The exotic world of quantum matter:
Spontaneous symmetry breaking and beyond.

Talk at GK 1694 meeting Bad Liebenzell

08.10.2012

Peter Wölfle

Institut für Theorie der Kondensierten Materie (TKM)
Institut für Nanotechnologie (INT)
Karlsruhe Institute of Technology

Karlsruhe Institute of Technology

Quantum matter: a definition

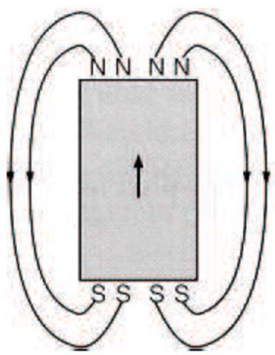
What is "Quantum Matter"?

Solid or liquid matter ("Condensed Matter")
showing quantum properties on the macroscopic scale

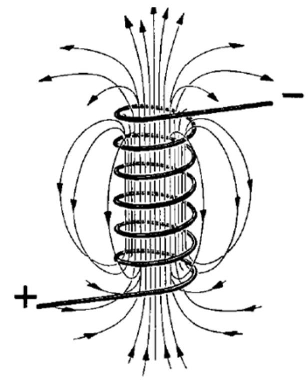
Magnetic Quantum Matter

Earliest example: ferromagnetic Iron, Nickel, Cobalt

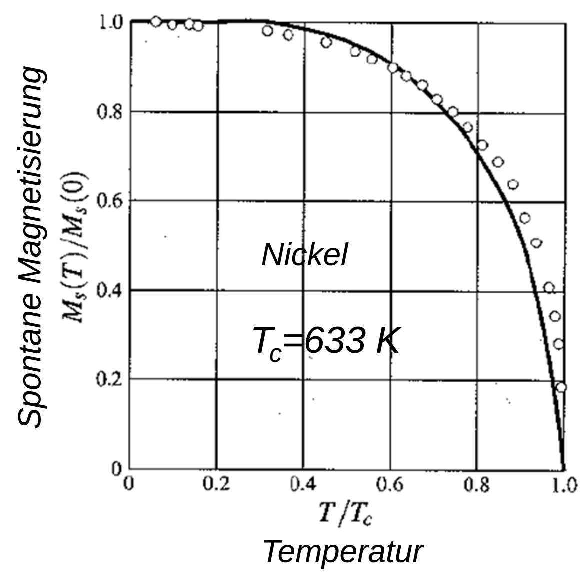
Magnetic field of a Permanent magnet



Magnetic field of an electromagnet



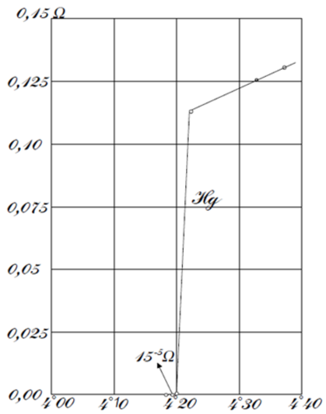
Permanent Ring Currents?
Not understandable within classical physics!



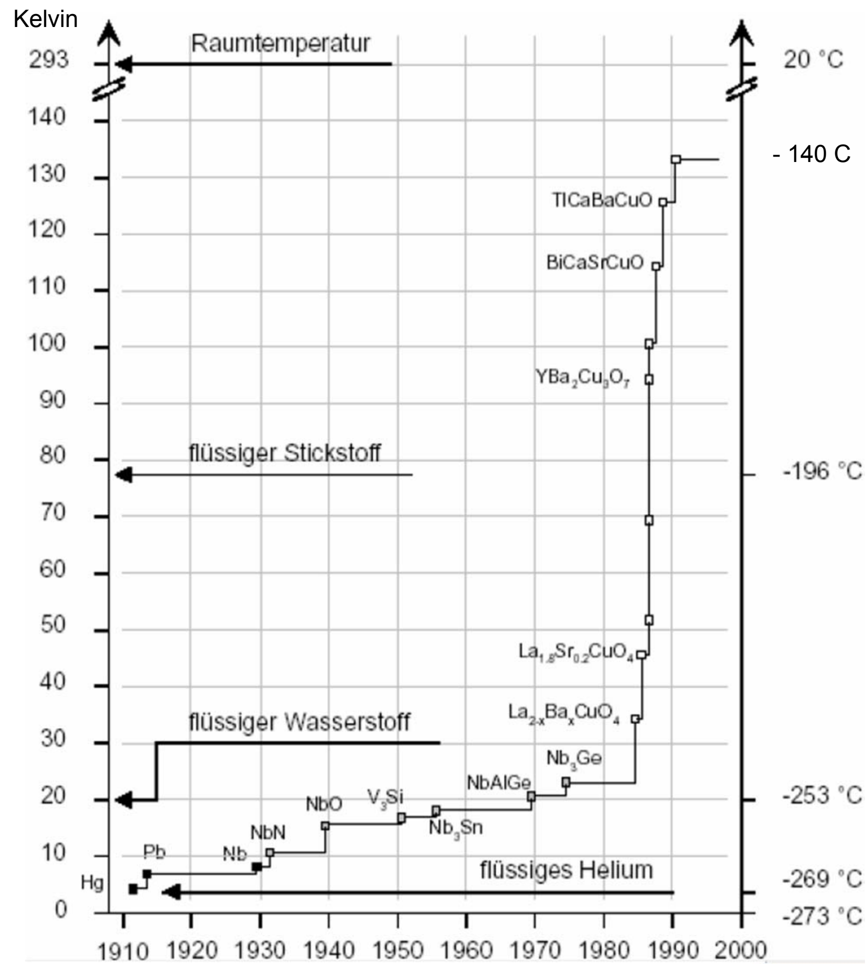
Phase transition Ferromagnet → Paramagnet

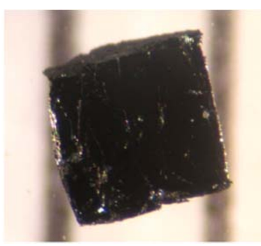
Superconductivity of Metals

Discovery of superconductivity
by Heike Kamerlingh Onnes 1911 (Nobel prize 1913)

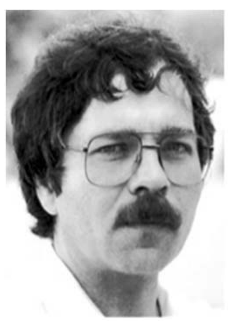


„High temperature"-Superconductivity:
Bednorz and Müller 1986 (Nobel prize 1987)





$Bi_2Sr_2Ca_2Cu_3O_{10}$



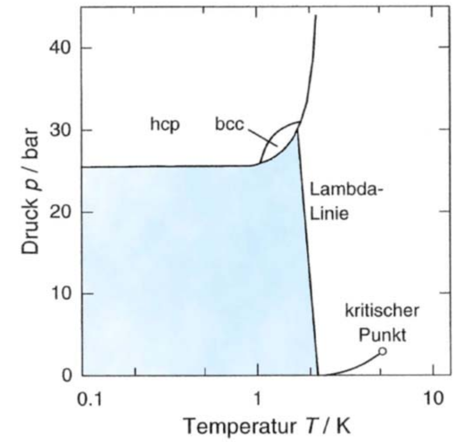
J. Georg Bednorz

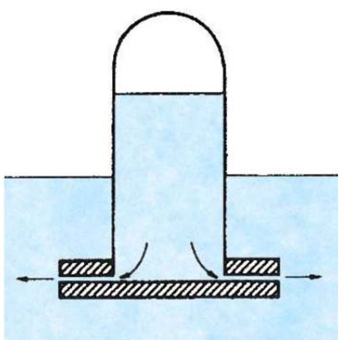
K. Alexander Müller

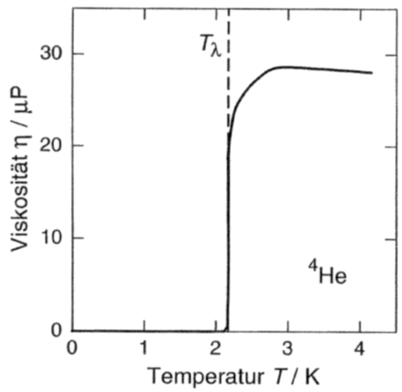
Superfluidity of Quantum Liquids: Helium 4

