BR($\overline{B} \to X_s\gamma$) in Two Higgs Doublet Models

arXiv:1208.2788

Thomas Hermann[1], Mikołaj Misiak[2] and Matthias Steinhauser[1]

[1]Institut für Theoretische Teilchenphysik, Karlsruhe Institute of Technology
[2]Institute of Theoretical Physics, Warsaw University

GK Workshop Bad Liebenzell 2012

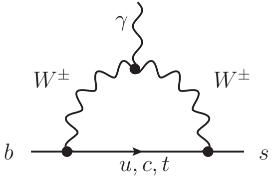
Karlsruher Institut für Technologie

SFB TR9

Outline

Introduction $\overline{B} \to X_s\gamma$

$$\Gamma\left(\overline{B} \to X_s\gamma\right)|_{E_\gamma > E_0} = \Gamma\left(b \to X_s^p\gamma\right)|_{E_\gamma > E_0} + \left(\begin{array}{c}\text{non-perturbative contributions} \\ \sim \pm 5\% \text{ [Benzke et al. 2010]}\end{array}\right)$$



$$\text{BR}(\overline{B} \to X_s\gamma) = \frac{\Gamma(\overline{B} \to X_s\gamma)}{\Gamma(\overline{B})}$$

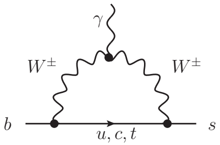
Exp. world average: measured at CLEO, BELLE and BABAR

$\text{BR}(\overline{B} \to X_s\gamma)|_{E_\gamma > 1.6\,\text{GeV}} = (3.37 \pm 0.23) \cdot 10^{-4}$ [Stone ICHEP 2012]

SM NNLO prediction:

$\text{BR}(\overline{B} \to X_s\gamma)|_{E_\gamma > 1.6\,\text{GeV}} = (3.15 \pm 0.23) \cdot 10^{-4}$ [Misiak et al. 2006]

Effective theory



different scales m_b and M_X
$M_X = M_W,\ M_t,\ M_{H^\pm}$

$\Rightarrow \quad \alpha \cdot \ln(M_X^2/m_b^2) \sim 1$
bad convergence of perturbation series

resummation of large logarithms

effective theory approach

integrating out heavy particles: $W^\pm$, top quark, $H^\pm$

$$\mathcal{L}_{full}$$
$$\Downarrow$$
$$\mathcal{L}_{eff} = \mathcal{L}_{\text{QCD x QED}}(u,d,s,c,b) + \frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{8} C_i Q_i$$

Effective theory

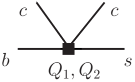
- Q_i dimension 5 and 6 operators

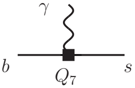
$$Q_1 = (\overline{s}_L \gamma_\mu T^a c_L)\,(\overline{c}_L \gamma^\mu T^a b_L)$$

$$\vdots$$

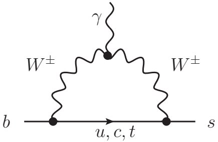
$$Q_7 = \frac{e}{16\pi^2} m_b\,(\overline{s}_L \sigma^{\mu\nu} b_R)\,F_{\mu\nu}$$

$$Q_8 = \frac{g}{16\pi^2} m_b\,(\overline{s}_L \sigma^{\mu\nu} T^a b_R)\,G^a_{\mu\nu}$$

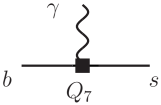




- $C_i(\mu)$ Wilson coefficients
  new coupling constants
  electroweak-scale physics in Wilson coefficients C_i



$\Rightarrow \quad C_7(\mu) \quad \times$



$+ \quad \ldots$

Calculations in the effective theory

1. Matching:
   Calculation of $C_i(\mu_0)$ at the matching scale
   $\mu_0 \sim M_W, M_t, \ldots$

Calculations in the effective theory

1. Matching:
   Calculation of $C_i(\mu_0)$ at the matching scale
   $\mu_0 \sim M_W, M_t, \ldots$
2. Running / Mixing:

$$C_i(\mu_0) \quad \Rightarrow \quad C_i(\mu_B), \qquad \mu_B \sim m_b$$

Renormalization group equation for Wilson coefficients

$$\mu \frac{d}{d\mu} C_i(\mu) = \gamma_{ji} \, C_j(\mu)$$

Resummation of large logarithms

Calculations in the effective theory

1. Matching:
   Calculation of $C_i(\mu_0)$ at the matching scale
   $\mu_0 \sim M_W, M_t, \ldots$
2. Running / Mixing:

$$C_i(\mu_0) \quad \Rightarrow \quad C_i(\mu_B), \qquad \mu_B \sim m_b$$

   Renormalization group equation for Wilson coefficients

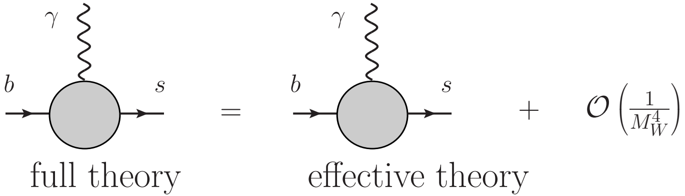
$$\mu \frac{d}{d\mu} C_i(\mu) = \gamma_{ji}\, C_j(\mu)$$

Resummation of large logarithms

3. On-shell matrix elements:
   Calculation of the on-shell matrix elements in the effective theory

Matching

Calculation of Wilson coefficients C_i at high scale μ_0



- Taylor expansion to second order in external momenta $\Rightarrow$ tadpole diagrams
- loop diagrams vanish in effective theory (massless tadpoles)

Two Higgs Doublet Models

- SM + additional Higgs doublet (two Higgs doublets)
- physical basis: h, H, A and $H^{\pm}$
- interaction between charged Higgs $H^{\pm}$ and quarks:

$$\mathcal{L} = (2\sqrt{2}G_F)^{1/2} \sum_{i,j=1}^{3} \overline{u}_i \left(A_u m_{u_i} V_{ij} P_L - A_d m_{d_j} V_{ij} P_R\right) d_j H^+ + h.c.$$

