The Standard Model with four generations

Graduiertenkollegs-Workshop in Bad Liebenzell, 7. Oktober 2011

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Institut für Theoretische Teilchenphysik

Outline

- The fourth family – pros and cons
- The SM4 parameters
- Constraints to the SM4
- First results of a global fit

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The fourth family

Another sequential generation of fermions

Leptons: $\begin{pmatrix} \nu_e \\ e \end{pmatrix}, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}, \begin{pmatrix} \nu_4 \\ \ell_4 \end{pmatrix}$

Quarks: $\begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} c \\ s \end{pmatrix}, \begin{pmatrix} t \\ b \end{pmatrix}, \begin{pmatrix} t' \\ b' \end{pmatrix}$

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The fourth family – pros ...

Some arguments in favour of the fourth family

- Baryogenesis (CP-violation and phase transition)

  [Hou '08, Carena et al. '04, Fok & Kribs '08, Kikukawa et al. '09]

- Solution to certain flavour physics problems

  [Hou '06, Soni '09]

- Unification of the gauge couplings

  [Hung '97]

- Softening of the Higgs mass bounds

  [Novikov et al. '02,'09, Frere et al. '04, Kribs et al. '07]

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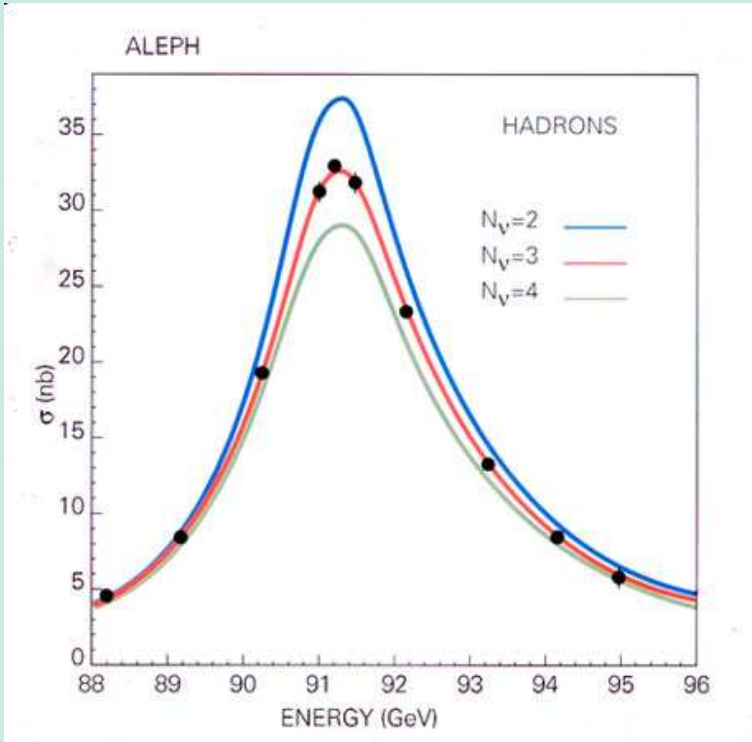
... and cons

Invisible Z decays

$N_\nu = 2.9840 \pm 0.0082$

[LEP '06]



... and cons

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$N_\nu^{\text{light}} = 2.9840 \pm 0.0082$

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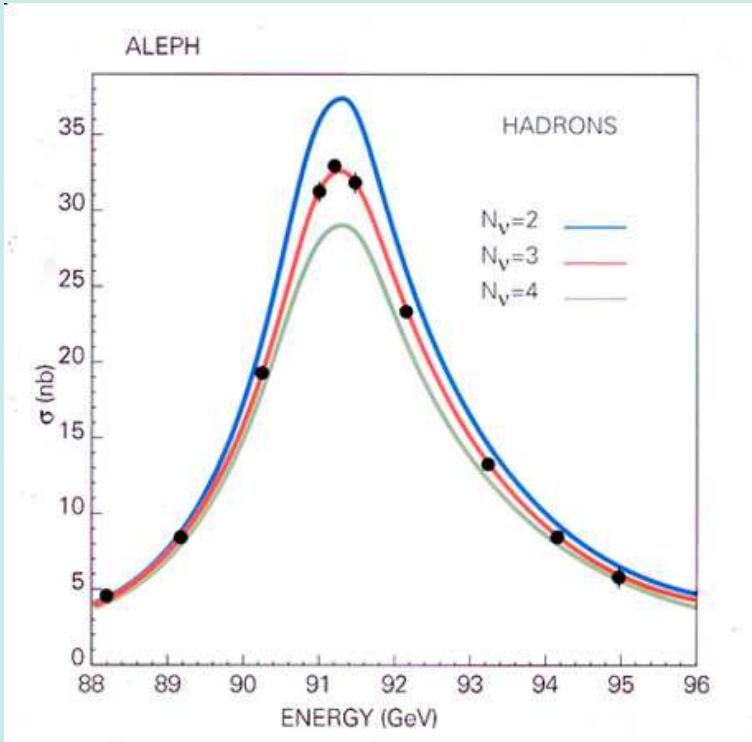
But neutrinos do have a mass.

[Super-Kamiokande '98]



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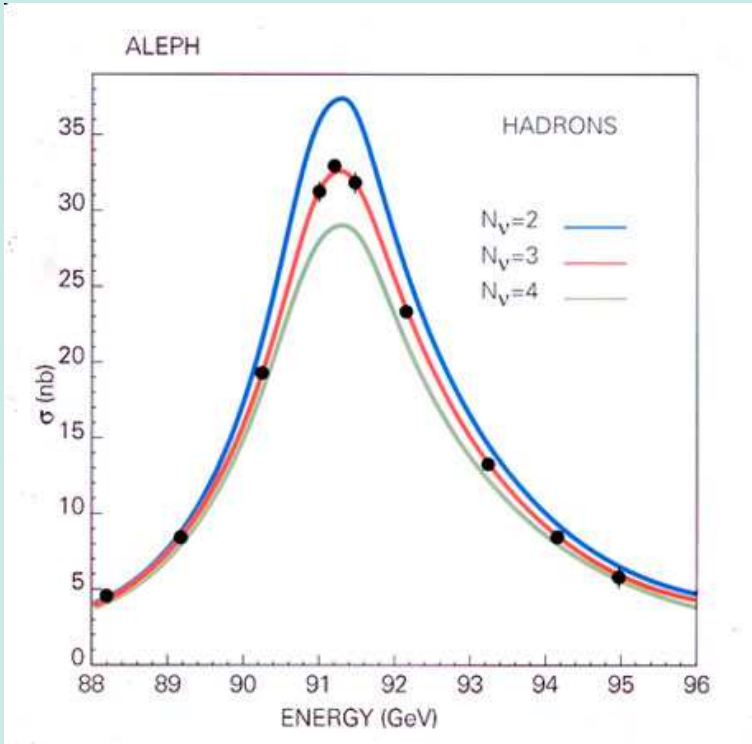
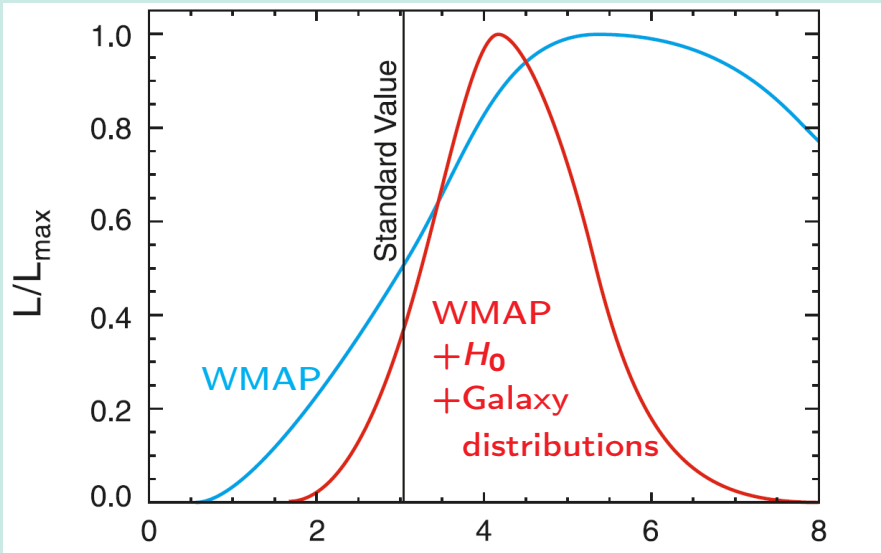