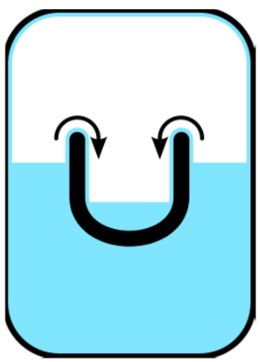
Under normal pressure Helium remains a liquid down to absolute zero

At temperatures T < 2.18 K Helium 4 becomes superfluid









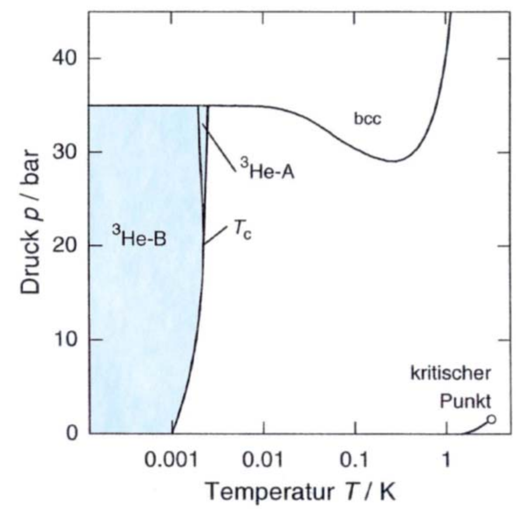
Pjotr Kapitza 1937 (Nobel prize 1978)

Superfluidity of Helium 3

Two stable Helium Isotopes, ^4He und ^3He (obtained through radioactive decay of Tritium)
The Isotopes differ only by their nuclear spin: $^4He=(2p+2n)$, Spin=0 ; $^3He=(2p+1n)$, Spin=1/2

Although the ^4He and ^3He atoms have identical chemical properties (electron shells), the two liquids behave entirely differently at temperatures < 3 K!

Superfluid Phases of ^3He appear at T < 2.6 mK:
D. Lee, D. Osheroff, R. Richardson 1971 (Nobel prize 1996)



Anisotropic, magnetic superfluid at temperatures of only 1/1000 of the transition temperature of ^4He

Theory: A. J. Leggett (Nobel prize 2003)

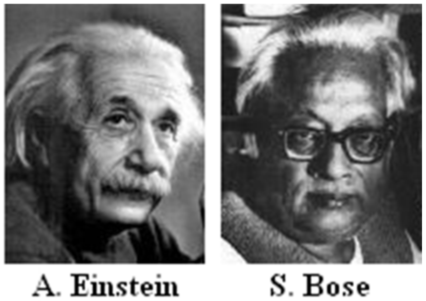
Karlsruhe Institute of Technology

Superfluidity of ultracold atomic gases

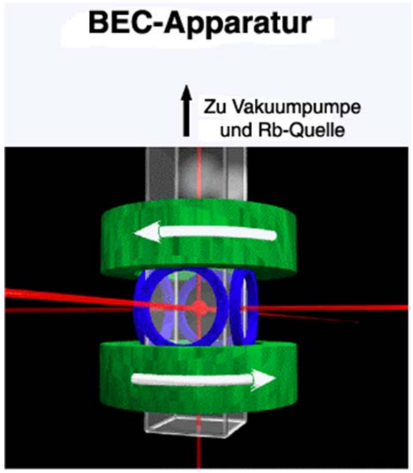
Bose-Einstein-condensate (BEC)

predicted 1925

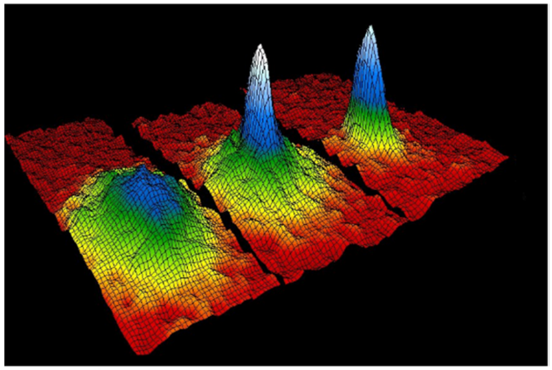
discovered 1995



Nobelpreisträger: Eric A. Cornell, Wolfgang Ketterle und Carl E. Wieman.

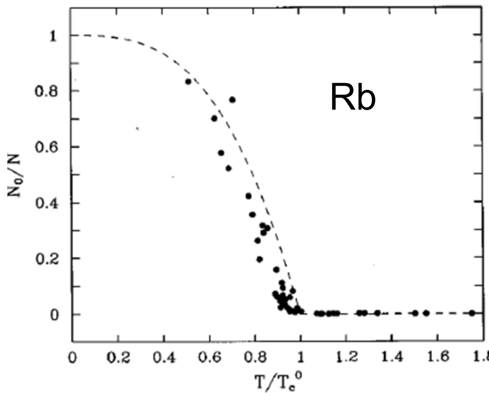


velocity distribution of atoms



Bose-Einstein-Kondensation bei 400, 200 und 50 Nanokelvin

number of atoms in condensate



T_c = 0,3 μK

Theory of Quantum Matter

Theoretical framework used to describe "Quantum matter"

1. Non-relativistic Quantum Mechanics of the constituents of condensed matter: electrons, nuclei, atoms

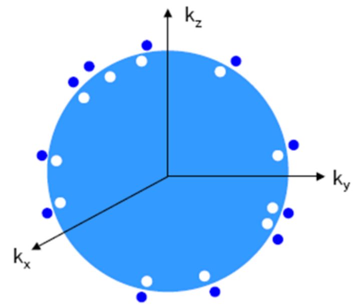
2. Quantum Statistics of Many-Particle Systems: Fermions, Bosons, Anyons

3. Collective Behavior: spontaneous ordering, excitations

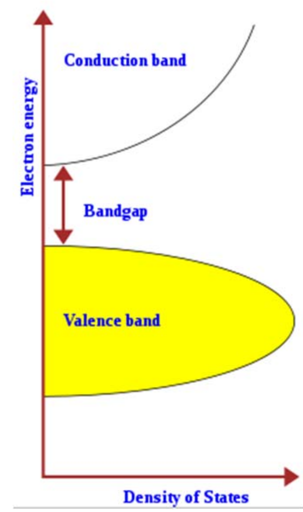
Quantum theory of electrons in solids

- ***Arnold Sommerfeld (1927) :***
  Electrons in Metals modeled as system of identical quantum particles with negligible interaction (Fermigas)

Ground state: Fermi sphere
in momentum space ($k<k_{F}$);
Weak excitations near the Fermi surface
($E_k = \hbar^2 k^2/2m - E_F \ll E_F$)



- ***Felix Bloch (1928) ; Born,Oppenheimer (1927):***
  Band structure of the energy spectrum of electrons in crystals
  Interaction with lattice vibrations



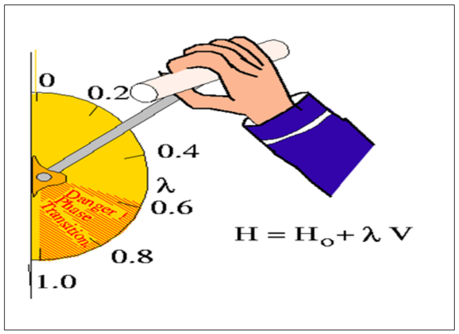
→ **Single particle theory of electrons in solids**

Explains many properties of normal metals qualitatively
(modern quantitative formulation by Density Functional Theory (DFT))

Problem: Coulomb interaction between electrons unimportant?

