

Decoupling of the Top Quark in $\Gamma(Z \rightarrow \text{hadrons})$

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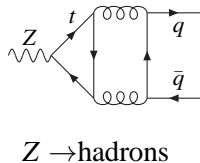
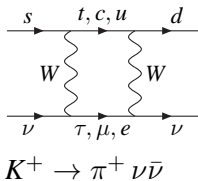
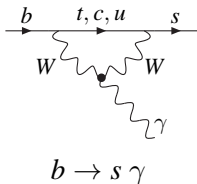
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- ① Decoupling of heavy particles
 - Why ?
 - How ?
- ② Decay rate of the Z-boson into hadrons: $\Gamma(Z \rightarrow \text{hadrons})$
 - Definition
 - Decoupling of the Top Quark
 - Results

Decoupling - Why?

Examples:



Problems with many scales

- **Technical:** Difficult (atm impossible) to calculate.
At high loop order only 1-scale problems possible.
- **conceptual:** Largely separated scales lead to big logarithms
→ bad convergence of the perturbation series ($\alpha \cdot \ln(m_h^2/m_l^2) \sim 1$)

$$\Gamma = \# + \alpha \left(\# \ln \frac{\mu^2}{m_h^2} + \# \ln \frac{\mu^2}{m_l^2} + \dots \right) \\ + \dots + \alpha^n \left(\# \ln^n \frac{\mu^2}{m_h^2} + \# \ln^n \frac{\mu^2}{m_l^2} + \dots \right)$$

Decoupling - How?

1. Decoupling (scale m_h)

Full Theory

(light and heavy particles)



Effective Theory

(only light particles + new couplings **C**)

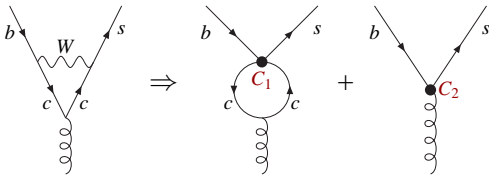
2. RGE running

high scale: $\mu = m_h$

C: ↓ **RGE**

low scale $\mu = m_l$

Decoupling:



3. Process (scale m_l)

Calculation of the process in the effective theory

→ Γ_{eff}

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Full Theory

(light and heavy particles)



Effective Theory

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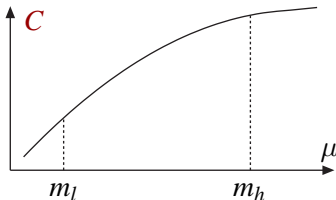
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high scale: $\mu = m_h$

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RGE running:



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Effective Theory

(only light particles + new couplings **C**)

2. RGE running

high scale: $\mu = m_h$

C: ↓ **RGE**

low scale $\mu = m_l$

Comparison:

$$\begin{aligned}\Gamma &= \# + \alpha \left(\# \ln \frac{\mu_h^2}{m_h^2} + \# \ln \frac{\mu^2}{\mu_h^2} + \# \ln \frac{\mu^2}{m_l^2} + \dots \right) \\ &+ \dots + \alpha^n \left(\# \ln^n \frac{\mu^2}{\mu_h^2} + \dots \right) \\ &= \mathbf{C}(m_h, \mu_h) \cdot \mathbf{RGE}(\mu_h, \mu) \cdot \mathbf{\Gamma}_{eff}(m_l, \mu)\end{aligned}$$

3. Process (scale m_l)

Calculation of the process in the effective theory

→ $\mathbf{\Gamma}_{eff}$

2. $\Gamma(Z \rightarrow \text{hadrons})$

Decay Rate of the Z-Boson into Hadrons

$$\Gamma(Z \rightarrow \text{hadrons}) = \sum_{f_{QCD}} \int d\Phi \left| \mathcal{M}(Z \rightarrow f_{QCD}) \right|^2$$