Cosmology: $N_\nu^{\text{eff}} = 4.34^{+0.86}_{-0.88}$

[7y WMAP '10]



... and cons

Electroweak precision observables

History of the PDG reviews [Erler/Langacker]:

- 1994: "one heavy generation of ordinary fermions is allowed at 95% CL"

... and cons

Electroweak precision observables

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- 1998: "an extra generation of ordinary fermions is now excluded at the 99.2% CL"

... and cons

Electroweak precision observables

History of the PDG reviews [Erler/Langacker]:

- 1994: "one heavy generation of ordinary fermions is allowed at 95% CL"
- 1998: "an extra generation of ordinary fermions is now excluded at the 99.2% CL"
- 2010: "an extra generation of ordinary fermions is excluded at the 6 σ level on the S parameter alone. This result assumes [...] that any new families are degenerate. [...] a fourth family is disfavored but not excluded by current data."

The SM4 parameters

The SM quark mixing matrix

$$c_{ij} \equiv \cos\theta_{ij}$$

$$s_{ij} \equiv \sin\theta_{ij}$$

$$V^{\mathrm{CKM3}} \equiv \begin{pmatrix} V_{ud}^{\mathrm{CKM3}} & V_{us}^{\mathrm{CKM3}} & V_{ub}^{\mathrm{CKM3}} \\ V_{cd}^{\mathrm{CKM3}} & V_{cs}^{\mathrm{CKM3}} & V_{cb}^{\mathrm{CKM3}} \\ V_{td}^{\mathrm{CKM3}} & V_{ts}^{\mathrm{CKM3}} & V_{tb}^{\mathrm{CKM3}} \end{pmatrix}$$

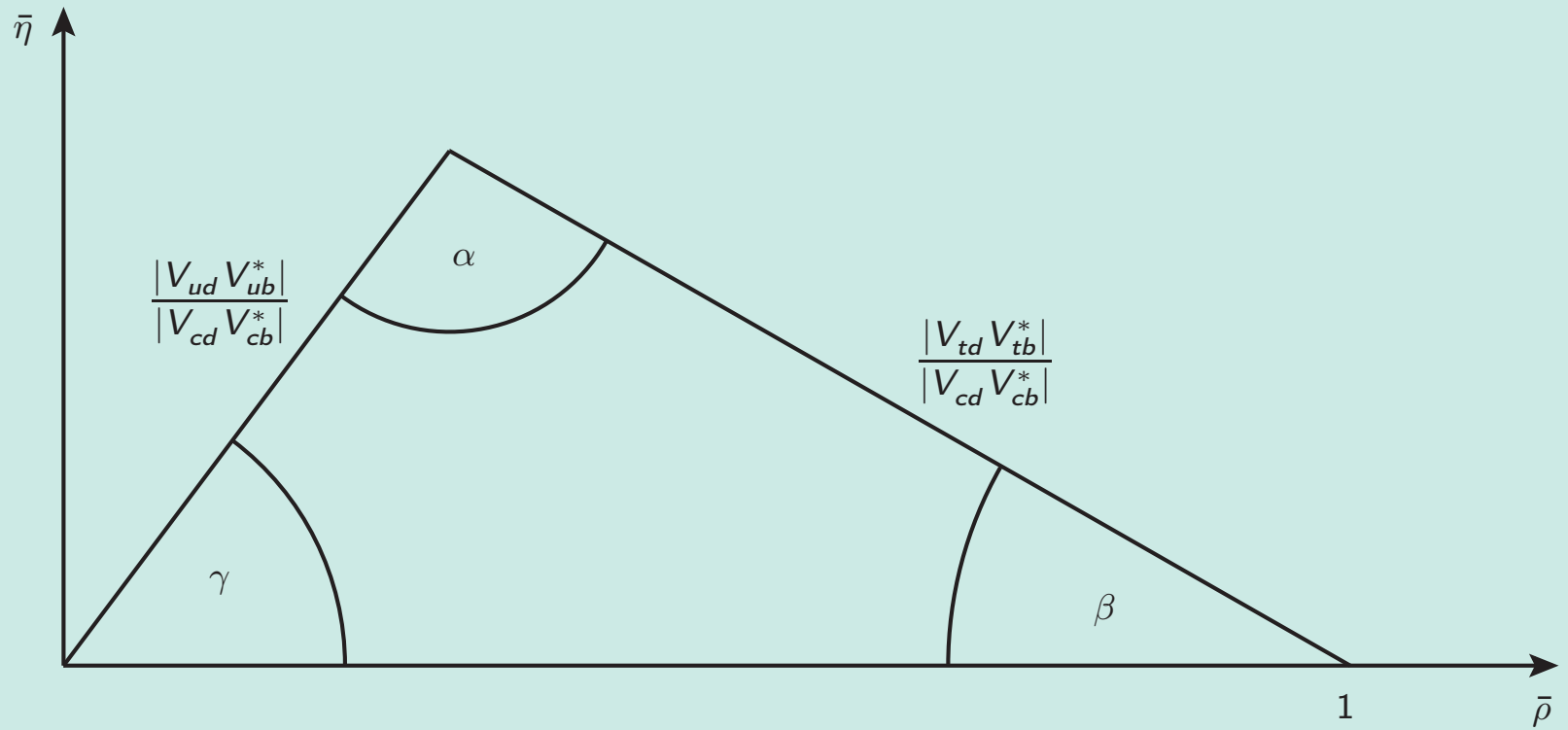
$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}$$

The SM4 parameters

The Wolfenstein parametrisation of the CKM matrix

$$V^{\mathrm{CKM3}} \approx \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$