Karlsruhe Institute of Technology

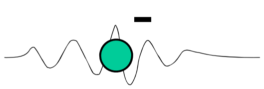
Landau's Fermi liquid theory



L.D. Landau (1957)
Nobel prize 1962

free electron

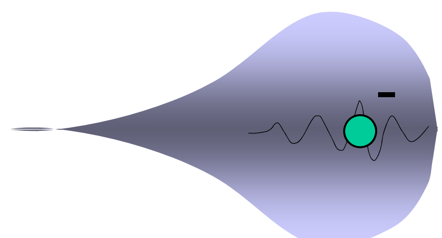
$$\left| e^{-} \right\rangle$$



interaction

→

adiabatic



$$\left| qp^{-} \right\rangle$$

"quasi particle"

Effect of interaction absorbed by a handful parameters
(effective mass, Landau parameters)

Karlsruhe Institute of Technology

Concepts of Quantum Matter I: quasi-particles

Description of the weakly excited states of a system of (strongly) interacting particles by mapping onto an effective model of

nearly free quasi-particles

Fermions (Spin ½) : Landau quasi-particles, Bogoliubov quasi-particles…

and/or

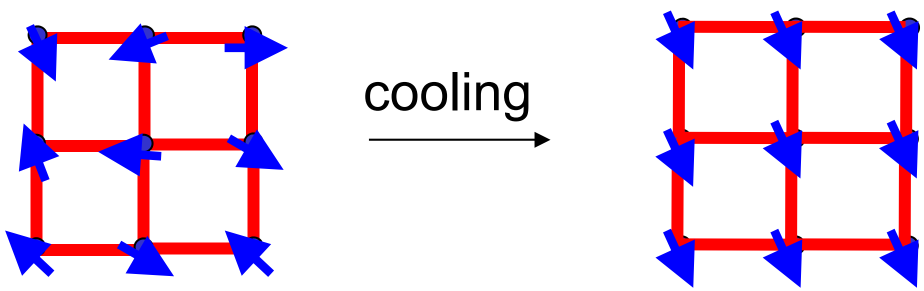
Bosons (Spin 0,1) : phonons (sound or shear waves)
plasmons (charge oscillations)
excitons (bound particle-hole pairs)
magnons (spin waves)
orbitons (orbital waves)

Concepts of Quantum Matter II: spontaneous symmetry breaking

- Interacting electron systems subject to cooling may develop a long-range ordered state below a critical temperature T_c

The symmetry of the ordered state is lower than that of the disordered state: spontaneous symmetry breaking

Example Ferromagnet: Orientation of magnetic moments



Emergence of preferred direction is breaking rotation symmetry in spin space

Concepts of quantum matter III: new quasiparticles as a consequence of spontaneous symmetry breaking

- **Existence of order parameter field, e.g. local magnetization M(r,t) of a ferromagnet**

- **"Elasticity" of order parameter field allows for oscillations/wave excitations:**
  - **"acoustical" : Goldstone modes of dispersion $\omega = ck$ or similar**
    (Spin waves; acoustical transverse phonons, Anderson-Bogoliubov mode, ..)
  - **"optical" : massive modes ω = const. , $k \to 0$**
    (optical phonons, Cooper pair oscillations, ..)

  → new quasi particles: bosons

- **Defects in order parameter field**
  (Domain walls of the magnetization, vortices in a superconductor or superfluid, ..)

  → new topological excitations

- **Gaps in the fermionic spectrum**
  (Bogoliubov quasiparticles in a superconductor, qps in a metallic ferromagnet, ..)

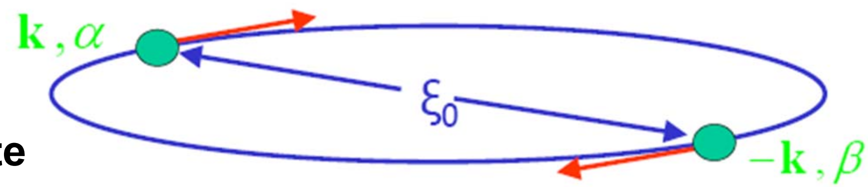
  → new fermionic qps: particle number not conserved

Theory of superconductivity I

BCS-Theory of electrons in a superconductor:
Electrons are bound into Cooper pairs (Quasi-Bosons; extension >> particle distance !) and form a quantum coherent condensate.
J. Bardeen, L. Cooper, R. Schrieffer (1957); Nobel prize *1971*

Conventional superconductors:

Orbital angular mom. L=0
Spin angular mom. S=0 → **unique state**

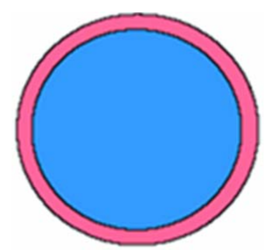


$A_{i\mu}$ **order parameter matrix**

Unconventional superconductors:

Orbital angular mom. L ≠ 0
And/or Spin S ≠ 0 → **(2L+1) x (2S+1) sub states**

Energy gap Δ in spectrum
of fermionic excitations $E_k = \sqrt{\xi_k^2 + \Delta^2}$
“Bogoliubov qp”



Superconductivity of Fermi systems is, similar to superfluidity of Bose systems a consequence of the quantum-mechanical entanglement of the “Bosons” (the Cooper pairs) in the condensate, encoded in the emergence of a “macroscopic quantum phase” $\Psi = |\Psi| e^{i\phi}$

”Spontaneous breaking of U(1) gauge symmetry”

Karlsruhe Institute of Technology

Higgs mechanism in superconductors

Excitations of the order parameter:

Phase mode (Goldstone)

gapless for neutral system (Anderson-Bogoliubov) $\omega = vq$

gapped for charged system by coupling to longitudinal el.magn. field $\omega = \omega_{plasma}$

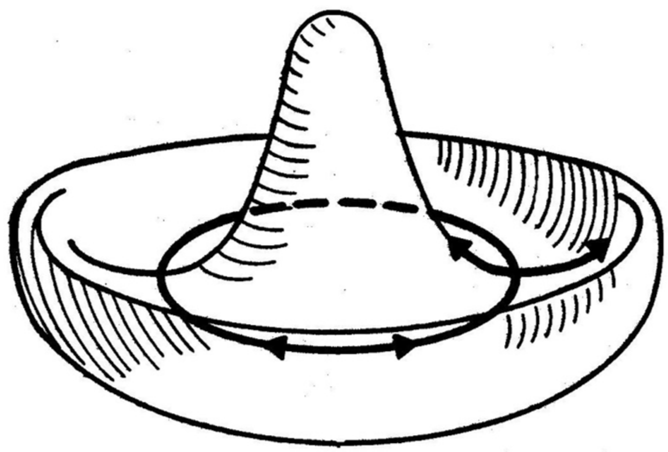
Amplitude mode (Higgs particle)

gapful for neutral and charged system; threshold at $\omega = 2\Delta$

no resonance

Under special conditions
Higgs particle well defined if

$$\omega = \omega_{Higgs} < 2\Delta$$



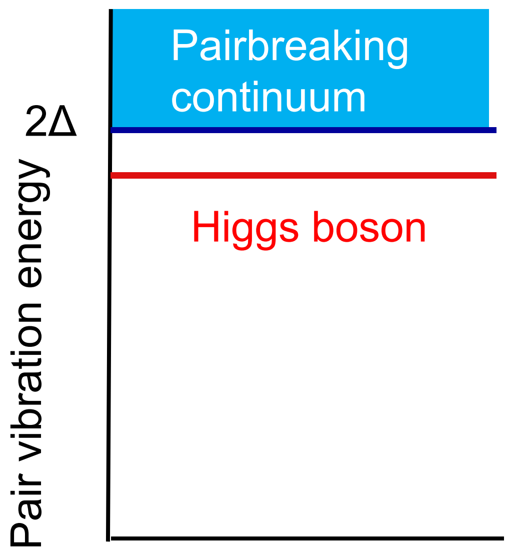
Higgs mechanism: transverse el.mag. field modes gapped $\omega \propto |\Psi| \propto 1/\lambda$
(magnetic penetration depth λ)

