Decoupling of the Top Quark in $\Gamma(Z \to \text{hadrons})$

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in collaboration with

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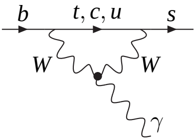
GK-Workshop / Bad Liebenzell   5.10.-7.10.2011

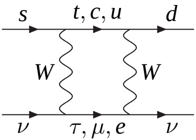
Outline

1. Decoupling of heavy particles
    - Why ?
    - How ?
2. Decay rate of the Z-boson into hadrons: $\Gamma(Z \to \text{hadrons})$
    - Definition
    - Decoupling of the Top Quark
    - Results

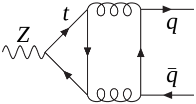
Examples:



$b \to s\,\gamma$



$K^+ \to \pi^+ \nu\bar{\nu}$



$Z \to$hadrons

Problems with many scales

- **Technical:** Difficult (atm impossible) to calculate.
  At high loop order only 1-scale problems possible.
- **conceptual:** Largly separated scales lead to big logarithms
  $\to$ bad convergence of the perturbation series ($\alpha \cdot \ln(m_h^2/m_l^2) \sim 1$)

$$\Gamma = \# + \alpha\left(\# \ln \frac{\mu^2}{m_h^2} + \# \ln \frac{\mu^2}{m_l^2} + \dots\right) + \dots + \alpha^n\left(\# \ln^n \frac{\mu^2}{m_h^2} + \# \ln^n \frac{\mu^2}{m_l^2} + \dots\right)$$

Decoupling - How?

1. Decoupling (scale m_h)

Full Theory
(light and heavy particles)
$\Downarrow$
Effective Theory
(only light particles + new couplings C)

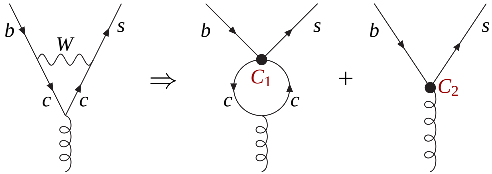
2. RGE running

high scale: $\mu = m_h$

C: $\downarrow$ RGE

low scale $\mu = m_l$

Decoupling:



3. Process (scale m_l)

Calculation of the process in the effective theory
$\rightarrow \Gamma_{eff}$

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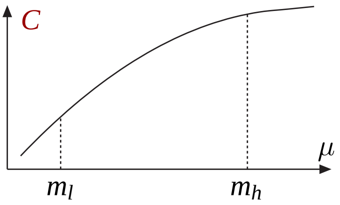

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Comparison:

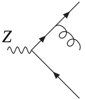
$$\Gamma = \# + \alpha\left(\#\ln\frac{\mu_h^2}{m_h^2} + \#\ln\frac{\mu^2}{\mu_h^2} + \#\ln\frac{\mu^2}{m_l^2} + \ldots\right) + \cdots + \alpha^n\left(\#\ln^n\frac{\mu^2}{\mu_h^2} + \ldots\right)$$
$$= C(m_h, \mu_h) \cdot RGE(\mu_h, \mu) \cdot \Gamma_{eff}(m_l, \mu)$$

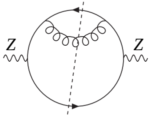
3. Process (scale m_l)

Calculation of the process in the effective theory
$\rightarrow \Gamma_{eff}$

2. $\Gamma(Z \to \text{hadrons})$

$$\Gamma(Z \to \text{hadrons}) = \sum_{f_{QCD}} \int d\Phi \left| \mathcal{M}(Z \to f_{QCD}) \right|^2$$