$$= \int d\Phi \left| \begin{array}{c} \text{Z} \\ \text{wavy} \\ \swarrow \quad \searrow \\ \text{quark} \quad \text{antiquark} \end{array} \right|^2 + \dots \xrightarrow{\text{Opt.Th.}} \text{Im} \left[\begin{array}{c} \text{Z} \\ \text{wavy} \\ \text{circle with quark loop} \\ \text{Z} \end{array} \right] + \dots$$

$$\begin{aligned} \Gamma(Z \rightarrow \text{hadrons}) &= \Gamma_0 \cdot \frac{4\pi}{s} \text{Im} \left(\Pi_1(-s - i\varepsilon) \right) \\ &= \Gamma_0 \cdot R(s) \quad \left(\Gamma_0 = \frac{G_F M_Z^3}{8\pi\sqrt{2}} \right) \end{aligned}$$

$$\begin{array}{c} \text{Z} \\ \text{wavy} \\ \mu \end{array} \text{ (QCD) } \begin{array}{c} \text{Z} \\ \text{wavy} \\ \nu \end{array} = g^{\mu\nu} \Pi_1(-q^2) + q^\mu q^\nu \Pi_2(-q^2)$$

Z-Propagator

The diagram shows the expansion of the Z boson propagator. On the left, a yellow circle labeled 'QCD' is connected to two wavy lines representing Z bosons. This is equal to a sum of diagrams: a tree-level propagator (two wavy lines), a one-loop diagram with a quark loop (circle with arrows and 'q'), a two-loop diagram with two quark loops (circle with two internal quark lines), and an ellipsis. Below this, two more diagrams are shown: a two-loop diagram with a top quark loop (triangle with 't, q' labels) and a three-loop diagram with two top quark loops (triangle with two internal top quark lines), followed by an ellipsis.

- scale $\sqrt{s} = M_Z \Rightarrow m_q = 0$ ($q = \{b, c, s, u, d\}$)
- 2 scales: $\sqrt{s} = M_Z$ and m_t (but $M_Z^2/(4m_t^2) \ll 1$)
- calculation up to α_s^4 (4-5 loops)
(only massive tadpoles and massless propagators possible)

\Rightarrow **Decoupling of the Top Quark**
(One 2-scale problem \rightarrow Two 1-scale problems)

Vector and Axial Vector Coupling

Z coupling to fermions:

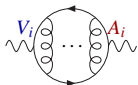
$$j_{Z_i}^\mu = g_i^V \overbrace{\bar{\psi}_i \gamma^\mu \psi_i}^{=V_i} + g_i^A \overbrace{\bar{\psi}_i \gamma^\mu \gamma_5 \psi_i}^{=A_i}$$

Incoherent sum of vector and axial vector coupling

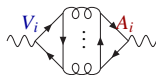
$$\begin{aligned}
 Z \text{ (wavy)} \text{---} \text{QCD} \text{ (yellow circle)} \text{---} Z \text{ (wavy)} &= \sum_q (g_q^V)^2 \text{ (vector loop)} + \sum_{i,j} g_i^V g_j^V \text{ (vector box)} \\
 &+ \sum_q (g_q^A)^2 \text{ (axial loop)} + \sum_{i,j} g_i^A g_j^A \text{ (axial box)}
 \end{aligned}$$

$i, j = \{t, b, c, s, u, d\}$

NO



and



diagrams

Decoupling of the Top Quark

$$= C_{2g}$$

$$= C_{3g}$$

$$= C_h$$

$$= C_\psi$$

vector coupling $V_i = \bar{\psi}_i \gamma^\mu \psi_i$

$$C_{2g} = C_{3g} = C_h = C_\psi = 0 \quad (\text{Ward identity})$$

axial vector coupling $A_i = \bar{\psi}_i \gamma^\mu \gamma_5 \psi_i$

$$C_{2g} = C_{3g} = 0, \quad C_h = \# \alpha_s^2 + \dots, \quad C_\psi = \# \alpha_s^3 + \dots$$