2HDM Type II (e.g. MSSM):

$$A_u = -\frac{1}{A_d} = \frac{1}{\tan\beta} \quad \text{with} \quad \tan\beta = \frac{<\phi_2^0>}{<\phi_1^0>}$$

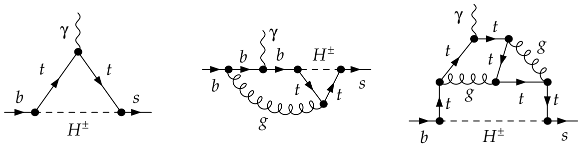
Wilson coefficients: $C_i = A_d\, A_u^{\star} \ldots + A_u\, A_u^{\star} \ldots$

1. Matching: 2HDM
2. Running/Mixing: same as in SM
3. On-shell matrix elements: same as in SM

Calculation of C_7 and C_8 to 3L

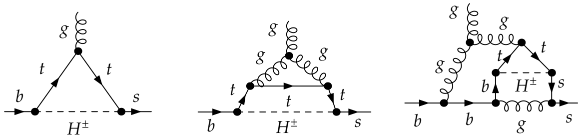
C_7: amputated 1PI Green function $b \to s\,\gamma$ ~ 350 diagrams for 3L

(a) (b) (c)



C_8: amputated 1PI Green function $b \to s\,g$ ~ 500 diagrams for 3L

(d) (e) (f)



Calculation of C_7 and C_8 to 3L

2 loops:

Wilson coefficients to 2 loops [Ciafaloni, Romanino and Strumia 1997] [Ciuchini, Degrassi, Gambino and Giudice 1997] [Borzumati and Greub 1998]

3 loops: [TH, Misiak, Steinhauser]

Three-loop vacuum integrals with two different mass scales (m_t and $M_{H^\pm}$), not all Masterintegrals are analytically known

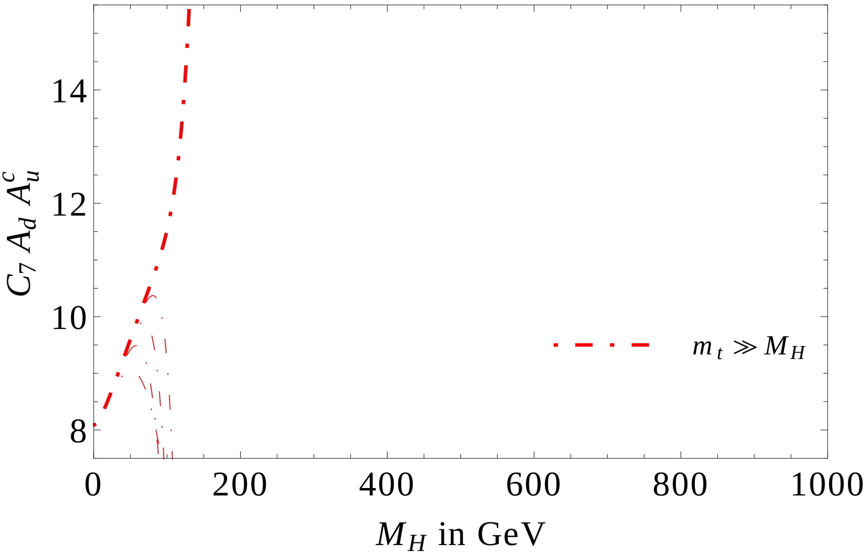
$\Rightarrow$ Expansion in three different mass regions

1. $M_{H^\pm} \gg m_t$: asymptotic expansion $(m_t/M_{H^\pm})^{10}$
2. $M_{H^\pm} \approx m_t$: ordinary Taylor expansion $\left(M^2_{H^\pm} - m_t^2\right)^{16}$
3. $M_{H^\pm} \ll m_t$: asymptotic expansion $(M_{H^\pm}/m_t)^{10}$

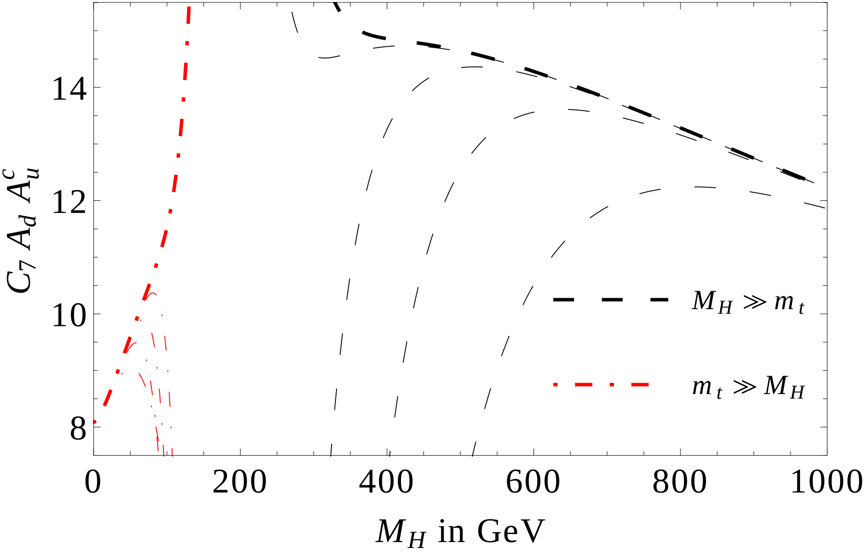
Asymptotic expansion: Q2E/EXP
[Harlander, Seidensticker, Steinhauser 1998 and Seidensticker 1999]

Three-loop massive tadpoles with one mass scale: MATAD [Steinhauser 2001]

