"Families": adding groups of fermions with the same quantum numbers (spin, gauge charges, ...) but different masses

Flavour related to Yukawa sector

- kinetic terms: flavour blind $\bar{\psi}_i \partial\!\!\!/ \psi_i$
- gauge interaction: flavour blind $\bar{\psi}_i D\!\!\!\!/ \psi_i$
- Yukawa interactions: responsible for masses, couplings differ for each family $y_{ij} \bar{\psi}_{L,i} \Phi \psi_{R,j}$

Yukawa sector of the Standard Model

Fermion content: $Q_{L,i}$, $u_{R,i}$, $d_{R,i}$, $L_{L,i}$, $\ell_{R,i}$

$$\mathcal{L}_{\mathsf{Y}} = y^d_{ij} \bar{Q}_{L,i} \Phi d_{R,j} + y^u_{ij} \bar{Q}_{L,i} \tilde{\Phi} u_{R,j} + y^e_{ij} \bar{L}_{L,i} \Phi \ell_{R,j} + \text{h. c.}$$

Quark Mixing

Quark mass matrices

$$m_{ij}^u = v\ y_{ij}^u,$$
$$m_{ij}^d = v\ y_{ij}^d.$$

Quark Mixing

Quark mass matrices

$$m^u_{ij} = v\, y^u_{ij},$$
$$m^d_{ij} = v\, y^d_{ij}.$$

Masses: diagonalize mass matrices

Bi-unitary transformations: $M \to UDV^\dagger$: $\quad D = U^\dagger M V$

Quark Mixing

Quark mass matrices

$$m_{ij}^{u} = v\, y_{ij}^{u},$$
$$m_{ij}^{d} = v\, y_{ij}^{d}.$$

Masses: diagonalize mass matrices

Bi-unitary transformations: $M \to UDV^{\dagger}$: $\quad D = U^{\dagger} M V$

Rotate fields in flavour space:

$$Q_{L,i} \to S_{ij}^{Q} Q_{L,j},$$
$$u_{R,i} \to S_{ij}^{u} u_{R,j},$$
$$d_{R,i} \to S_{ij}^{d} d_{R,j}.$$

Quark Mixing

Quark mass matrices

$$m_{ij}^u = v\, y_{ij}^u,$$
$$m_{ij}^d = v\, y_{ij}^d.$$

Masses: diagonalize mass matrices

Bi-unitary transformations: $M \to UDV^\dagger$: $\quad D = U^\dagger M V$

Rotate fields in flavour space:

$$Q_{L,i} \to S_{ij}^Q Q_{L,j},$$
$$u_{R,i} \to S_{ij}^u u_{R,j},$$
$$d_{R,i} \to S_{ij}^d d_{R,j}.$$

$$\mathcal{L}_\mathsf{Y}^q = \bar{Q}_{L,i} S_{ij'}^{Q*} y_{j'j}^d \Phi S_{jk}^d d_{R,k} + \bar{Q}_{L,i} S_{ij'}^{Q*} y_{j'j}^u \tilde{\Phi} S_{jk}^u u_{R,k} + \text{ h. c.}$$

Charged Current and CKM matrix

How the fermion mixing enters the charged current $\mathcal{L}_{CC} = -\frac{ig}{\sqrt{2}} W_\mu^+ J_L^\mu$?

- Masses:

$$\mathcal{L}_{\mathsf{Y}}^q = \bar{Q}_{L,i} S_{ij'}^{Q*} y_{j'j}^d \Phi S_{jk}^d d_{R,k} + \bar{Q}_{L,i} S_{ij'}^{Q*} y_{j'j}^u \tilde{\Phi} S_{jk}^u u_{R,k} + \text{ h. c.}$$

Charged Current and CKM matrix

How the fermion mixing enters the charged current $\mathcal{L}_{CC} = -\frac{ig}{\sqrt{2}} W_\mu^+ J_L^\mu$?

- Masses:

$$\mathcal{L}_{\mathsf{Y}}^q = \bar{Q}_{L,i} S_{ij'}^{Q*} y_{j'j}^d \Phi S_{jk}^d d_{R,k} + \bar{Q}_{L,i} S_{ij'}^{Q*} y_{j'j}^u \tilde{\Phi} S_{jk}^u u_{R,k} + \text{ h. c.}$$

- $\mathbf{S}^Q$ and $\mathbf{S}^u$ fixed by $\tilde{\mathbf{m}}_u = \mathbf{S}^{Q\dagger} \mathbf{m}_u \mathbf{S}^u = \mathrm{diag}(m_u, m_c, m_t)$

Charged Current and CKM matrix

How the fermion mixing enters the charged current $\mathcal{L}_{CC} = -\frac{ig}{\sqrt{2}} W^+_\mu J^\mu_L$?