PW Anderson, 1958

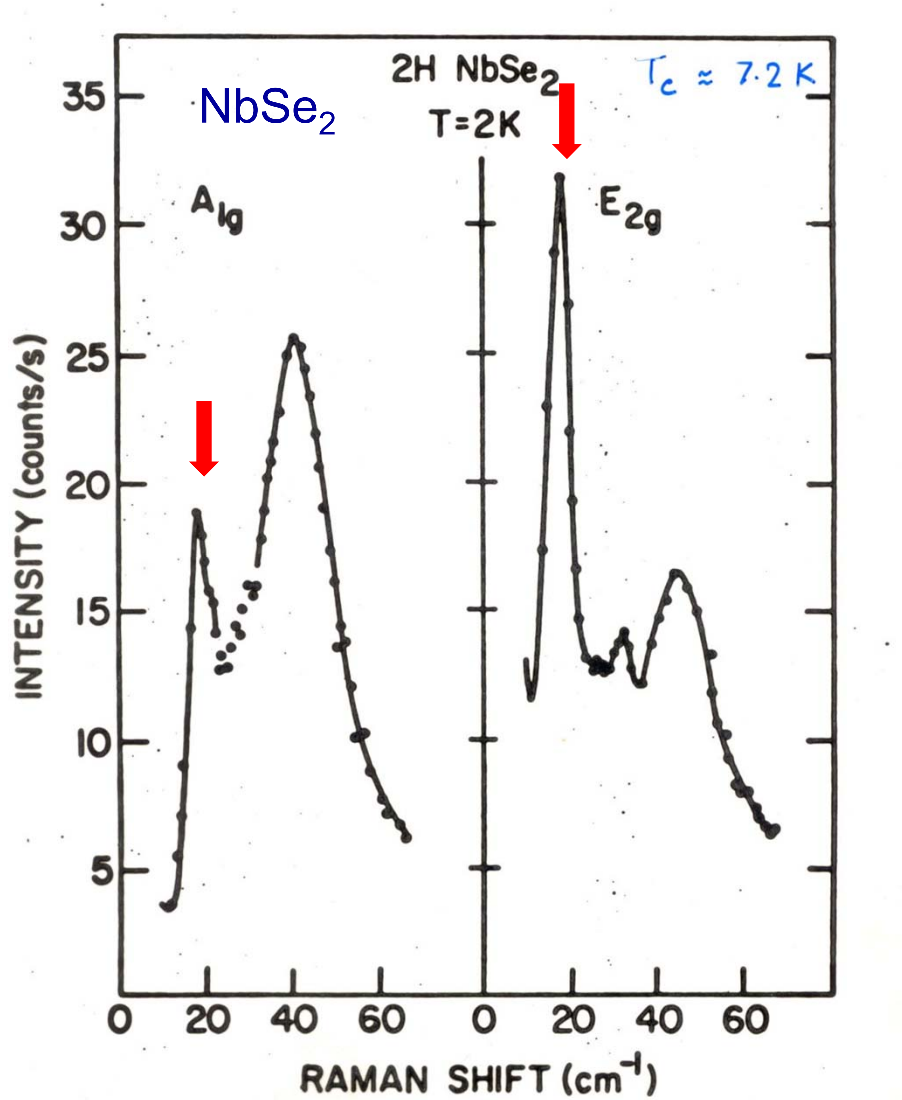
Karlsruhe Institute of Technology

Higgs boson in the Raman spectrum of $NbSe_2$

Coupling of OP to optical phonons
in Charge Density Wave material pulls
Higgs boson energy down
into energy gap



Th: P. B.Littlewood and C. M. Varma, 1982



Exp: R. Sooryakumar and M. V. Klein, 1980

Karlsruhe Institute of Technology

p-wave pairing states in liquid Helium 3

Orbital angular momentum L=1
Spin triplet state *(Pauli)* S=1

→ **3x3 substates** $\mathbf{A}_{j\mu}$ **order parameter**

$SO(3)_L \times SO(3)_S \times U(1)$

Pseudo-isotropic state

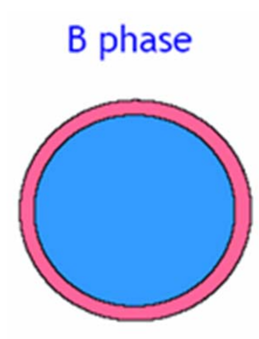
(Balian and Werthamer, 1963)

$$\mathbf{A}_{j\mu} = \Delta\, \mathbf{R}_{j\mu}(\mathbf{n}, \theta) e^{i\phi}$$

spin-orbit rotation matrix R,
broken symmetry with respect to
relative spin-orbit rotations

isotropic energy gap Δ

anisotropic dynamics

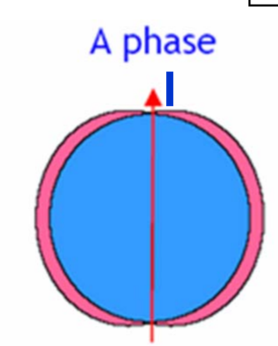


Axial state

(Anderson and Morel, 1961)

$$\mathbf{A}_{j\mu} = \Delta_0\, (\mathbf{n}_j + i\mathbf{m}_j)\mathbf{d}_\mu$$

Preferred directions:
m, n, l=mxn, in orbital space
d perp. **S,** in spin space



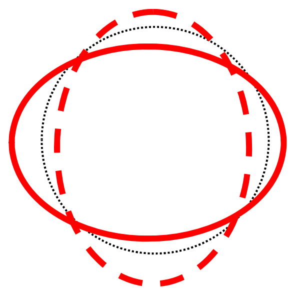
anisotropic energy gap

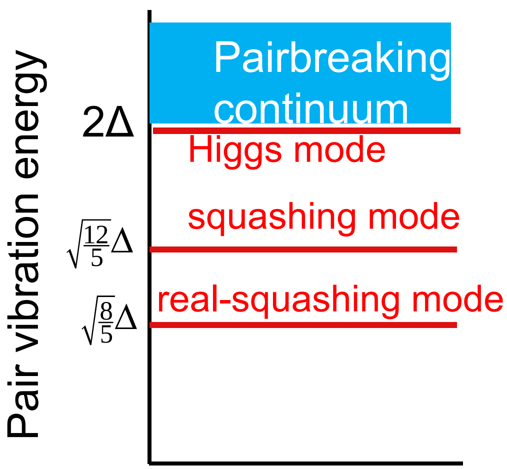
Vollhardt, Wölfle, "The superfluid phases of Helium 3"

KIT Karlsruhe Institute of Technology

Collective modes and ultrasound in the B-phase

Two “squashing” modes,
each five-fold degenerate:
quadrupolar oscillations
of the isotropic gap