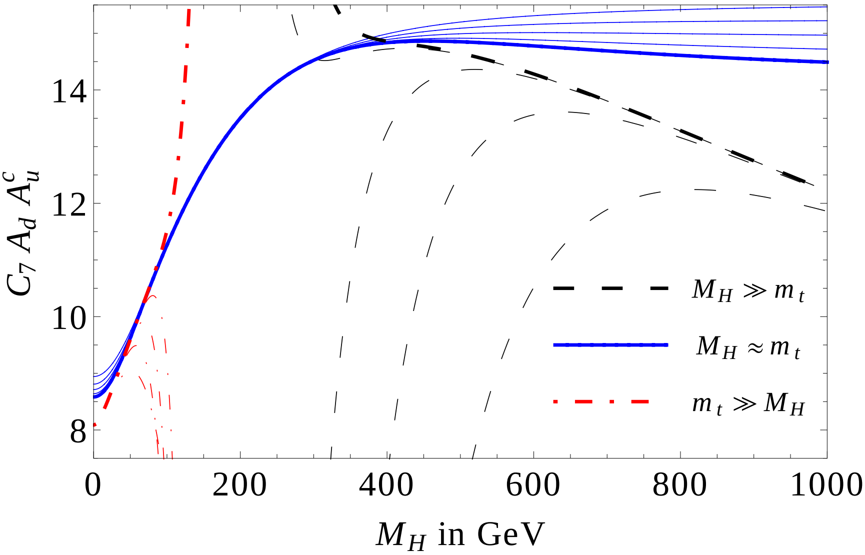
Results for C_7: $A_d A_u^\star$



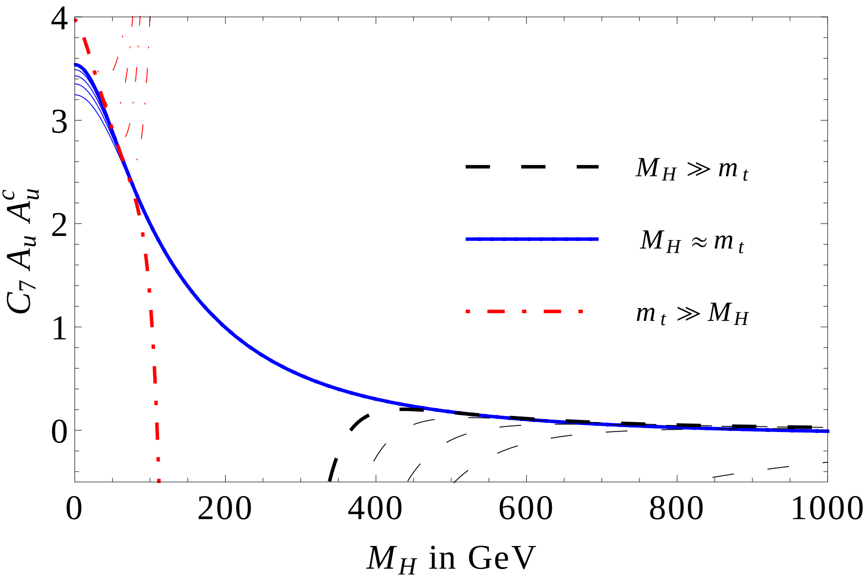
Results for C_7: $A_d A_u^\star$



Results for C_7: $A_d A_u^\star$

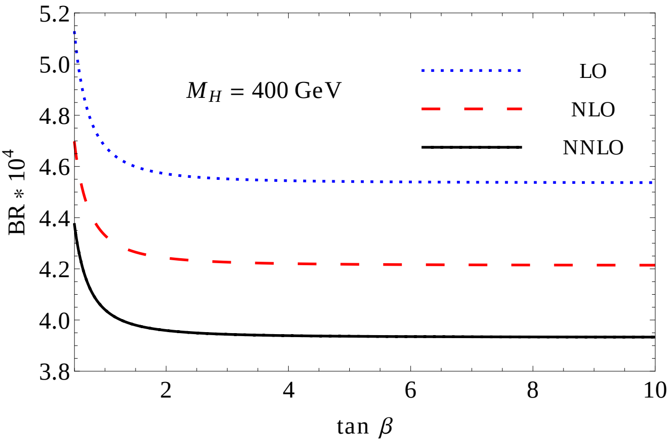


Results for C_7: $A_u A_u^\star$



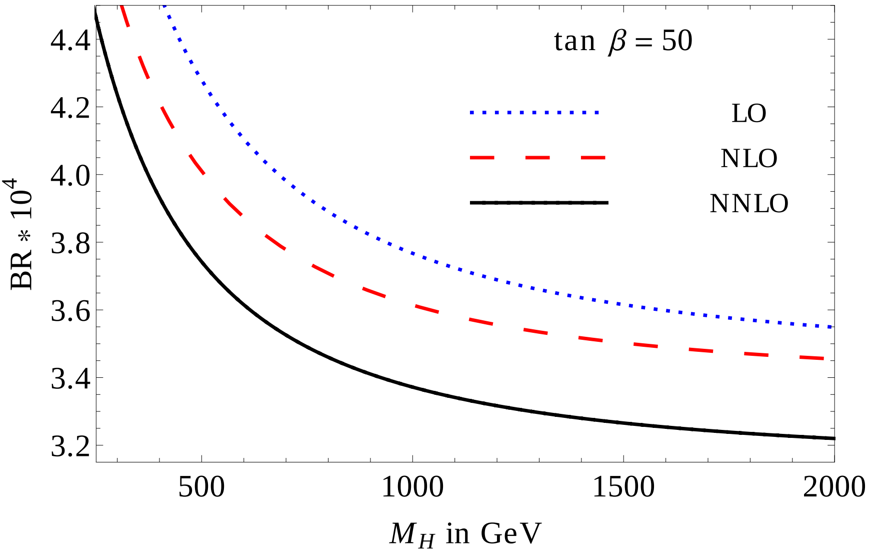
Branching ratio in 2HDM Type II

New: Wilson coefficients for 2HDM
Mixing and on-shell matrix elements [Misiak, Steinhauser 2006]