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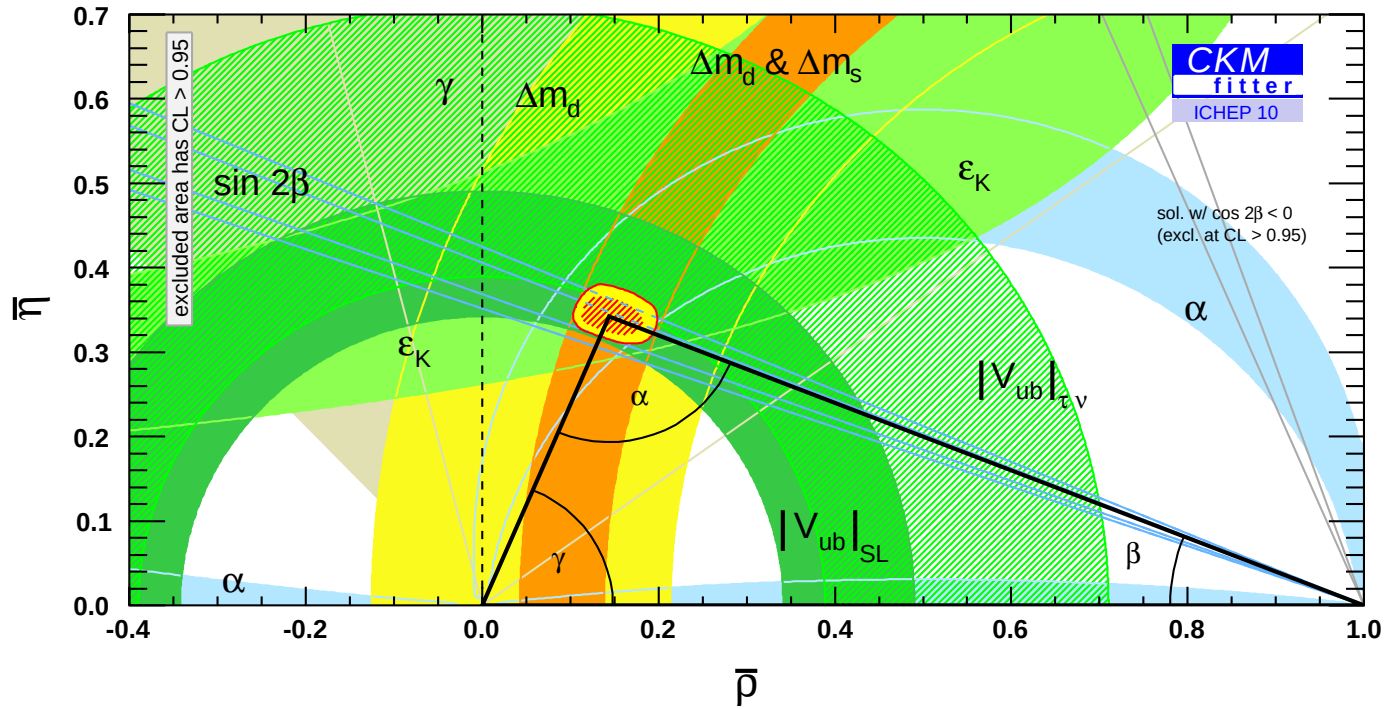

The SM4 parameters

The 4×4 CKM matrix

$$V^{\mathrm{CKM4}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & c_{34} & s_{34} \\ 0 & 0 & -s_{34} & c_{34} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{24} & 0 & s_{24} e^{-i\delta_{24}} \\ 0 & 0 & 1 & 0 \\ 0 & -s_{24} e^{i\delta_{24}} & 0 & c_{24} \end{pmatrix}$$

$$\cdot \begin{pmatrix} c_{14} & 0 & 0 & s_{14} e^{-i\delta_{14}} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s_{14} e^{i\delta_{14}} & 0 & 0 & c_{14} \end{pmatrix} \begin{pmatrix} & & & 0 \\ & V^{\mathrm{CKM3}} & & 0 \\ & & & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$c_{ij} \equiv \cos\theta_{ij}$

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The SM4 parameters

The 4×4 CKM matrix

$$V^{\mathrm{CKM4}} = \begin{pmatrix}
c_{12}c_{13}c_{14} & c_{13}c_{14}s_{12} & c_{14}s_{13}e^{-i\delta_{13}} & s_{14}e^{-i\delta_{14}} \\
\begin{array}{l}-c_{23}c_{24}s_{12} \\ -c_{12}c_{24}s_{13}s_{23}e^{i\delta_{13}} \\ -c_{12}c_{13}s_{14}s_{24}e^{i(\delta_{14}-\delta_{24})}\end{array} & \begin{array}{l}c_{12}c_{23}c_{24} \\ -c_{24}s_{12}s_{13}s_{23}e^{i\delta_{13}} \\ -c_{13}s_{12}s_{14}s_{24}e^{i(\delta_{14}-\delta_{24})}\end{array} & \begin{array}{l}c_{13}c_{24}s_{23} \\ -s_{13}s_{14}s_{24}e^{-i(\delta_{13}+\delta_{24}-\delta_{14})}\end{array} & c_{14}s_{24}e^{-i\delta_{24}} \\
\begin{array}{l}-c_{12}c_{23}c_{34}s_{13}e^{i\delta_{13}} \\ +c_{34}s_{12}s_{23} \\ -c_{12}c_{13}c_{24}s_{14}s_{34}e^{i\delta_{14}} \\ +c_{23}s_{12}s_{24}s_{34}e^{i\delta_{24}} \\ +c_{12}s_{13}s_{23}s_{24}s_{34}e^{i(\delta_{13}+\delta_{24})}\end{array} & \begin{array}{l}-c_{12}c_{34}s_{23} \\ -c_{23}c_{34}s_{12}s_{13}e^{i\delta_{13}} \\ -c_{12}c_{23}s_{24}s_{34}e^{i\delta_{24}} \\ -c_{13}c_{24}s_{12}s_{14}s_{34}e^{i\delta_{14}} \\ +s_{12}s_{13}s_{23}s_{24}s_{34}e^{i(\delta_{13}+\delta_{24})}\end{array} & \begin{array}{l}c_{13}c_{23}c_{34} \\ -c_{13}s_{23}s_{24}s_{34}e^{i\delta_{24}} \\ -c_{24}s_{13}s_{14}s_{34}e^{i(\delta_{14}-\delta_{13})}\end{array} & c_{14}c_{24}s_{34} \\
\begin{array}{l}-c_{12}c_{13}c_{24}c_{34}s_{14}e^{i\delta_{14}} \\ +c_{12}c_{23}s_{13}s_{34}e^{i\delta_{13}} \\ +c_{23}c_{34}s_{12}s_{24}e^{i\delta_{24}} \\ -s_{12}s_{23}s_{34} \\ +c_{12}c_{34}s_{13}s_{23}s_{24}e^{i(\delta_{13}+\delta_{24})}\end{array} & \begin{array}{l}-c_{12}c_{23}c_{34}s_{24}e^{i\delta_{24}} \\ +c_{12}s_{23}s_{34} \\ -c_{13}c_{24}c_{34}s_{12}s_{14}e^{i\delta_{14}} \\ +c_{23}s_{12}s_{13}s_{34}e^{i\delta_{13}} \\ +c_{34}s_{12}s_{13}s_{23}s_{24}e^{i(\delta_{13}+\delta_{24})}\end{array} & \begin{array}{l}-c_{13}c_{23}s_{34} \\ -c_{13}c_{34}s_{23}s_{24}e^{i\delta_{24}} \\ -c_{24}c_{34}s_{13}s_{14}e^{i(\delta_{14}-\delta_{13})}\end{array} & c_{14}c_{24}c_{34}
\end{pmatrix}$$

$c_{ij} \equiv \cos\theta_{ij}$

$s_{ij} \equiv \sin\theta_{ij}$

The SM4 parameters

The "old" parameters

Quarks:	$m_u, m_d, m_s, m_c, m_b, m_t,$	$\theta_{12}, \theta_{13}, \theta_{23},$	φ_{13}
Leptons:	$m_{\nu_e}, m_{\nu_\mu}, m_{\nu_\tau}, m_e, m_\mu, m_\tau,$	$\theta^\ell_{12}, \theta^\ell_{13}, \theta^\ell_{23},$	φ^ℓ_{13}
Higgs:	m_H		