- Masses:

$$\mathcal{L}^q_{\mathsf{Y}} = \bar{Q}_{L,i} S^{Q*}_{ij'} y^d_{j'j} \Phi S^d_{jk} d_{R,k} + \bar{Q}_{L,i} S^{Q*}_{ij'} y^u_{j'j} \tilde{\Phi} S^u_{jk} u_{R,k} + \text{ h. c.}$$

- $\mathbf{S}^Q$ and $\mathbf{S}^u$ fixed by $\tilde{\mathbf{m}}_u = \mathbf{S}^{Q\dagger} \mathbf{m}_u \mathbf{S}^u = \mathrm{diag}(m_u, m_c, m_t)$
- Charged current:

$$J^\mu_L = \bar{u}_{L,i} \gamma^\mu d_{L,i} + \text{ h. c.}$$

Charged Current and CKM matrix

How the fermion mixing enters the charged current $\mathcal{L}_{CC} = -\frac{ig}{\sqrt{2}} W_\mu^+ J_L^\mu$?

- Masses:

$$\mathcal{L}_{\mathsf{Y}}^q = \bar{Q}_{L,i} S_{ij'}^{Q*} y_{j'j}^d \Phi S_{jk}^d d_{R,k} + \bar{Q}_{L,i} S_{ij'}^{Q*} y_{j'j}^u \tilde{\Phi} S_{jk}^u u_{R,k} + \text{ h. c.}$$

- $\mathbf{S}^Q$ and $\mathbf{S}^u$ fixed by $\tilde{\mathbf{m}}_u = \mathbf{S}^{Q\dagger} \mathbf{m}_u \mathbf{S}^u = \mathrm{diag}(m_u, m_c, m_t)$
- Charged current:

$$J_L^\mu = \bar{u}_{L,i} S_{ij}^{Q*} \gamma^\mu S_{ji}^Q d_{L,i} + \text{ h. c.}$$

Charged Current and CKM matrix

How the fermion mixing enters the charged current $\mathcal{L}_{CC} = -\frac{ig}{\sqrt{2}} W^+_\mu J^\mu_L$?

- Masses:

$$\mathcal{L}^q_{\mathsf{Y}} = \bar{Q}_{L,i} S^{Q*}_{ij'} y^d_{j'j} \Phi S^d_{jk} d_{R,k} + \bar{Q}_{L,i} S^{Q*}_{ij'} y^u_{j'j} \tilde{\Phi} S^u_{jk} u_{R,k} + \text{ h. c.}$$

- $\mathbf{S}^Q$ and $\mathbf{S}^u$ fixed by $\tilde{\mathbf{m}}_u = \mathbf{S}^{Q\dagger} \mathbf{m}_u \mathbf{S}^u = \mathrm{diag}(m_u, m_c, m_t)$
- Charged current:

$$J^\mu_L = \bar{u}_{L,i} S^{Q*}_{ij} \gamma^\mu S^Q_{ji} d_{L,i} + \text{ h. c.}$$

CKM matrix *(Cabibbo, Kobayashi, Maskawa)*

- still have to diagonalize $\mathbf{m}_d$: $\mathbf{S}^{Q\dagger} \mathbf{m}_d \mathbf{S}^d \rightarrow \tilde{\mathbf{m}}_d = \mathbf{S}^{Q\dagger} \mathbf{V}^\dagger \mathbf{m}_d \mathbf{S}^d$

How the fermion mixing enters the charged current $\mathcal{L}_{CC} = -\frac{ig}{\sqrt{2}} W_\mu^+ J_L^\mu$?

- Masses:

$$\mathcal{L}_{\mathsf{Y}}^q = \bar{Q}_{L,i} S_{ij'}^{Q*} y_{j'j}^d \Phi S_{jk}^d d_{R,k} + \bar{Q}_{L,i} S_{ij'}^{Q*} y_{j'j}^u \tilde{\Phi} S_{jk}^u u_{R,k} + \text{ h. c.}$$

- $\mathbf{S}^Q$ and $\mathbf{S}^u$ fixed by $\tilde{\mathbf{m}}_u = \mathbf{S}^{Q\dagger} \mathbf{m}_u \mathbf{S}^u = \mathrm{diag}(m_u, m_c, m_t)$
- Charged current:

$$J_L^\mu = \bar{u}_{L,i} S_{ij}^{Q*} \gamma^\mu S_{ji}^Q d_{L,i} + \text{ h. c.}$$