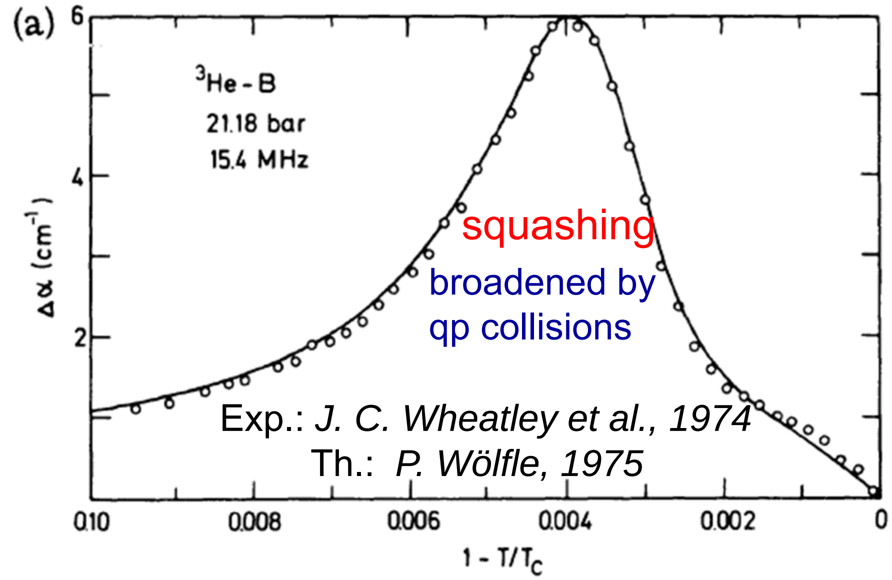


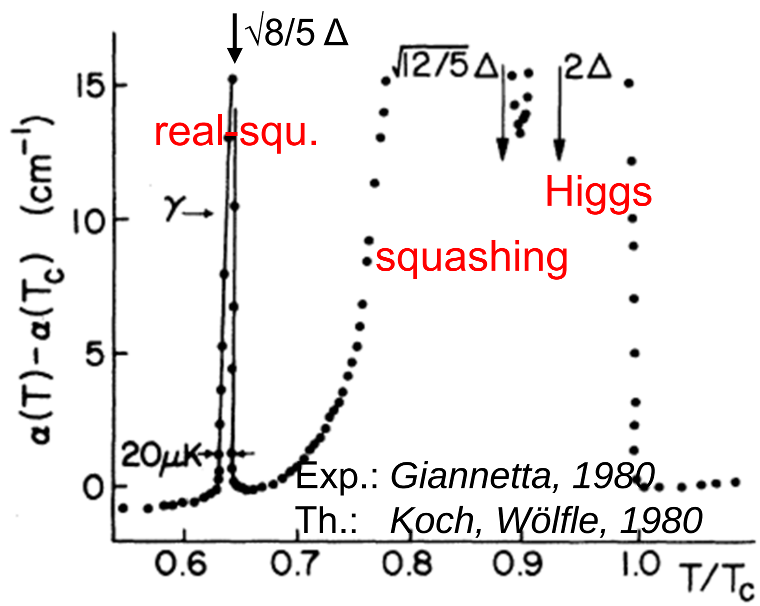


Note supersymmetry relation:

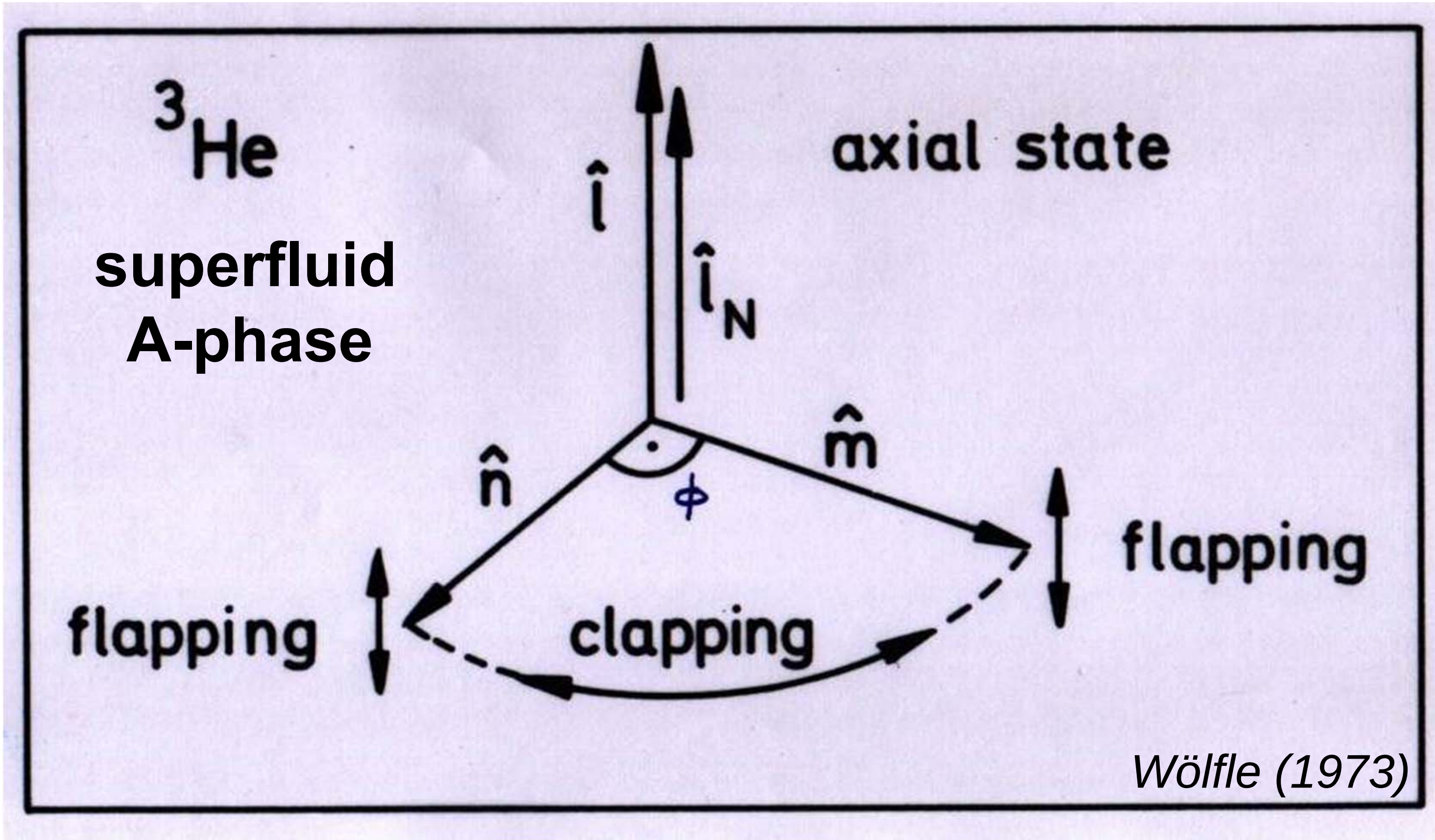
$$(\omega_{sq})^2 + (\omega_{rsq})^2 = (2\Delta)^2 = (2\omega_{fermion})^2$$

Nambu 1976





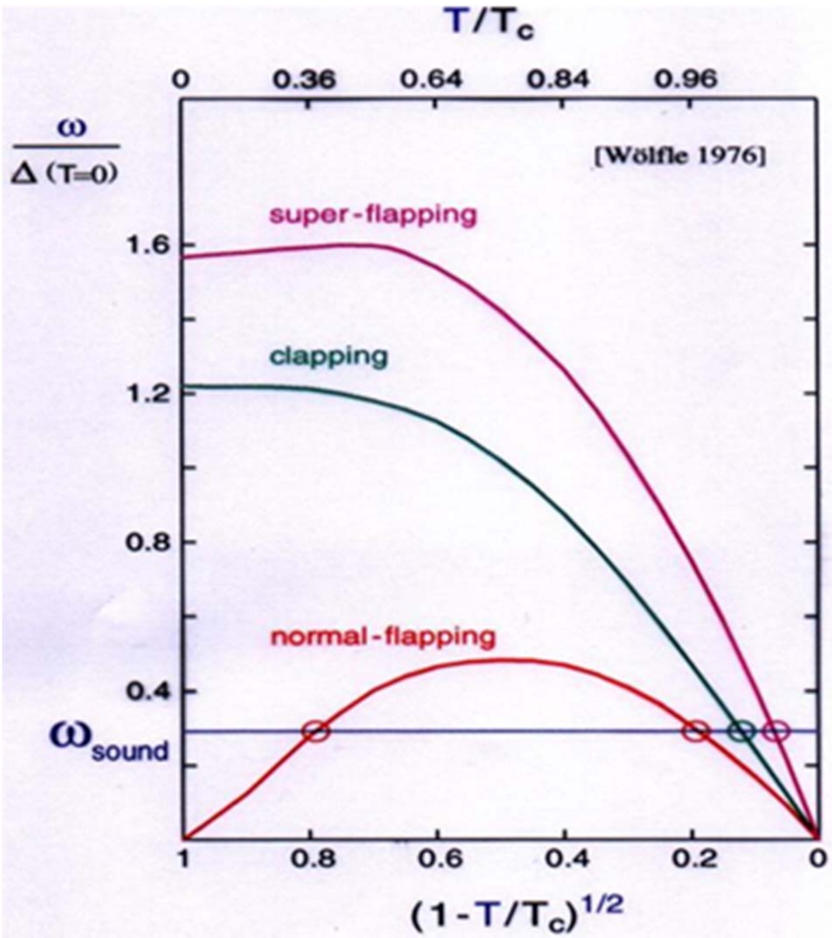
Oscillations of the order parameter structure in the A-Phase of superfluid Helium 3