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The SM4 parameters

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Quarks:	$m_u, m_d, m_s, m_c, m_b, m_t,$	$\theta_{12}, \theta_{13}, \theta_{23},$	φ_{13}
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Higgs:	m_H		

The new parameters

Quarks:	$m_{b'}, m_{t'},$	$\theta_{14}, \theta_{24}, \theta_{34},$	$\varphi_{14}, \varphi_{24}$
Leptons:	$m_{\nu_4}, m_{\ell_4},$	$\theta^\ell_{14}, \theta^\ell_{24}, \theta^\ell_{34},$	$\varphi^\ell_{14}, \varphi^\ell_{24}$

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The SM4 parameters

The "old" parameters

Quarks:	$m_u, m_d, m_s, m_c, m_b, m_t,$	$\theta_{12}, \theta_{13}, \theta_{23},$	φ_{13}
Leptons:	$m_{\nu_e}, m_{\nu_\mu}, m_{\nu_\tau}, m_e, m_\mu, m_\tau,$	$\theta^\ell_{12}, \theta^\ell_{13}, \theta^\ell_{23},$	φ^ℓ_{13}
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14 free parameters

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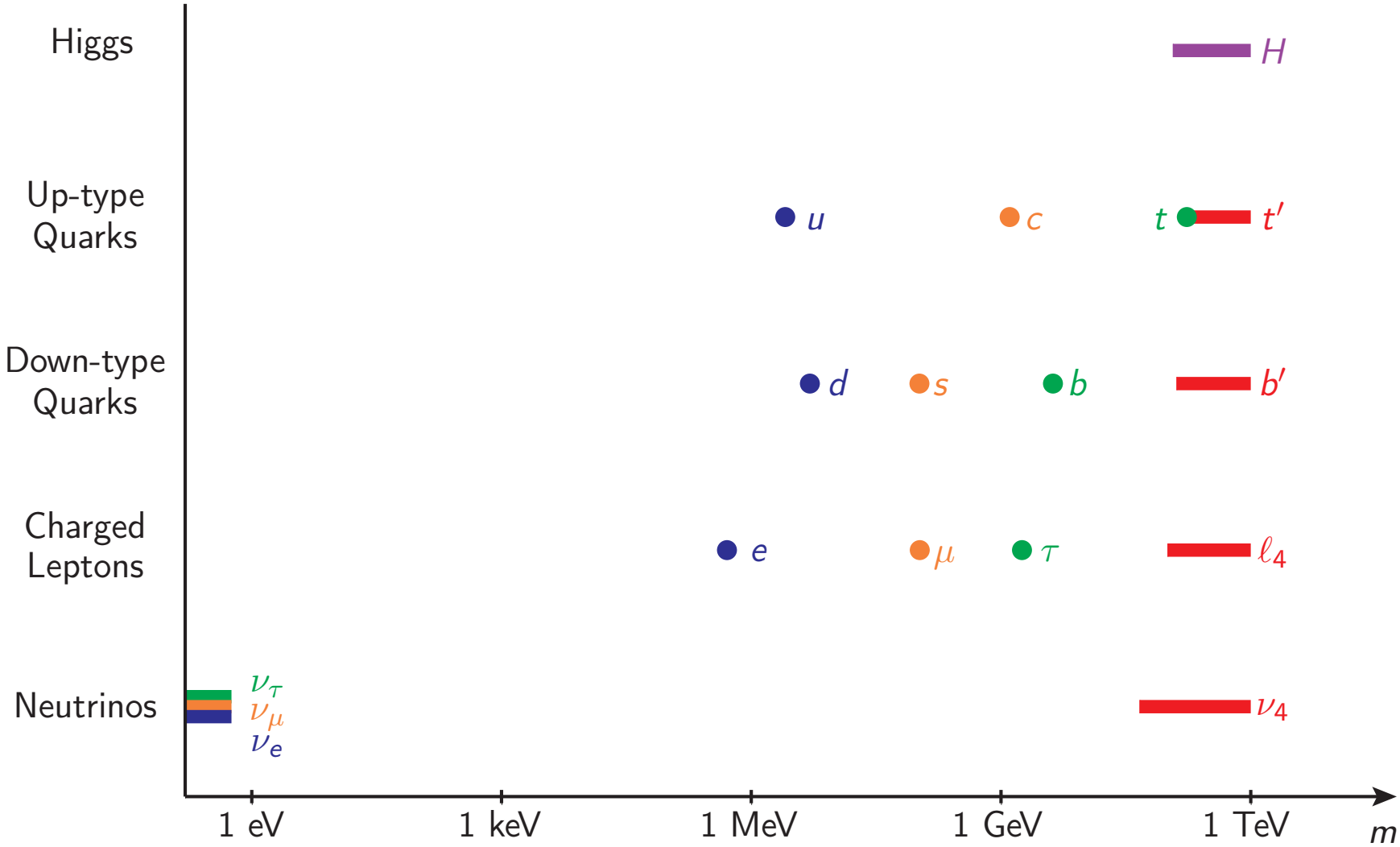
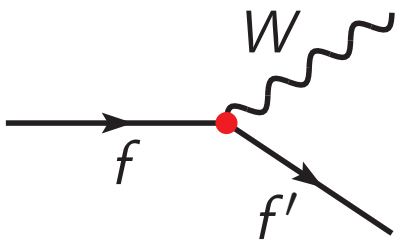
Constraints to the SM4

1. Masses



Constraints to the SM4

2. Tree-level observables



$$V_{CKM4} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} & V_{ub'} \\ V_{cd} & V_{cs} & V_{cb} & V_{cb'} \\ V_{td} & V_{ts} & V_{tb} & V_{tb'} \\ V_{t'd} & V_{t's} & V_{t'b} & V_{t'b'} \end{pmatrix}$$

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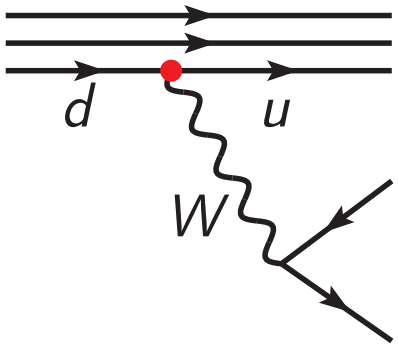
Constraints to the SM4

2. Tree-level observables

β decays



$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} & V_{ub'} \\ V_{cd} & V_{cs} & V_{cb} & V_{cb'} \\ V_{td} & V_{ts} & V_{tb} & V_{tb'} \\ V_{t'd} & V_{t's} & V_{t'b} & V_{t'b'} \end{pmatrix}$$