CKM matrix *(Cabibbo, Kobayashi, Maskawa)*

- still have to diagonalize $\mathbf{m}_d$: $\mathbf{S}^{Q\dagger} \mathbf{m}_d \mathbf{S}^d \to \tilde{\mathbf{m}}_d = \mathbf{S}^{Q\dagger} \mathbf{V}^\dagger \mathbf{m}_d \mathbf{S}^d$

$$J_L^\mu = \bar{u}_{L,i} S_{ij}^{Q*} \gamma^\mu V_{jj'} S_{j'i}^Q d_{L,i} + \text{ h. c.}$$

Charged Current and CKM matrix

How the fermion mixing enters the charged current $\mathcal{L}_{CC} = -\frac{ig}{\sqrt{2}} W_\mu^+ J_L^\mu$?

- Masses:

$$\mathcal{L}_\mathsf{Y}^q = \bar{Q}_{L,i} S_{ij'}^{Q*} y_{j'j}^d \Phi S_{jk}^d d_{R,k} + \bar{Q}_{L,i} S_{ij'}^{Q*} y_{j'j}^u \tilde{\Phi} S_{jk}^u u_{R,k} + \text{ h. c.}$$

- $\mathbf{S}^Q$ and $\mathbf{S}^u$ fixed by $\tilde{\mathbf{m}}_u = \mathbf{S}^{Q\dagger} \mathbf{m}_u \mathbf{S}^u = \mathrm{diag}(m_u, m_c, m_t)$
- Charged current:

$$J_L^\mu = \bar{u}_{L,i} S_{ij}^{Q*} \gamma^\mu S_{ji}^Q d_{L,i} + \text{ h. c.}$$

CKM matrix *(Cabibbo, Kobayashi, Maskawa)*

- still have to diagonalize $\mathbf{m}_d$: $\mathbf{S}^{Q\dagger} \mathbf{m}_d \mathbf{S}^d \to \tilde{\mathbf{m}}_d = \mathbf{S}^{Q\dagger} \mathbf{V}^\dagger \mathbf{m}_d \mathbf{S}^d$

$$J_L^\mu = \bar{u}'_{L,i} \gamma^\mu V_{ij} d'_{L,j} + \bar{d}'_{L,i} V_{ij}^\dagger \gamma^\mu u'_{L,j}$$

What about Leptons?

In the SM: no lepton mixing!

$$\mathcal{L}_Y^\ell = y_{ij}^e \bar{L}_{L,i} \Phi \ell_{R,j} + \text{ h. c.}$$

What about Leptons?

In the SM: no lepton mixing!

$$\mathcal{L}_Y^\ell = y_{ij}^e \bar{L}_{L,i} \Phi \ell_{R,j} + \text{ h. c.}$$

- Rotate lepton doublet: $L_{L,i} \to S_{ij}^L L_{L,j}$, and singlet: $\ell_{R,i} \to S_{ij}^\ell \ell_{R,j}$.

$$\mathcal{L}_Y^\ell = \bar{L}_{L,i} S_{ij'}^{L*} y_{j'j}^e \Phi S_{jk}^\ell \ell_{R,k} + \text{ h. c.}$$

What about Leptons?

In the SM: no lepton mixing!

$$\mathcal{L}_Y^\ell = y_{ij}^e \bar{L}_{L,i} \Phi \ell_{R,j} + \text{ h. c.}$$

- Rotate lepton doublet: $L_{L,i} \to S_{ij}^L L_{L,j}$,
  and singlet: $\ell_{R,i} \to S_{ij}^\ell \ell_{R,j}$.

$$\mathcal{L}_Y^\ell = \bar{L}_{L,i} S_{ij'}^{L*} y_{j'j}^e \Phi S_{jk}^\ell \ell_{R,k} + \text{ h. c.}$$

- charged current:

$$\bar{\nu}_{\ell,i} \gamma^\mu \ell_{L,i} \to \bar{\nu}_{\ell,i} S_{ij}^{L*} \gamma^\mu S_{jk}^L \ell_{L,k}$$

What about Leptons?