$= \int d\Phi \Bigg|$ 

$\Bigg|^2 + \; \ldots \; \xrightarrow[\text{Opt.Th.}]{} \; Im\Bigg[$ 

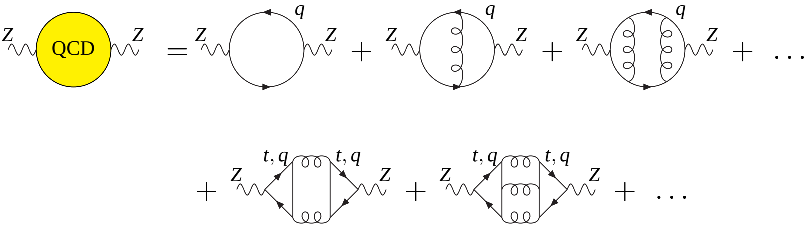
$\Bigg] + \; \ldots$

$$\Gamma(Z \to \text{hadrons}) = \Gamma_0 \cdot \frac{4\pi}{s} Im\Big(\Pi_1(-s - i\,\varepsilon)\Big)$$

$$= \Gamma_0 \cdot R(s) \qquad \left(\Gamma_0 = \frac{G_F M_Z^3}{8\pi\sqrt{2}} \right)$$

$= g^{\mu\nu}\Pi_1(-q^2) + q^\mu q^\nu \Pi_2(-q^2)$

Z-Propagator



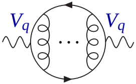
- scale $\sqrt{s} = M_z \quad \Rightarrow \quad m_q = 0 \quad (q = \{b, c, s, u, d\})$
- 2 scales: $\sqrt{s} = M_Z$ and m_t (but $M_Z^2/(4m_t^2) \ll 1$)
- calculation up to α_s^4 (4-5 loops)
  (only massive tadpoles and
  massless propagators possible)

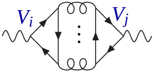
$\Rightarrow$ Decoupling of the Top Quark
(One 2-scale problem $\rightarrow$ Two 1-scale problems)

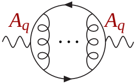
Z coupling to fermions:

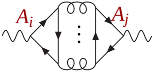
$$j^\mu_{Z_i} = g_i^V \overbrace{\bar{\psi}_i\, \gamma^\mu\, \psi_i}^{=V_i} + g_i^A \overbrace{\bar{\psi}_i\, \gamma^\mu \gamma_5\, \psi_i}^{=A_i}$$

Incoherent sum of vector and axial vector coupling

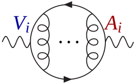
$= \sum_q (g_q^V)^2$ 

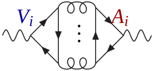
$+ \sum_{i,j} g_i^V\, g_j^V$ 

$+ \sum_q (g_q^A)^2$ 

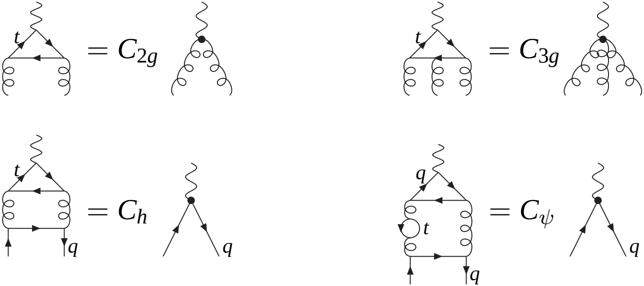
$+ \sum_{i,j} g_i^A\, g_j^A$ 

$i, j = \{t, b, c, s, u, d\}$

NO 

and 

diagrams



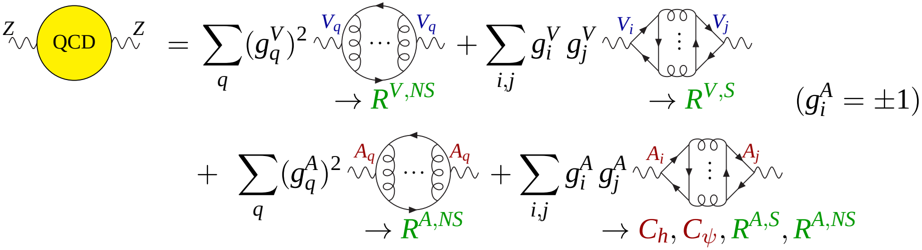
vector coupling $V_i = \overline{\psi}_i \, \gamma^\mu \, \psi_i$