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Constraints to the SM4

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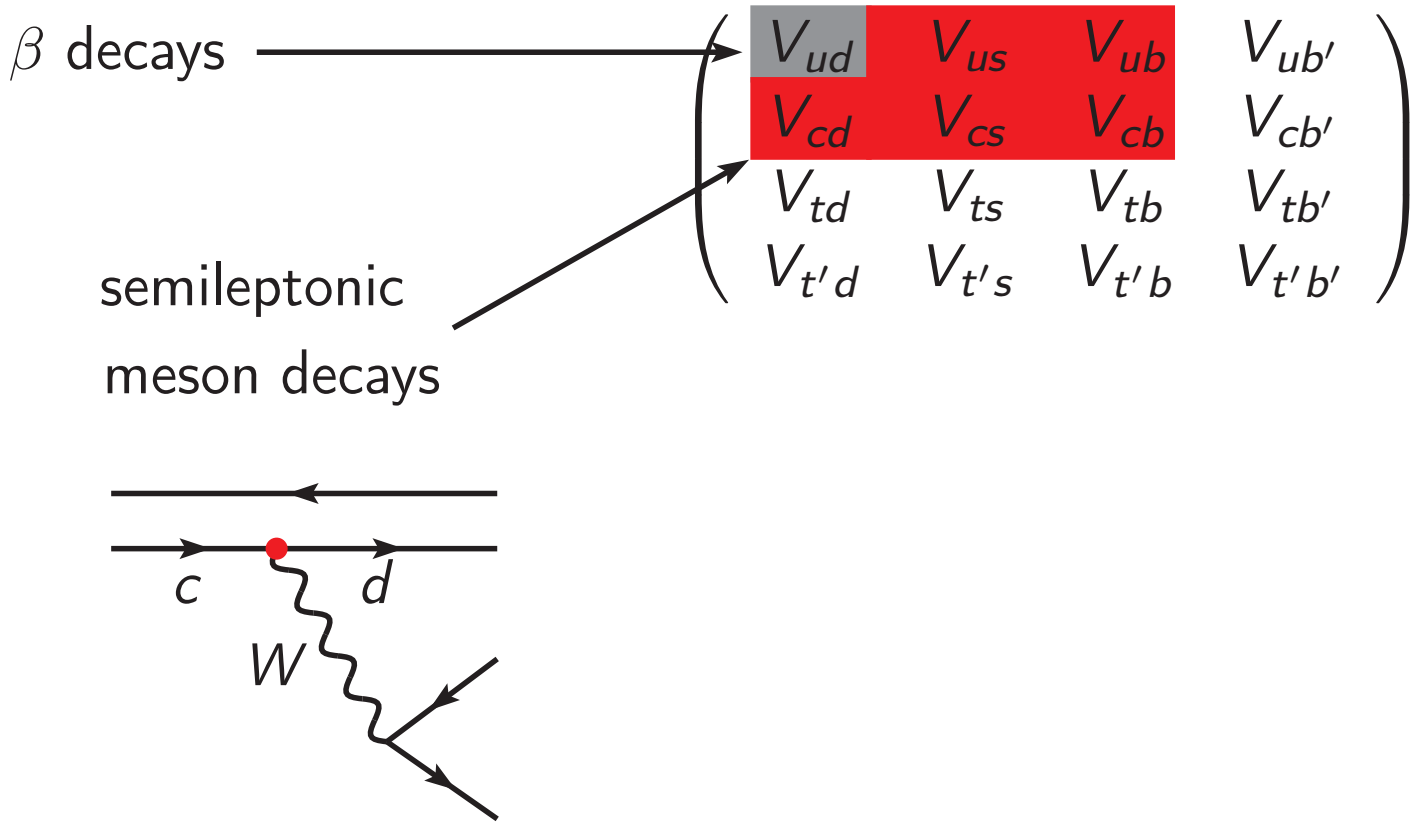

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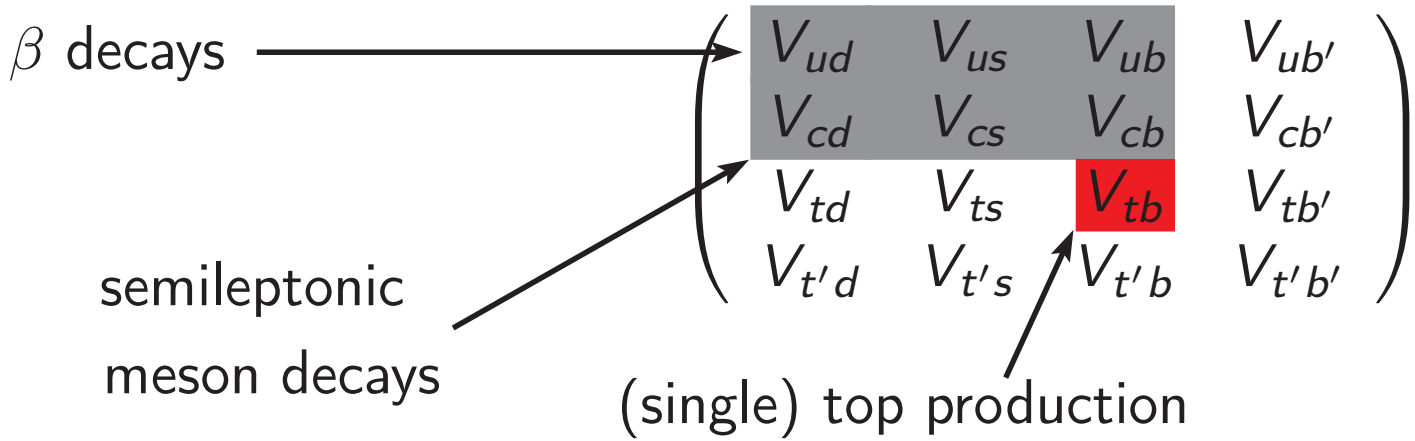

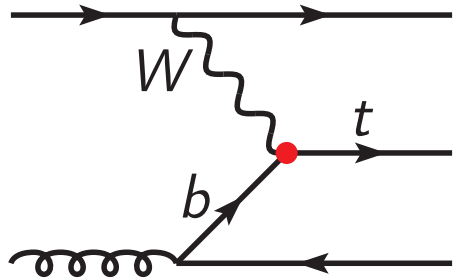

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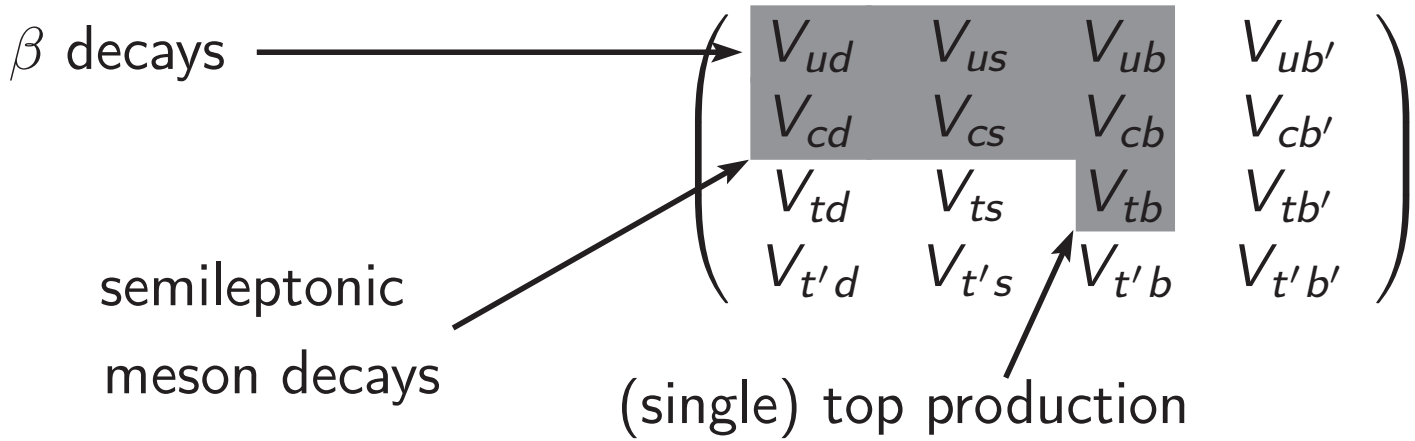