$$A_{j\mu} = (n_j + i m_j) d_\mu , \quad \hat{n} \cdot \hat{m} = 0 \quad .$$

Karlsruhe Institute of Technology

Collective Modes and ultrasound in the A-phase



Collective mode peaks in ultrasound absorption

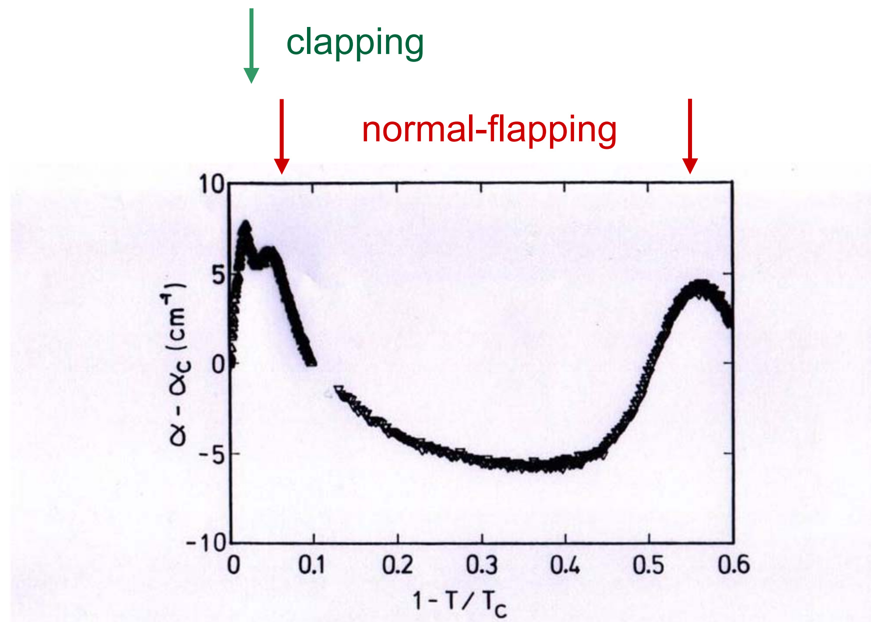


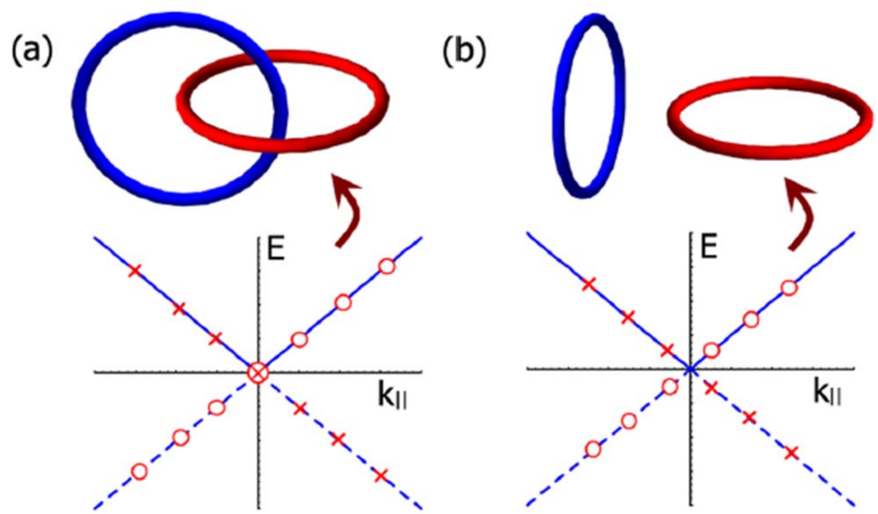
Fig. 38. Normal flapping resonances and clapping resonance at 34.2 MHz with an angle of 45° between *H* (= 14 mT) and *q* (triangle), and with an angle of 63° and *H* = 0.45 *T* after Ling et al (1987a, 1989) (inverted triangle).

Majorana fermions at the core of vortices in He3-B

Superfluid He3-B is a topologically non-trivial superfluid, supporting ring vortices of vorticity 1 (winding of the R-matrix)

In the vortex core fermionic excitations may exist, which appear in time-reversal invariant pairs (Dirac)

If the rings are linked, topology requires the existence of a zero mode ➡ Majorana fermions



X-L Qi, T L. Hughes, S. Raghu, S-C Zhang, 2009

“Standard model” of Condensed Matter Theory

Theory of Fermi or Bose liquid

+ spontaneous symmetry breaking

= most successful concept of quantum matter

Beyond the “Standard Model”

Quantum fluctuations in reduced dimensions may destroy

- Landau quasi particles
- Long range order

Examples:

- Electrons in 1d (Quantum wire): Separation of Charge and Spin
  ➡ Landau quasi particle decays into Spinon und Holon
- Quantum Hall effect in 2d:
  ➡ Landau qp decays into “fractional” quasi particles
- Topological insulators
- Frustrated magnetic systems
- High temperature cuprate superconductors?

Karlsruhe Institute of Technology

Quantum Hall effect

- **Integer QHE:** *K. von Klitzing, M. Pepper, G. Dorda (1980)*

- **Fractional QHE:** *D.C. Tsui, H.L. Störmer, A.C. Gossard (1982)*

  Theory: *R.B. Laughlin (1983)*

Nobel prizes:
1985 *K. von Klitzing*
1998 *R.B. Laughlin, H.L. Störmer, D.C. Tsui*