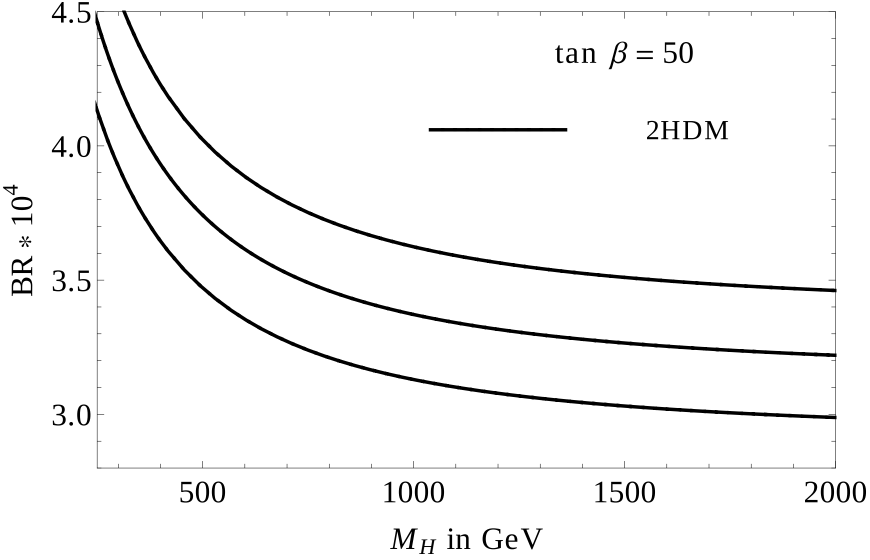


global exclusion limit for M_{H^+}: $\tan(\beta) = 50$ good choice

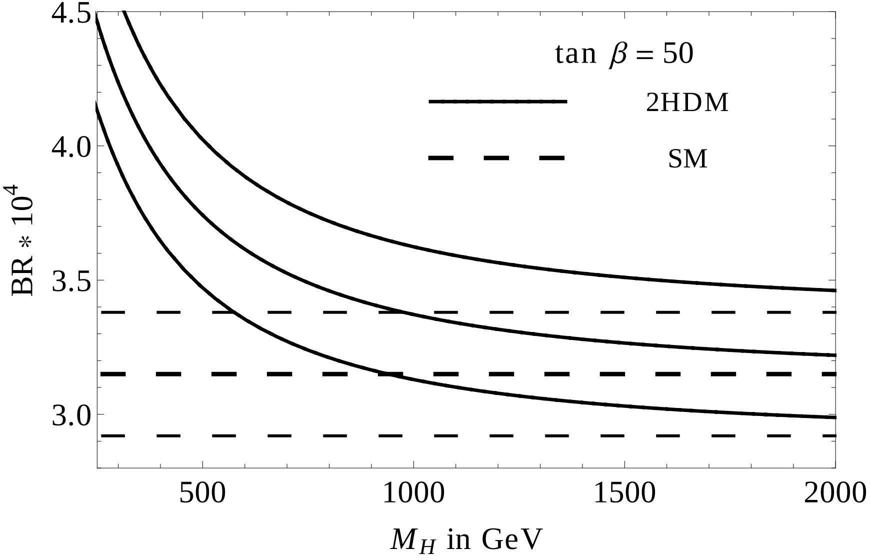
Branching ratio in 2HDM Type II



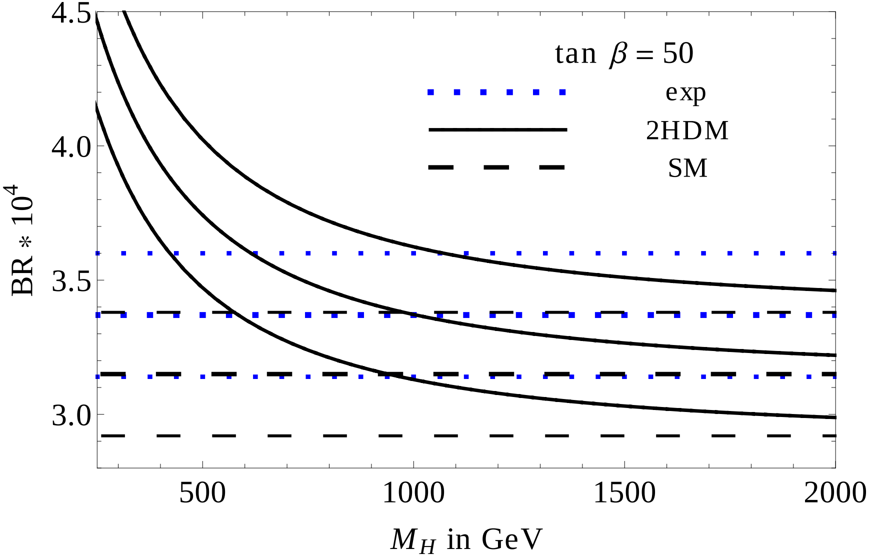
Uncertainty band in 2HDM Type II



Uncertainty band in 2HDM Type II



Uncertainty band in 2HDM Type II



Uncertainty band in 2HDM Type II

$\tan(\beta) = 50$ and $M_{H^\pm} = 400$ GeV

1. **approx. of $\langle s\gamma|Q_2|b\rangle$:** $\pm 3\%$ [Misiak, Steinhauser 2006]
2. **scale uncertainty:** $+4.2\%, -3.7\%$
   - $\mu_0 = 80\,\text{GeV}, 160\,\text{GeV}, 800\,\text{GeV} \Rightarrow -0.4\%$
   - $\mu_b = 1.25\,\text{GeV}, 2.5\,\text{GeV}, 5\,\text{GeV} \Rightarrow +3.0\%, -3.5\%$
   - $\mu_c = 1.224\,\text{GeV}, 1.5\,\text{GeV}, 4.68\,\text{GeV} \Rightarrow +2.9\%, -1.2\%$
3. **parameter uncertainty:** $\pm 2.3\%$
   - BR$(B \to X_c e\overline{\nu})_{\text{exp}}$: $\pm 1.6\%$
   - semileptonic phase space ratio C and $m_c(m_c)$: $\pm 1.1\%$ correlated [Hoang, Manohar 2006]
   - $\alpha_s(M_Z)$: $\pm 0.9\%$
   - ...
4. **non-perturbative contributions:** $\pm 5\%$