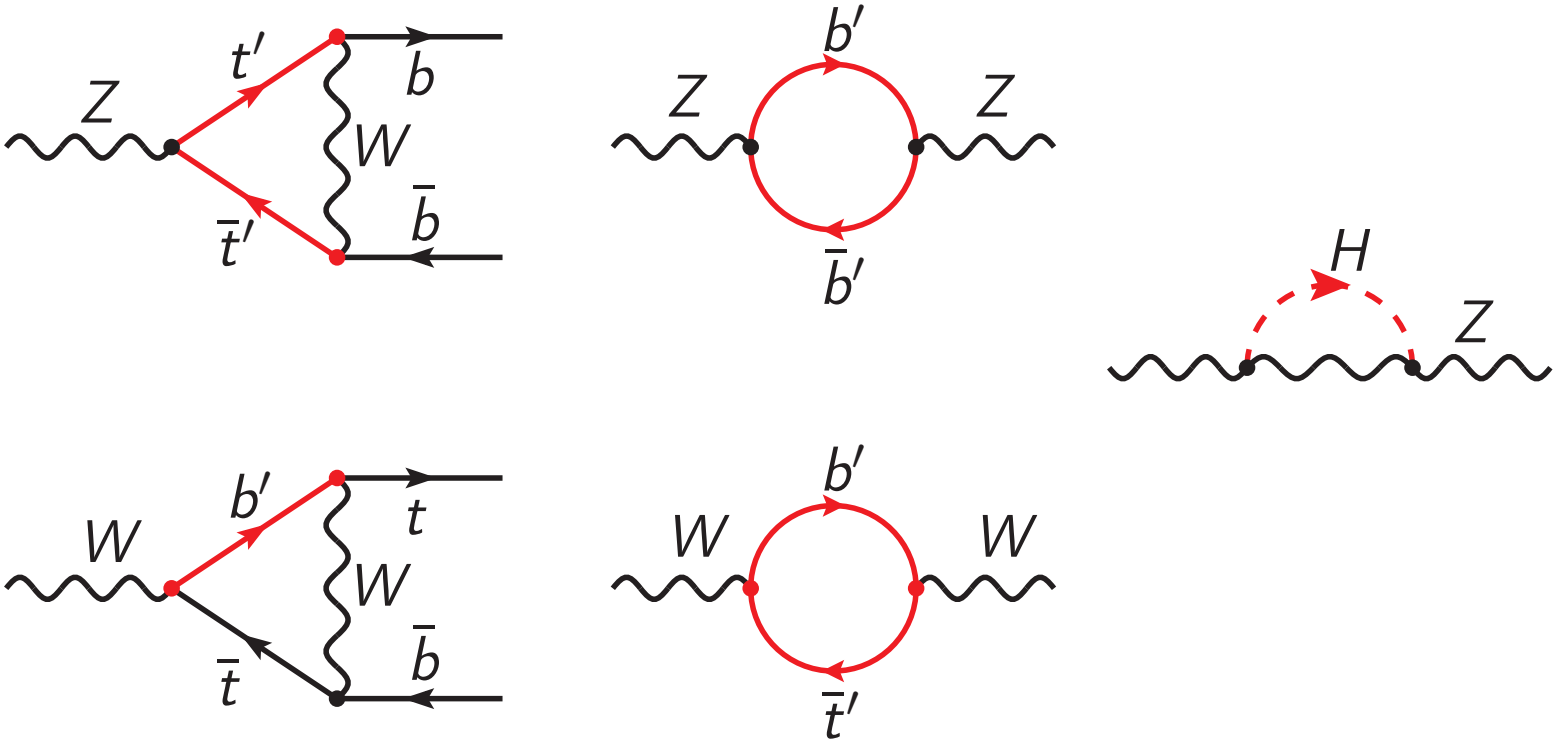
$\gamma \equiv \arg \dfrac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*}$ from CKMfitter fits

$W \to \ell\nu$ decays

Constraints to the SM4

3. Electroweak precision observables

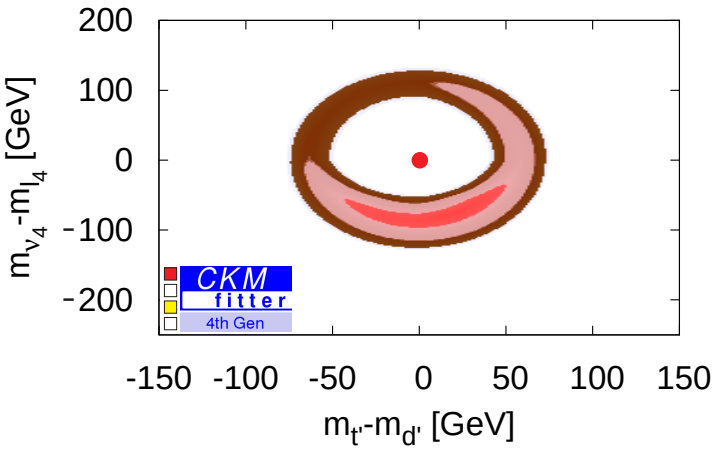
SM4 loop contributions to Z and W processes (examples)

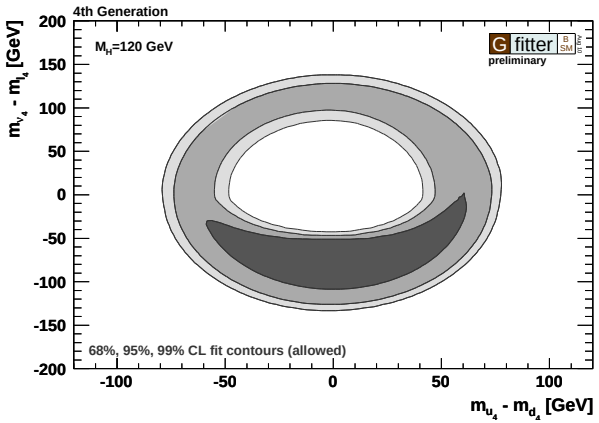


Constraints to the SM4

3. Electroweak precision observables

Fermion mass differences





(m_H fixed, V_{CKM4} diagonal)

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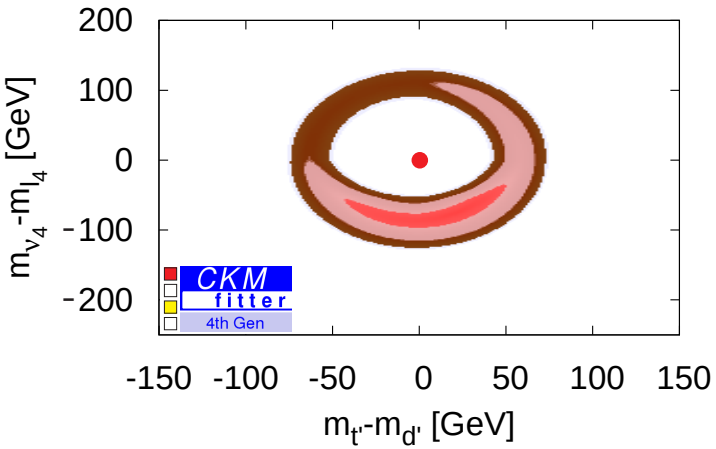
Constraints to the SM4