In the SM: no lepton mixing!

$$\mathcal{L}_Y^\ell = y^e_{ij} \bar{L}_{L,i} \Phi \ell_{R,j} + \text{ h. c.}$$

- Rotate lepton doublet: $L_{L,i} \to S^L_{ij} L_{L,j}$,
  and singlet: $\ell_{R,i} \to S^\ell_{ij} \ell_{R,j}$.

$$\mathcal{L}_Y^\ell = \bar{L}_{L,i} S^{L*}_{ij'} y^e_{j'j} \Phi S^\ell_{jk} \ell_{R,k} + \text{ h. c.}$$

- charged current:

$$\bar{\nu}_{\ell,i} \gamma^\mu \ell_{L,i} \to \bar{\nu}_{\ell,i} S^{L*}_{ij} \gamma^\mu S^L_{jk} \ell_{L,k}$$

- $\mathbf{S}^L$ unitary: $\mathbf{S}^{L\dagger} \mathbf{S}^L = \mathbf{1} \quad \hookrightarrow \quad S^{L*}_{ij} S^L_{jk} = \delta_{ik}$

What about Leptons?

In the SM: no lepton mixing!

$$\mathcal{L}_Y^\ell = y_{ij}^e \bar{L}_{L,i} \Phi \ell_{R,j} + \text{ h. c.}$$

- Rotate lepton doublet: $L_{L,i} \to S_{ij}^L L_{L,j}$,
  and singlet: $\ell_{R,i} \to S_{ij}^\ell \ell_{R,j}$.

$$\mathcal{L}_Y^\ell = \bar{L}_{L,i} S_{ij'}^{L*} y_{j'j}^e \Phi S_{jk}^\ell \ell_{R,k} + \text{ h. c.}$$

- charged current:

$$\bar{\nu}_{\ell,i} \gamma^\mu \ell_{L,i} \to \bar{\nu}_{\ell,i} S_{ij}^{L*} \gamma^\mu S_{jk}^L \ell_{L,k}$$

- $\mathbf{S}^L$ unitary: $\mathbf{S}^{L\dagger}\mathbf{S}^L = \mathbf{1} \quad \hookrightarrow \quad S_{ij}^{L*} S_{jk}^L = \delta_{ik}$
- redefine lepton fields:

$$J_L^\mu = \bar{\nu}_{\ell,i} S_{ij}^{L*} \gamma^\mu S_{jk}^L \ell_{L,k} = \bar{\nu}'_{\ell,i} \delta_{ij} \gamma^\mu \ell_{L,j}$$

Righthanded Neutrinos

How to generate lepton mixing?

- No lepton mixing in the SM $\Leftrightarrow$ righthanded neutrino fields.

Righthanded Neutrinos

How to generate lepton mixing?

- No lepton mixing in the SM $\Leftrightarrow$ righthanded neutrino fields.
- Introduce them:

$$\mathcal{L}_Y^\ell = y_{ij}^e \bar{L}_{L,i} \Phi \ell_{R,j} + y_{ij}^\nu \bar{L}_{L,i} \Phi \nu_{R,j} + \text{ h. c.}$$

Righthanded Neutrinos

How to generate lepton mixing?

- No lepton mixing in the SM $\Leftrightarrow$ righthanded neutrino fields.
- Introduce them:

$$\mathcal{L}_Y^\ell = y_{ij}^e \bar{L}_{L,i} \Phi \ell_{R,j} + y_{ij}^\nu \bar{L}_{L,i} \Phi \nu_{R,j} + \text{ h. c.}$$

- Masses and mixings similar to CKM mechanism.

Righthanded Neutrinos

How to generate lepton mixing?

- No lepton mixing in the SM $\Leftrightarrow$ righthanded neutrino fields.
- Introduce them:

$$\mathcal{L}_Y^\ell = y_{ij}^e \bar{L}_{L,i} \Phi \ell_{R,j} + y_{ij}^\nu \bar{L}_{L,i} \Phi \nu_{R,j} + \text{ h. c.}$$

- Masses and mixings similar to CKM mechanism.
- "Problem": neutrinos only have small masses

Righthanded Neutrinos

How to generate lepton mixing?

- No lepton mixing in the SM $\Leftrightarrow$ righthanded neutrino fields.
- Introduce them:

$$\mathcal{L}_Y^\ell = y_{ij}^e \bar{L}_{L,i} \Phi \ell_{R,j} + y_{ij}^\nu \bar{L}_{L,i} \Phi \nu_{R,j} + \text{ h. c.}$$

- Masses and mixings similar to CKM mechanism.
- "Problem": neutrinos only have small masses
  - $m_\nu / m_e = y_\nu / y_e \approx 9.7 \times 10^{-8}$;
    $m_\nu / m_\tau = y_\nu / y_\tau \approx 2.8 \times 10^{-11}$;

Righthanded Neutrinos

How to generate lepton mixing?