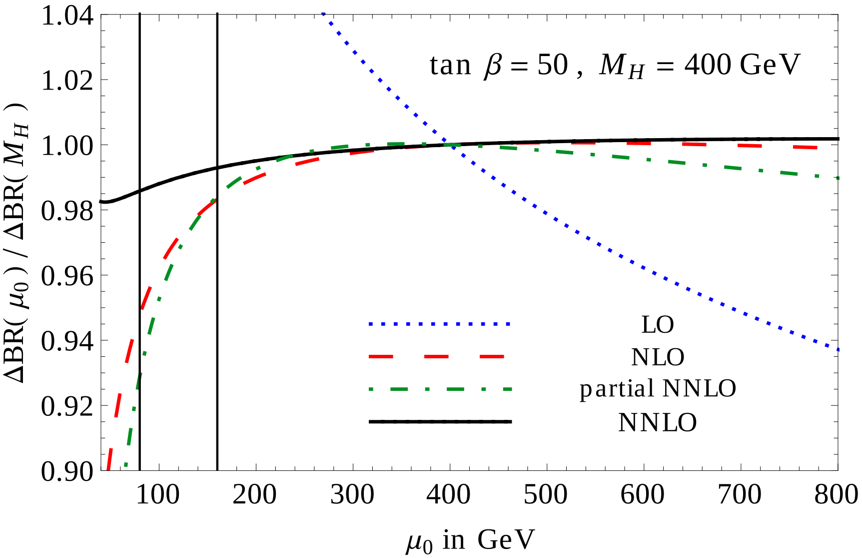
uncertainties added in quadrature: $\Rightarrow$ $+7.5\%, -7.3\%$

Lower bound

$$M_{H^+} \geq 380\,\text{GeV with } 95\%\ \text{CL}$$

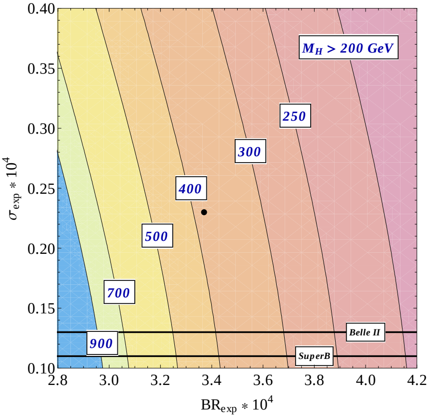
Matching scale dependence in 2HDM Type II

Partial NNLO in 2HDM Type II without $C_{7,8}^{(2)\text{eff, 2HDM}}(\mu_0)$ [Misiak et al. 2006]

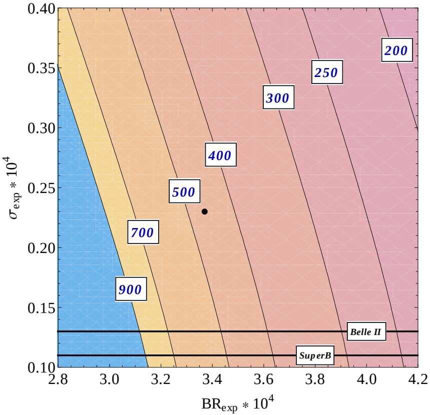


Exclusion limit for M_{H^+} in 2HDM Type II

current state



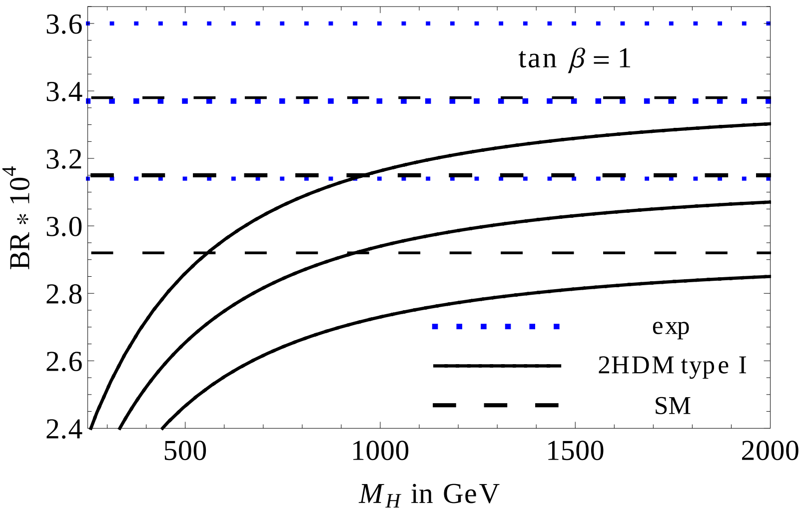
reduced theory error (factor 1/2)



- $\text{BR} = (3.37 \pm 0.23) \cdot 10^{-4}$ [Stone ICHEP 2012]