$$C_{2g} = C_{3g} = C_h = C_\psi = 0 \qquad \text{(Ward identity)}$$

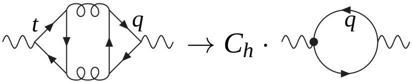
axial vector coupling $A_i = \overline{\psi}_i \, \gamma^\mu \gamma_5 \, \psi_i$

$$C_{2g} = C_{3g} = 0, \quad C_h = \#\alpha_s^2 + \ldots, \quad C_\psi = \#\alpha_s^3 + \ldots$$

Results



Example:



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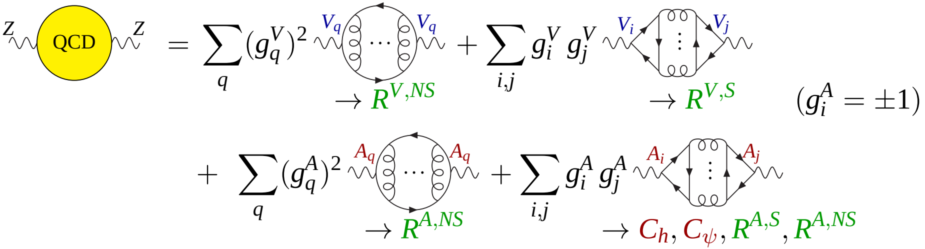

$$
\begin{aligned}
R^V &= \sum_q (g_q^V)^2 R^{V,NS} + \Big(\sum_q g_q^V\Big)^2 R^{V,S} \\
R^A &= 5 R^{A,NS} + (C_h - C_\psi)^2 5 (R^{A,NS} + 5 R^{A,S}) \\
&\quad - 2 (C_h - C_\psi)(R^{A,NS} + 5 R^{A,S}) + R^{A,S}
\end{aligned}
$$

$$\Gamma(Z \to \text{hadrons}) = \Gamma_0 \cdot R = \Gamma_0 \cdot \big(R^V + R^A\big)$$

Results

$R^{V,NS}$, $R^{A,NS}$, $R^{V,S}$ und $R^{A,S}$: Baikov, Chetyrkin
C_h und C_ψ: JR

$$R = R^V + R^A = 6.73 + 6.73\,\frac{\alpha_s}{\pi} + 5.12\left(\frac{\alpha_s}{\pi}\right)^2 - 113.07\left(\frac{\alpha_s}{\pi}\right)^3 - 571.61\left(\frac{\alpha_s}{\pi}\right)^4$$

$$\Gamma(Z \to \text{hadrons}) = \Gamma_0 \cdot R = 1738\ MeV \qquad \text{(QCD corrections)}$$

$$\Gamma(Z \to \text{hadrons}) = 1744 \pm 2\ MeV \qquad \text{(PDG)}$$

Electroweak corrections and mass corrections (m_q/M_Z) are missing.

Conclusion

Very high precission in α_s
$\Rightarrow$ Very exact prediction of α_s with very small error

Summary

1. Decoupling of heavy particles
   - technical and conceptual problems with many scales
   - Solution:

     1. Decoupling of heavy particles (m_h)
        $\rightarrow$ effective theory with new couplings
     2. Using RGE running ($m_h \rightarrow m_l$)
     3. Calculation of the process in the effective theory (m_l)

2. Decay rate of the Z boson into hadrons: $\Gamma(Z \rightarrow \text{hadrons})$
   - $\Gamma(Z \rightarrow \text{hadrons}) \propto Im(\text{Z-Propagator})$
   - Decoupling of the top quark in vector and axial vector coupling
   - $\Gamma(Z \rightarrow \text{hadrons}) = \Gamma_0 \, R \mathrel{\hat{=}} \Gamma_0 (R^{NS,S} + C_{h,\psi} \cdot R^{NS,S})$
     with very high precission in α_s