Results

$$\begin{aligned}
 Z \text{ (wavy)} \text{ (QCD)} \text{ (wavy)} &= \sum_q (g_q^V)^2 \text{ (diagram with } V_q \text{ and } R^{V,NS}) + \sum_{i,j} g_i^V g_j^V \text{ (diagram with } V_i, V_j \text{ and } R^{V,S}) \\
 &+ \sum_q (g_q^A)^2 \text{ (diagram with } A_q \text{ and } R^{A,NS}) + \sum_{i,j} g_i^A g_j^A \text{ (diagram with } A_i, A_j \text{ and } C_h, C_\psi, R^{A,S}, R^{A,NS})
 \end{aligned}
 \quad (g_i^A = \pm 1)$$

Example:

$$\text{(diagram with } t, q \text{ and } C_h) \rightarrow C_h \cdot \text{(diagram with } q \text{ and } C_h)$$

Results

$$\begin{aligned}
 Z \text{ (wavy)} \text{---} \text{QCD} \text{---} Z &= \sum_q (g_q^V)^2 \text{ (diagram with } V_q \text{) } + \sum_{i,j} g_i^V g_j^V \text{ (diagram with } V_i, V_j \text{)} \\
 &\rightarrow R^{V,NS} \qquad \qquad \qquad \rightarrow R^{V,S} \qquad (g_i^A = \pm 1) \\
 + \sum_q (g_q^A)^2 \text{ (diagram with } A_q \text{)} &+ \sum_{i,j} g_i^A g_j^A \text{ (diagram with } A_i, A_j \text{)} \\
 &\rightarrow R^{A,NS} \qquad \qquad \qquad \rightarrow C_h, C_\psi, R^{A,S}, R^{A,NS}
 \end{aligned}$$

$$R^V = \sum_q (g_q^V)^2 R^{V,NS} + \left(\sum_q g_q^V \right)^2 R^{V,S}$$

$$\begin{aligned}
 R^A &= 5 R^{A,NS} + (C_h - C_\psi)^2 5 (R^{A,NS} + 5 R^{A,S}) \\
 &\quad - 2(C_h - C_\psi)(R^{A,NS} + 5 R^{A,S}) + R^{A,S}
 \end{aligned}$$

$$\Gamma(Z \rightarrow \text{hadrons}) = \Gamma_0 \cdot R = \Gamma_0 \cdot (R^V + R^A)$$

$R^{V,NS}$, $R^{A,NS}$, $R^{V,S}$ und $R^{A,S}$: Baikov, Chetyrkin
 C_h und C_ψ : JR

$$R = R^V + R^A = 6.73 + 6.73 \frac{\alpha_s}{\pi} + 5.12 \left(\frac{\alpha_s}{\pi}\right)^2 - 113.07 \left(\frac{\alpha_s}{\pi}\right)^3 - 571.61 \left(\frac{\alpha_s}{\pi}\right)^4$$

$$\Gamma(Z \rightarrow \text{hadrons}) = \Gamma_0 \cdot R = 1738 \text{ MeV} \quad (\text{QCD corrections})$$

$$\Gamma(Z \rightarrow \text{hadrons}) = 1744 \pm 2 \text{ MeV} \quad (\text{PDG})$$

Electroweak corrections and mass corrections (m_q/M_Z) are missing.

Conclusion

Very high precision in α_s
 \Rightarrow Very exact prediction of α_s with very small error

- ① Decoupling of heavy particles
 - technical and conceptual problems with many scales
 - Solution:
 1. Decoupling of heavy particles (m_h)
→ effective theory with new couplings
 2. Using RGE running ($m_h \rightarrow m_l$)
 3. Calculation of the process in the effective theory (m_l)
- ② Decay rate of the Z boson into hadrons: $\Gamma(Z \rightarrow \text{hadrons})$
 - $\Gamma(Z \rightarrow \text{hadrons}) \propto \text{Im}(Z\text{-Propagator})$
 - Decoupling of the top quark in vector and axial vector coupling
 - $\Gamma(Z \rightarrow \text{hadrons}) = \Gamma_0 R \hat{=} \Gamma_0 (R^{NS,S} + C_{h,\psi} \cdot R^{NS,S})$
with very high precision in α_s