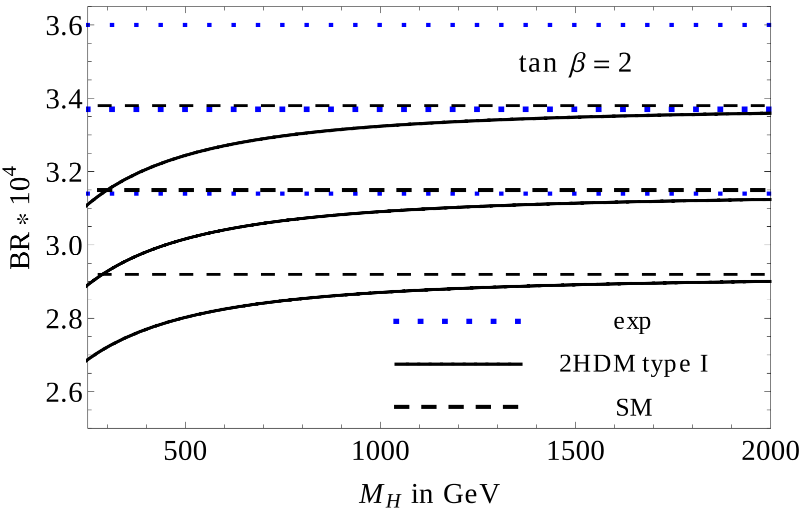
Belle II and SuperB [Meadows et al. 2011]

Branching ratio in 2HDM Type I



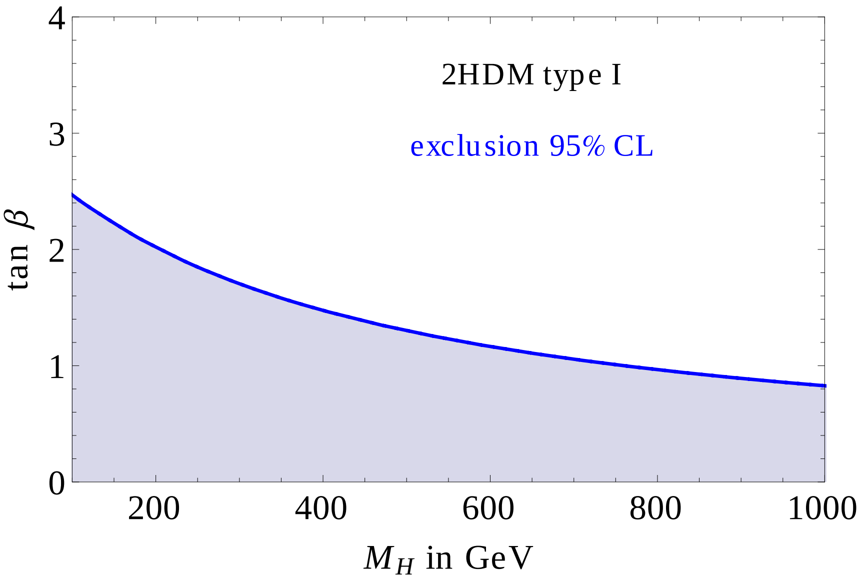
2HDM contributions are $\tan\beta$ suppressed in Type I

Branching ratio in 2HDM Type I



2HDM contributions are $\tan\beta$ suppressed in Type I

Exclusion in 2HDM Type I



Conclusion

Conclusion:

- C_7 and C_8 to three-loop order in Two Higgs Doublet Models
- consistent NNLO estimation in 2HDMs
- reduction of matching scale dependence
- lower bound in 2HDM Type II:

$$M_{H^+} \geq 380\,\text{GeV with } 95\% \text{ CL}$$

Outlook:

Most 3L vacuum integrals with two masses are known analytically, the rest numerically

[J. Grigo diploma thesis 2012]
[Grigo, Hoff, Marquard, Steinhauser 2012]

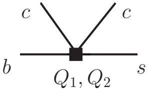
$\Rightarrow$ check for different expansions

Effective theory

current-current operators:

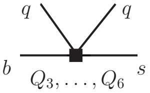
[Chetyrkin, Misiak, Muenz 1998]

$$Q_1 = (\bar{s}_L \gamma_\mu T^a c_L)\,(\bar{c}_L \gamma^\mu T^a b_L)$$
$$Q_2 = (\bar{s}_L \gamma_\mu c_L)\,(\bar{c}_L \gamma^\mu b_L)$$



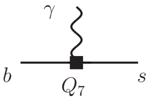
QCD penguin operators:

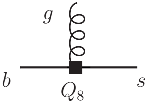
$$Q_3 = (\bar{s}_L \gamma_\mu b_L) \sum (\bar{q}\,\gamma^\mu q)$$
$$Q_4 = (\bar{s}_L \gamma_\mu T^a b_L) \sum (\bar{q}\,\gamma^\mu T^a q)$$
$$Q_5 = (\bar{s}_L \gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} b_L) \sum (\bar{q}\,\gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} q)$$
$$Q_6 = (\bar{s}_L \gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} T^a b_L) \sum (\bar{q}\,\gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} T^a q)$$



dipole operators:

$$Q_7 = \frac{e}{16\pi^2} m_b\,(\bar{s}_L \sigma^{\mu\nu} b_R)\,F_{\mu\nu}$$
$$Q_8 = \frac{g}{16\pi^2} m_b\,(\bar{s}_L \sigma^{\mu\nu} T^a b_R)\,G^a_{\mu\nu}$$





BR($\overline{B} \to X_s\gamma$) in the SM

$$\Gamma\left(b \to X_s^p \gamma\right)|_{E_\gamma > E_0} = \frac{G_F^2 m_b^5 \alpha_{em}}{32\pi^4} \left|V_{ts}^\star V_{tb}\right|^2 \sum_{i,j=1}^{8} C_i(\mu_B) C_j(\mu_B) G_{ij}(E_0, \mu_B)$$

Matrix elements G_{ij}:

LO: $G_{ij} = \delta_{i7}\delta_{j7} +$ (small tree-level)

NLO: G_{ij} complete [Ali, Buras, Czarnecki, Greub, Hurth, Misiak, Pott, Urban, Wyler 1991-2002]

NNLO:

- G_{77} fully known [Blokland, Czarnecki, Misiak, Slusarczyk, Tkachov 2005] [Melnikov, Mitov 2005] [Asatrian et al 2006-2007]
- G_{78} fully known [Asatrian et al. 2010]
- G_{ij} for $i, j \in (1, 2, 7, 8)$ partly known (BLM approx. and more) [Bieri, Greub, Steinhauser 2003] [Ligeti, Luke, Manohar, Wise 1999] [Asatrian, Boughezal, Czakon, Ewerth, Ferroglia, Gabrielyan, Greub, Haisch, Misiak, Poradzinski, Schutzmeier 2007-2011]
- beyond BLM: $m_c \gg m_b/2$ limit, interpolation to physical value m_c
  $\Rightarrow$ uncertainty of 3% to BR [Misiak, Steinhauser 2006]

Electroweak contributions NLO: [Gambino, Haisch 2001]

Non-perturbative power corrections:

[Bigi et al. 1992]
[Falk, Luke, Savage 1993]
[Voloshin 1996]
[Buchalla, Isidori, Rey 1997]
[Bauer 1997]
[Gambino, Ewerth, Nandi 2009]
[Benzke et al. 2010]

Normalization:

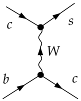
$$\mathsf{BR}(\overline{B} \to X_s \gamma)|_{E_\gamma > E_0} = \frac{\mathsf{BR}(\overline{B} \to X_c e \overline{\nu})_{\mathsf{exp}}}{C_{\mathsf{fit}}} \left(\frac{\Gamma\left(\overline{B} \to X_s \gamma\right)|_{E_\gamma > E_0}}{|V_{cb}/V_{ub}|^2\, \Gamma(\overline{B} \to X_u e \overline{\nu})} \right)_{\mathsf{th}}$$

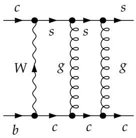
semileptonic phase space ratio

$$C = \left| \frac{V_{ub}}{V_{cb}} \right|^2 \frac{\Gamma(\overline{B} \to X_c e \overline{\nu})}{\Gamma(\overline{B} \to X_u e \overline{\nu})}$$

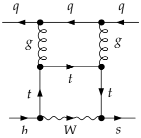
Wilson coefficients in the SM

2 loop Wilson coefficients: [Bobeth, Misiak, Urban 2000]

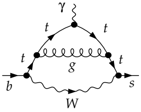


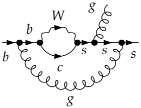


$\Rightarrow C_1^c, C_2^c$



$\Rightarrow C_3^t, \ldots, C_6^t$



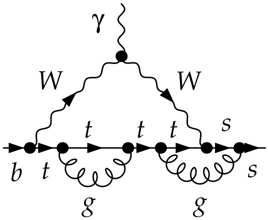


$\Rightarrow C_7^t, C_8^c$

subtleties: evanescent operators, non-physical operators off-shell

Wilson coefficients in the SM

3 loop Wilson coefficients: [Misiak, Steinhauser 2004]



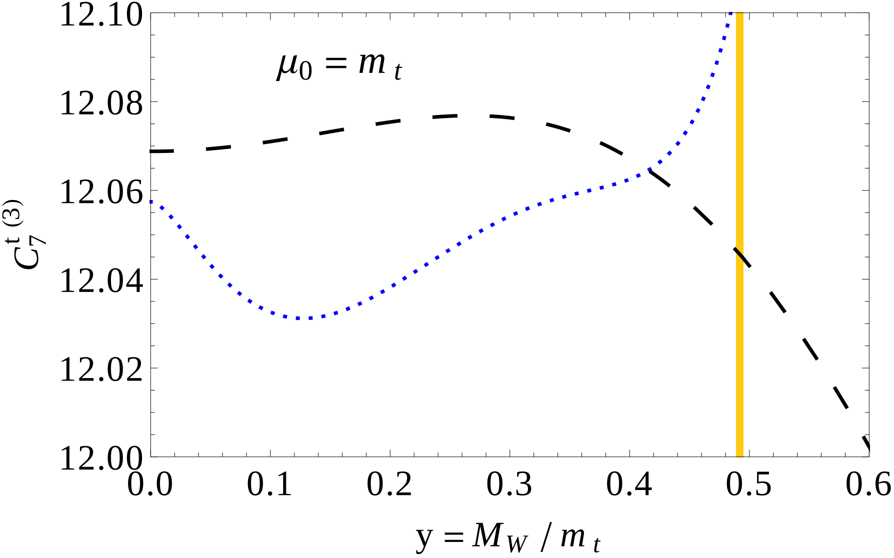
$\Rightarrow C_7^t \; (C_8^t)$

1. $m_t \gg M_W$: asymptotic expansion $(M_W/m_t)^8$
2. $m_t \approx M_W$: ordinary Taylor expansion $\left(M_W^2 - m_t^2\right)^8$

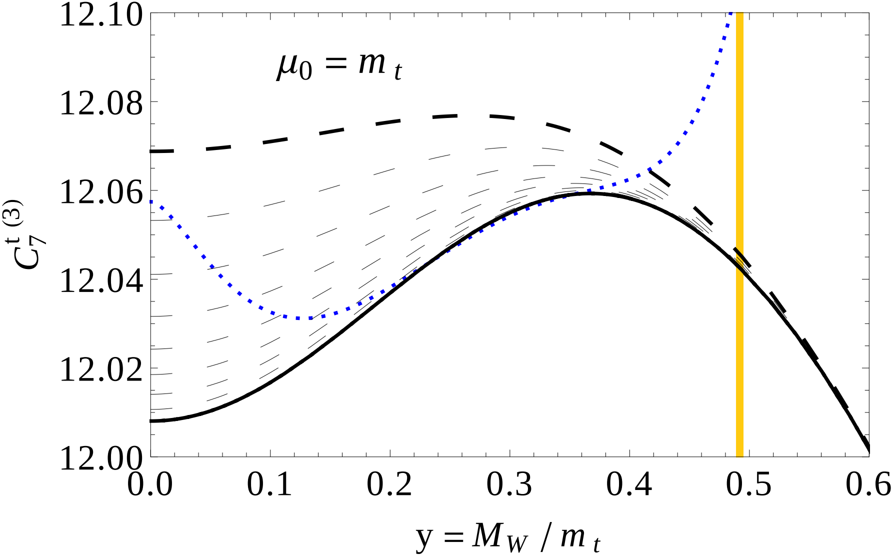
improved calculation: [TH, Misiak, Steinhauser]