KIT Karlsruhe Institute of Technology

Quantum Hall effect set-up

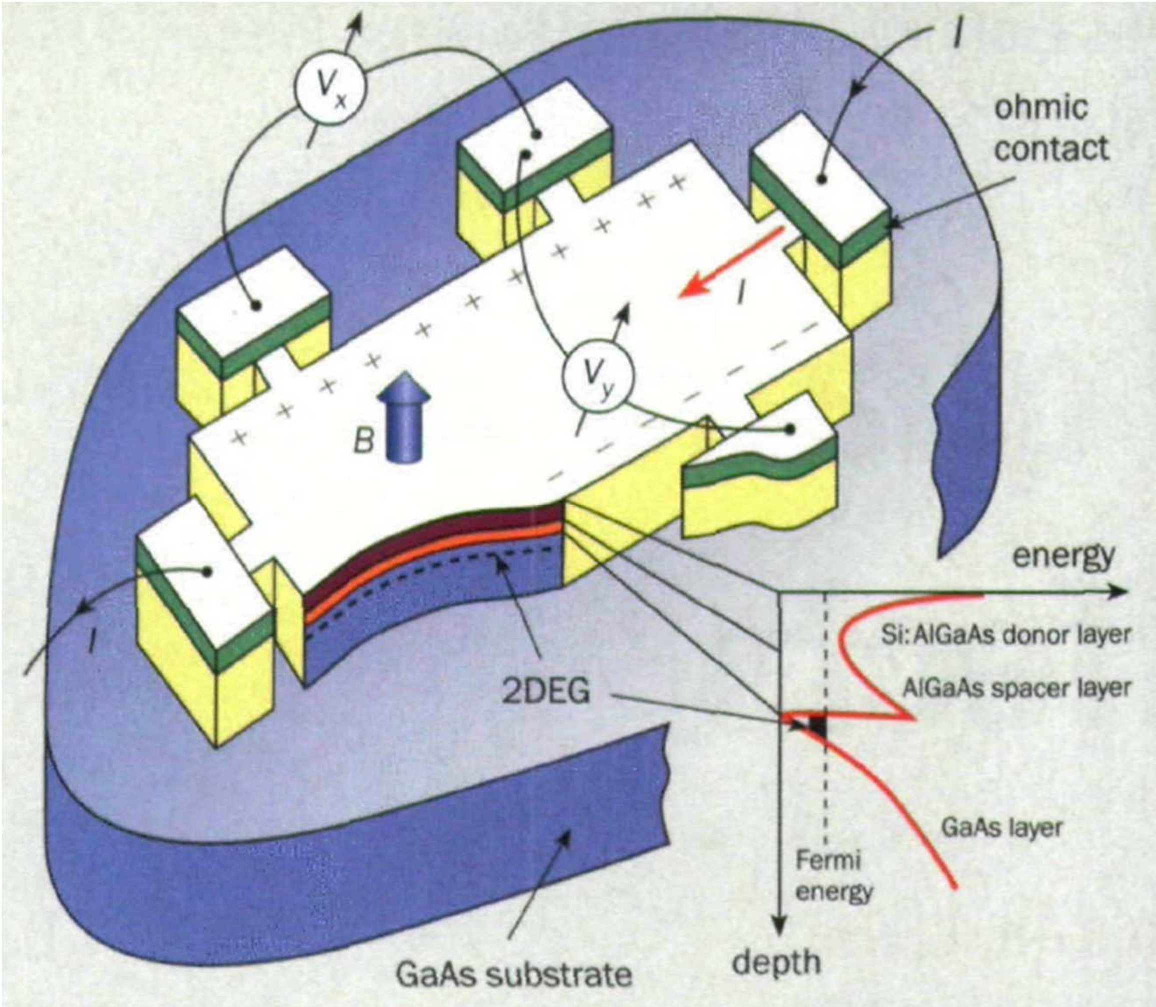
Measurement of the electrical resistance in a magnetic field

Longitudinal resistance:

$R_{xx}=V_x/I$

Hall resistance:

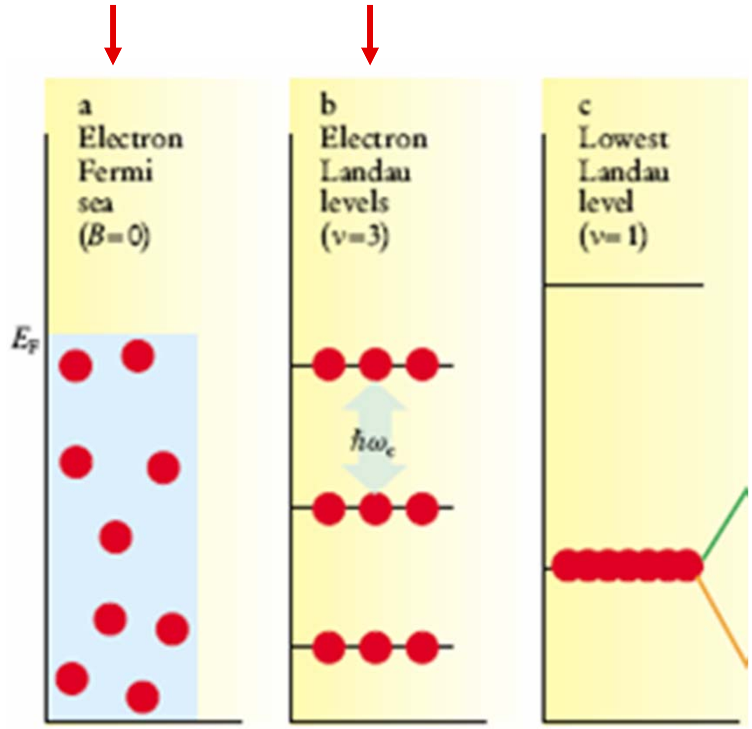
$R_{xy}=V_y/I$



Karlsruhe Institute of Technology

Energy spectrum of 2d electrons in magnetic field B

Spectrum is: continuous / discrete



ν : Filling factor of Landau levels

L.D. Landau (1930)

Quantum Hall effect

Plateaus in Hall resistance at multiples of "Quantum resistance"