3. Electroweak precision observables

Fermion mass differences



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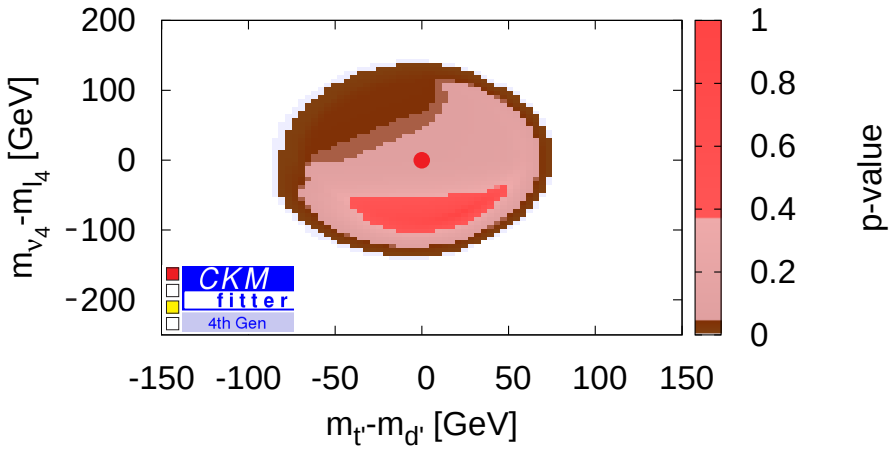

(m_H fixed)

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Constraints to the SM4

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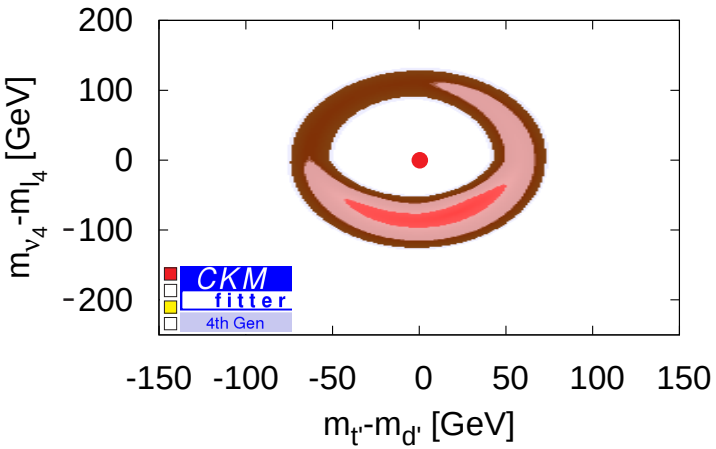
Fermion mass differences



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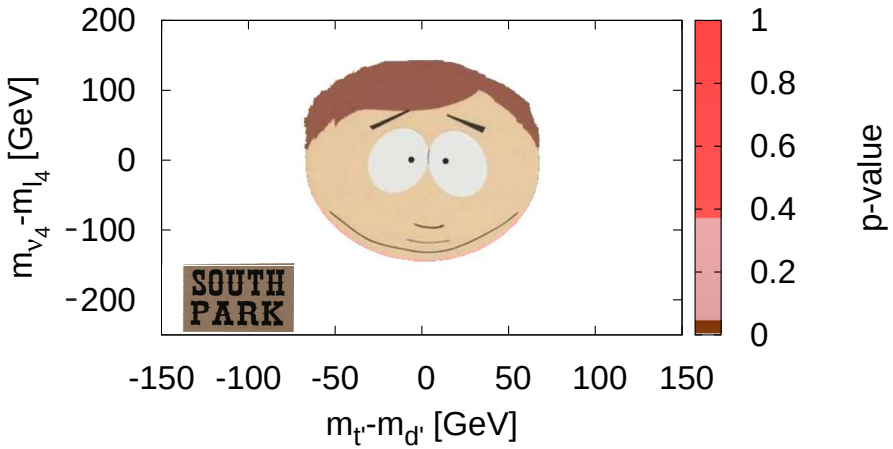

(Cartman)

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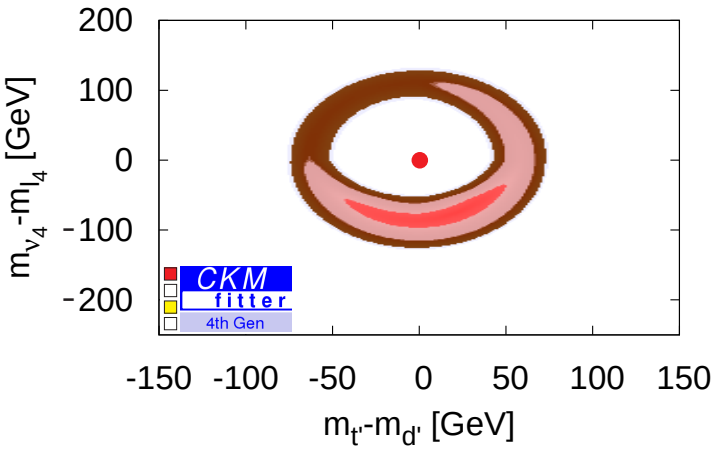
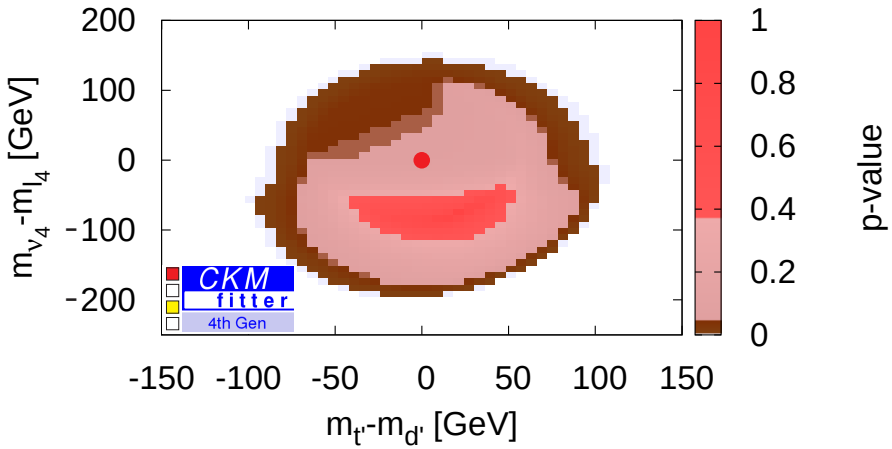
Constraints to the SM4

3. Electroweak precision observables

Fermion mass differences



(m_H fixed, V_{CKM4} diagonal)



(m_H free)