$m_t \approx M_W$: ordinary Taylor expansion $\left(M_W^2 - m_t^2\right)^{16}$

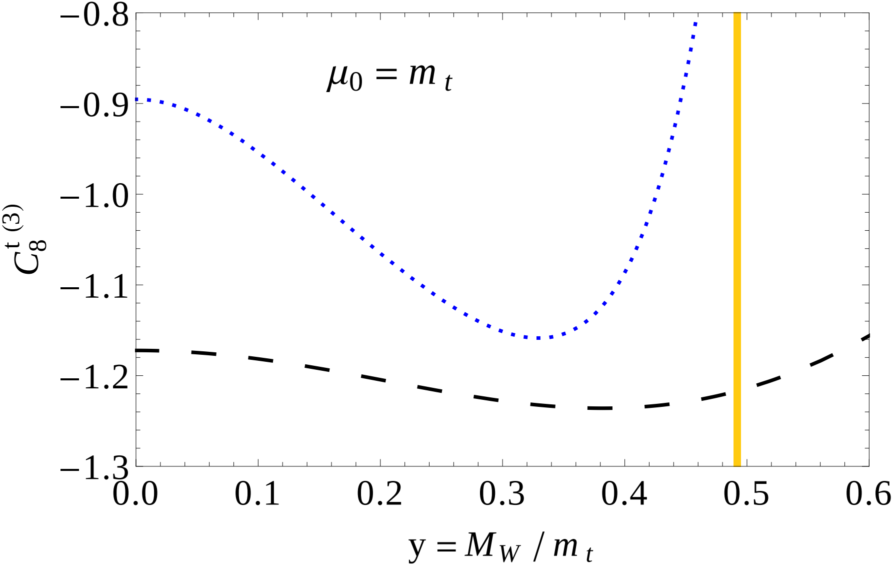
$C_7(\mu_0 = m_t)$ 3L in the SM, 2004



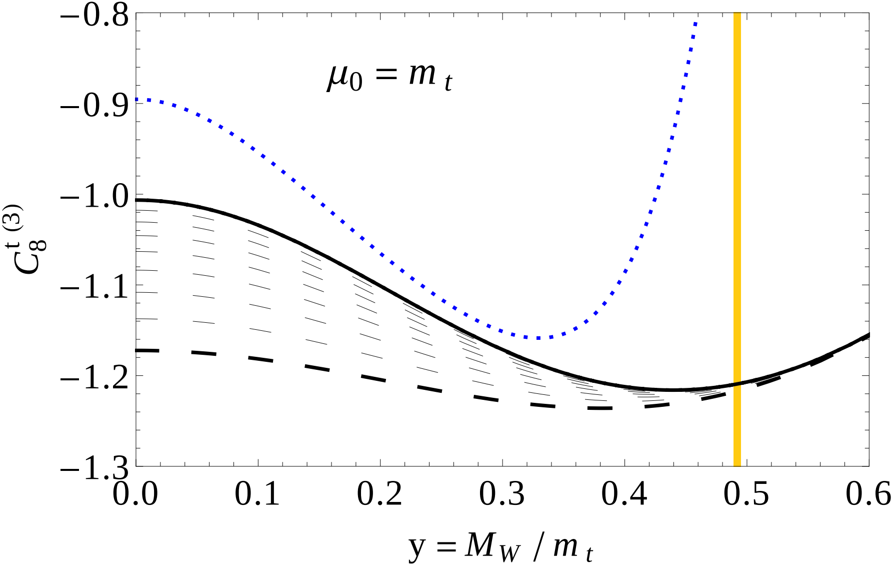
$C_7(\mu_0 = m_t)$ 3L in the SM, 2012



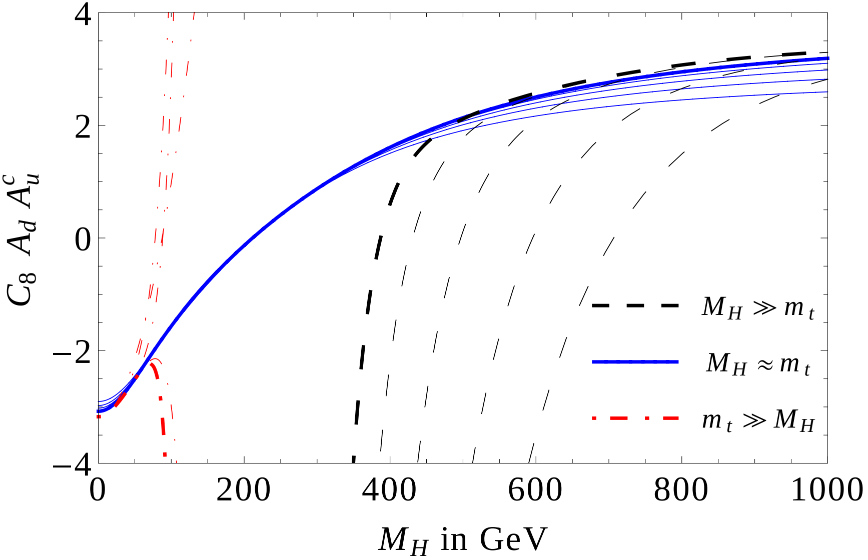
$C_8(\mu_0 = m_t)$ 3L in the SM, 2004



$C_8(\mu_0 = m_t)$ 3L in the SM, 2012



Results for C_8: $A_d A_u^\star$



Results for C_8: $A_u A_u^\star$