- No lepton mixing in the SM $\Leftrightarrow$ righthanded neutrino fields.
- Introduce them:

$$\mathcal{L}_Y^\ell = y_{ij}^e \bar{L}_{L,i} \Phi \ell_{R,j} + y_{ij}^\nu \bar{L}_{L,i} \Phi \nu_{R,j} + \text{ h. c.}$$

- Masses and mixings similar to CKM mechanism.
- "Problem": neutrinos only have small masses
  - $m_\nu / m_e = y_\nu / y_e \approx 9.7 \times 10^{-8}$;
    $m_\nu / m_\tau = y_\nu / y_\tau \approx 2.8 \times 10^{-11}$;
  - Neutrino Yukawa couplings must be drastically smaller

Righthanded Neutrinos

How to generate lepton mixing?

- No lepton mixing in the SM $\Leftrightarrow$ righthanded neutrino fields.
- Introduce them:

$$\mathcal{L}_Y^\ell = y_{ij}^e \bar{L}_{L,i} \Phi \ell_{R,j} + y_{ij}^\nu \bar{L}_{L,i} \Phi \nu_{R,j} + \text{ h. c.}$$

- Masses and mixings similar to CKM mechanism.
- "Problem": neutrinos only have small masses
  - $m_\nu / m_e = y_\nu / y_e \approx 9.7 \times 10^{-8}$;
    $m_\nu / m_\tau = y_\nu / y_\tau \approx 2.8 \times 10^{-11}$;
  - Neutrino Yukawa couplings must be drastically smaller

Righthanded Neutrinos

How to generate lepton mixing?

- No lepton mixing in the SM $\Leftrightarrow$ righthanded neutrino fields.
- Introduce them:

$$\mathcal{L}_Y^\ell = y_{ij}^e \bar{L}_{L,i} \Phi \ell_{R,j} + y_{ij}^\nu \bar{L}_{L,i} \Phi \nu_{R,j} + \text{ h. c.}$$

- Masses and mixings similar to CKM mechanism.
- "Problem": neutrinos only have small masses
  - $m_\nu / m_e = y_\nu / y_e \approx 9.7 \times 10^{-8}$;
    $m_\nu / m_\tau = y_\nu / y_\tau \approx 2.8 \times 10^{-11}$;
  - Neutrino Yukawa couplings must be drastically smaller

The Way out:

- rh neutrinos are SM singlets: may have Majorana mass

$$\mathcal{L}_{Y+M}^\ell = y_{ij}^e \bar{L}_{L,i} \Phi \ell_{R,j} + y_{ij}^\nu \bar{L}_{L,i} \Phi \nu_{R,j} + \frac{1}{2} \nu_{R,i}^T C M_{ij} \nu_{R,j} + \text{ h. c.}$$

PMNS Mixing

$$\mathcal{L}^{\ell}_{Y+M} = y^{e}_{ij}\bar{L}_{L,i}\Phi\ell_{R,j} + y^{\nu}_{ij}\bar{L}_{L,i}\Phi\nu_{R,j} + \frac{1}{2}\nu^{T}_{R,i}CM_{ij}\nu_{R,j} + \text{ h. c.}$$

PMNS Mixing

$$\mathcal{L}^{\ell}_{Y+M} = y^{e}_{ij}\bar{L}_{L,i}\Phi\ell_{R,j} + y^{\nu}_{ij}\bar{L}_{L,i}\Phi\nu_{R,j} + \frac{1}{2}\nu^{T}_{R,i}CM_{ij}\nu_{R,j} + \text{ h. c.}$$

- Rotate:

$$L_{L,i} \to S^{L}_{ij}L_{L,j},$$
$$\ell_{R,i} \to S^{\ell}_{ij}\ell_{R,j},$$
$$\nu_{R,i} \to S^{\nu}_{ij}\nu_{R,j}$$

PMNS Mixing

$$\mathcal{L}^{\ell}_{Y+M} = y^e_{ij}\bar{L}_{L,i}\Phi\ell_{R,j} + y^{\nu}_{ij}\bar{L}_{L,i}\Phi\nu_{R,j} + \frac{1}{2}\nu^T_{R,i}CM_{ij}\nu_{R,j} + \text{ h. c.}$$

- Rotate:
$$L_{L,i} \to S^L_{ij}L_{L,j},$$
$$\ell_{R,i} \to S^{\ell}_{ij}\ell_{R,j},$$
$$\nu_{R,i} \to S^{\nu}_{ij}\nu_{R,j}$$