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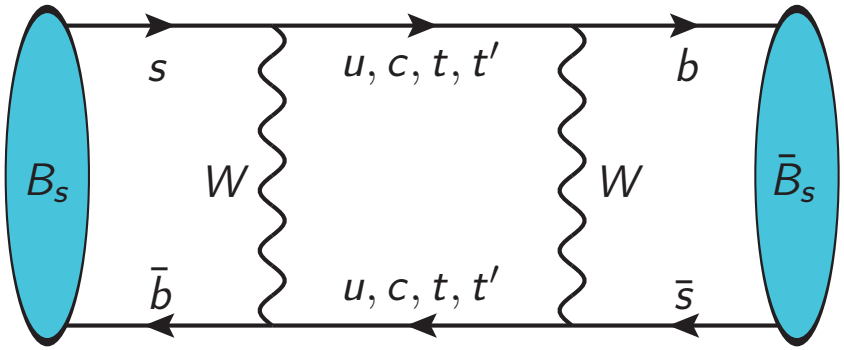
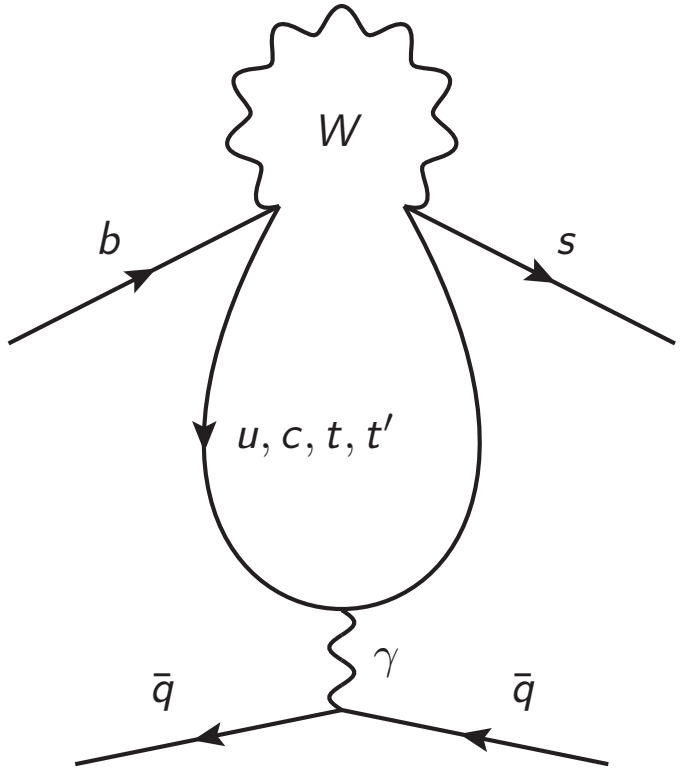
Constraints to the SM4

4. Flavour observables





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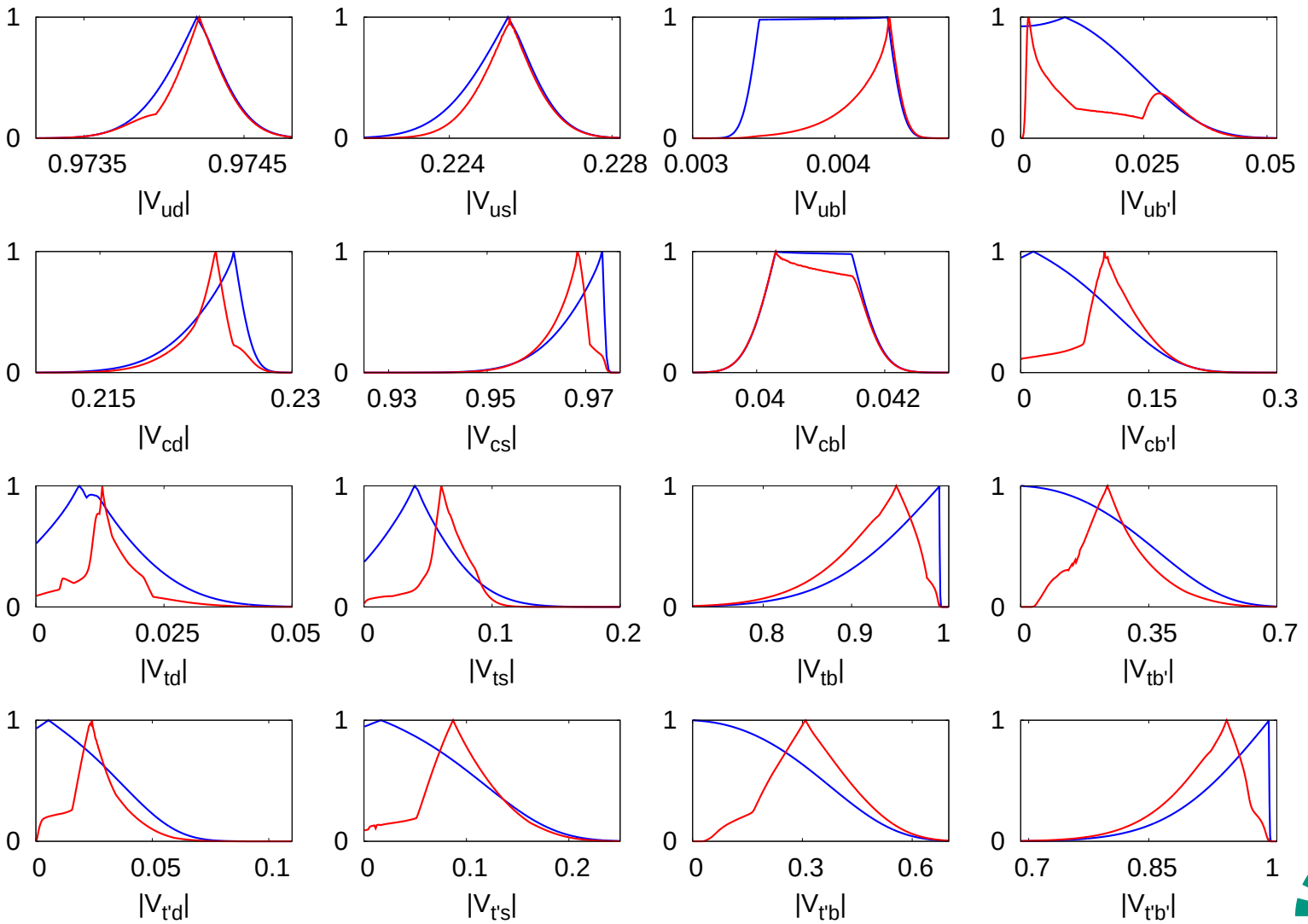
First results of o global fit

Moduli of the CKM matrix without and with EW and flavour constraints:



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Outlook

- Global fit combining all observables
- Allowing for non-trivial PMNS structure
- Waiting for new experimental results

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Back-up slides

Inputs I

$$m_{t'} \in [150, 1000] \text{ GeV}$$
$$m_{b'} \in [128, 1000] \text{ GeV}$$
$$m_{\nu_4} \in [46, 1000] \text{ GeV}$$
$$m_{\ell_4} \in [100, 1000] \text{ GeV}$$

$$m_H \in [116, 1000] \text{ GeV}$$

$$|V_{ud}| = 0.97421^{+0.00034}_{-0.00029}$$
$$|V_{us}| = 0.2254 \pm 0.0013$$
$$|V_{ub}| = \left(3.92 \pm 0.09\ _{\text{(stat)}} \pm 0.45\ _{\text{(sys)}}\right) \cdot 10^{-3}$$
$$|V_{cd}| = 0.230 \pm 0.011$$
$$|V_{cs}| = 0.98 \pm 0.01\ _{\text{(stat)}} \pm 0.1\ _{\text{(sys)}}$$
$$|V_{cb}| = \left(40.89 \pm 0.37\ _{\text{(stat)}} \pm 0.59\ _{\text{(sys)}}\right) \cdot 10^{-3}$$
$$|V_{tb}| = 1.0 \pm 0.099$$

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Inputs II

S	0.03 ± 0.09	0.867	
T		0.07 ± 0.08	
U			
$\mathcal{B}(W \to e\nu)$	0.1075 ± 0.0013	0.110	-0.195
$\mathcal{B}(W- > \mu\nu)$		0.1057 ± 0.0015	-0.132
$\mathcal{B}(W- > \tau\nu)$			0.1125 ± 0.0020

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