- $\mathbf{y}^e \to \mathbf{S}^{L\dagger}\mathbf{y}^e\mathbf{S}^e = \tilde{\mathbf{y}}^e$
- $\mathbf{M} \to \mathbf{S}^{\nu\dagger}\mathbf{M}\mathbf{S}^{\nu}$
- $\mathbf{y}^{\nu} \to \mathbf{S}^{L\dagger}\mathbf{y}^{\nu}\mathbf{S}^{\nu}$ arbitrary

PMNS Mixing

$$\mathcal{L}^{\ell}_{Y+M} = y^e_{ij}\bar{L}_{L,i}\Phi\ell_{R,j} + y^\nu_{ij}\bar{L}_{L,i}\Phi\nu_{R,j} + \frac{1}{2}\nu^T_{R,i}CM_{ij}\nu_{R,j} + \text{ h. c.}$$

- Rotate:
$$L_{L,i} \to S^L_{ij}L_{L,j},$$
$$\ell_{R,i} \to S^\ell_{ij}\ell_{R,j},$$
$$\nu_{R,i} \to S^\nu_{ij}\nu_{R,j}$$
- $\mathbf{y}^e \to \mathbf{S}^{L\dagger}\mathbf{y}^e\mathbf{S}^e = \tilde{\mathbf{y}}^e$
- $\mathbf{M} \to \mathbf{S}^{\nu\dagger}\mathbf{M}\mathbf{S}^\nu$
- $\mathbf{y}^\nu \to \mathbf{S}^{L\dagger}\mathbf{y}^\nu\mathbf{S}^\nu$ arbitrary

Neutrino mass matrix:

Seesaw: $\mathbf{m}_\nu = -v^2\mathbf{y}^\nu\mathbf{M}^{-1}\mathbf{y}^{\nu T} \quad \hookrightarrow \quad \tilde{\mathbf{m}}_\nu = \mathbf{U}^T_\nu\mathbf{m}_\nu\mathbf{U}_\nu$

PMNS Mixing

$$\mathcal{L}^{\ell}_{Y+M} = y^e_{ij}\bar{L}_{L,i}\Phi\ell_{R,j} + y^{\nu}_{ij}\bar{L}_{L,i}\Phi\nu_{R,j} + \frac{1}{2}\nu^T_{R,i}CM_{ij}\nu_{R,j} + \text{ h. c.}$$

- Rotate:

$$L_{L,i} \to S^L_{ij}L_{L,j},$$
$$\ell_{R,i} \to S^{\ell}_{ij}\ell_{R,j},$$
$$\nu_{R,i} \to S^{\nu}_{ij}\nu_{R,j}$$

- $\mathbf{y}^e \to \mathbf{S}^{L\dagger}\mathbf{y}^e\mathbf{S}^e = \tilde{\mathbf{y}}^e$
- $\mathbf{M} \to \mathbf{S}^{\nu\dagger}\mathbf{M}\mathbf{S}^{\nu}$
- $\mathbf{y}^{\nu} \to \mathbf{S}^{L\dagger}\mathbf{y}^{\nu}\mathbf{S}^{\nu}$ arbitrary

Neutrino mass matrix:

Seesaw: $\mathbf{m}_\nu = -v^2\mathbf{y}^{\nu}\mathbf{M}^{-1}\mathbf{y}^{\nu T} \quad \hookrightarrow \quad \tilde{\mathbf{m}}_\nu = \mathbf{U}^T_\nu\mathbf{m}_\nu\mathbf{U}_\nu$

PMNS matrix *(Pontecorvo, Maki, Nakagawa, Sakata)*

$$J^\mu_L = \bar{\nu}_{\ell,i}S^{L*}_{ij}U^*_{\nu,jk}\gamma^\mu S^L_{kl}\ell_{L,l} = \bar{\nu}'_{\ell,i}U^*_{\nu,ij}\gamma^\mu\ell_{L,j}$$

Conclusion

- Spontaneous Symmetry Breaking: theory has some symmetry which the ground state does not respect
- Existence of some "order parameter" (which vanishes, if symmetry is exact)
- Condensed matter physics: ferromagnetism, superfluidity, superconductivity
- Gauge boson masses forbidden by gauge invariance
- "Higgs mechanism": masses in a gauge invariant way
- Electroweak Standard Model: $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$
- Fermion masses via Yukawa interactions
- Fermion mixing via Yukawa interactions