$$R_Q = \frac{h}{e^2}$$

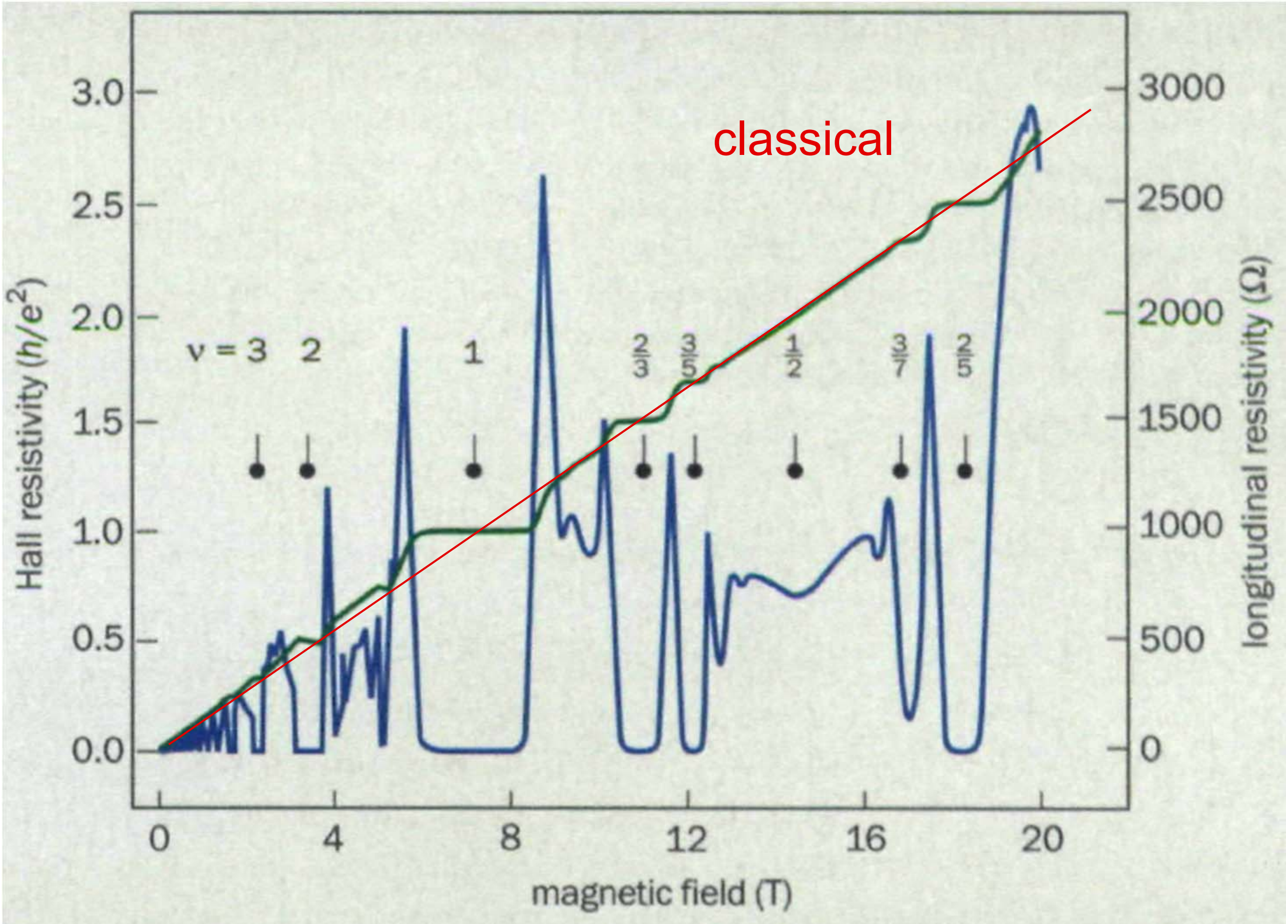
Integer QHE

$$R_{xy}^{in} = \frac{1}{\nu} R_Q, \quad \nu = 1, 2, ..$$

Fractional QHE

$$R_{xy}^{fract} = \frac{1}{\nu_p^q} R_Q,$$

$$\nu_p^q = \frac{p}{pq+1}, \qquad q = 2, 4, .. \qquad p = 1, \pm 2, ..$$



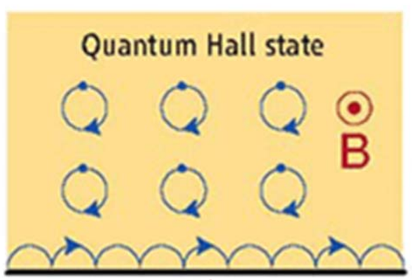
V. Umansky und J. Smet (2000)

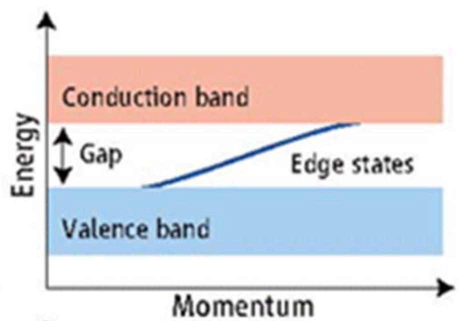
Integer Quantum Hall effect: edge states

In the region of large electron density:
Coulomb interaction between electrons negligible (screening)

Disorder localizes electrons in the bulk of the QHE sample.
Charge transport may occur only via “edge channels”, forming exactly one-dimensional, spin-polarized ideal “quantum wires” , of quantized conductance:

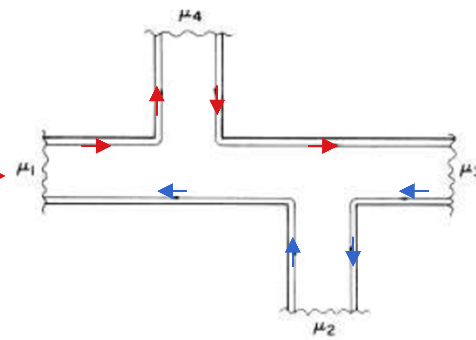
$$G_Q = 1/R_Q = \frac{e^2}{h}$$





Dissipation free transport

single edge channel:

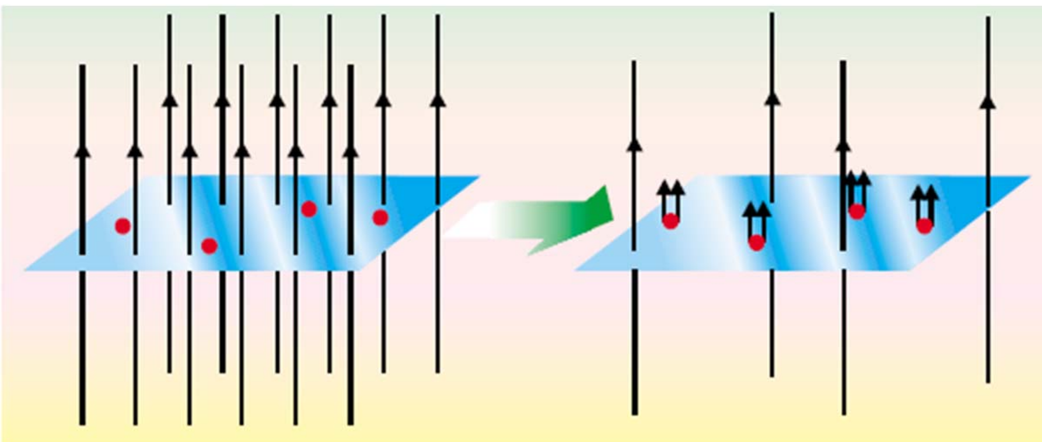


ν edge channels:

$$R_{xy}^{in} = \frac{1}{\nu} R_Q, \quad \nu = 1,2,..$$

Fractional Quantum Hall effect: “Composite Fermion” = Electron + q flux quanta

In case of small electron density:
Coulomb interaction of electrons dominates



q=2

↑ : **magnetic flux quantum h/e**

Composite Fermion “absorbs” part or all of the applied magnetic flux:

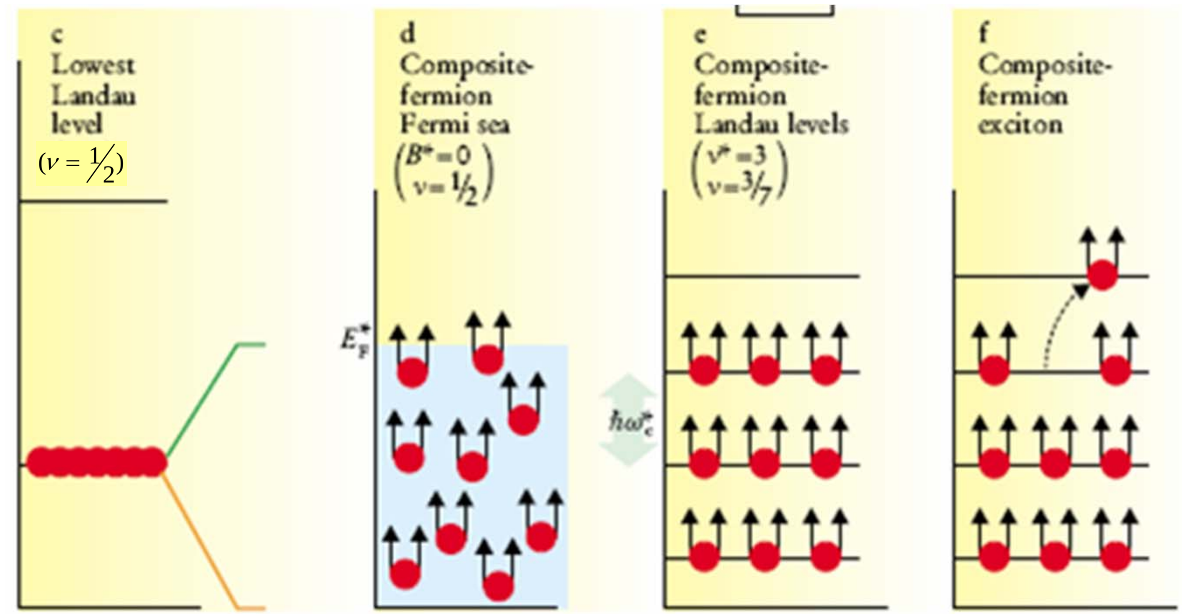
Effective magnetic field: $B_{eff} = B(1-2\nu)$

- **Fermi liquid at** $B_{eff} = 0, \quad \nu = \frac{1}{2}$

J. Jain (1989),
Read, Halperin, Lee (1995)

Karlsruhe Institute of Technology

QHE of "Composite Fermions"



- **integer QHE of CFs corresponds to fractional QHE of electrons**

$$\nu^* = p, \quad \nu = \frac{p}{pq+1}, \quad p \text{ integer}$$

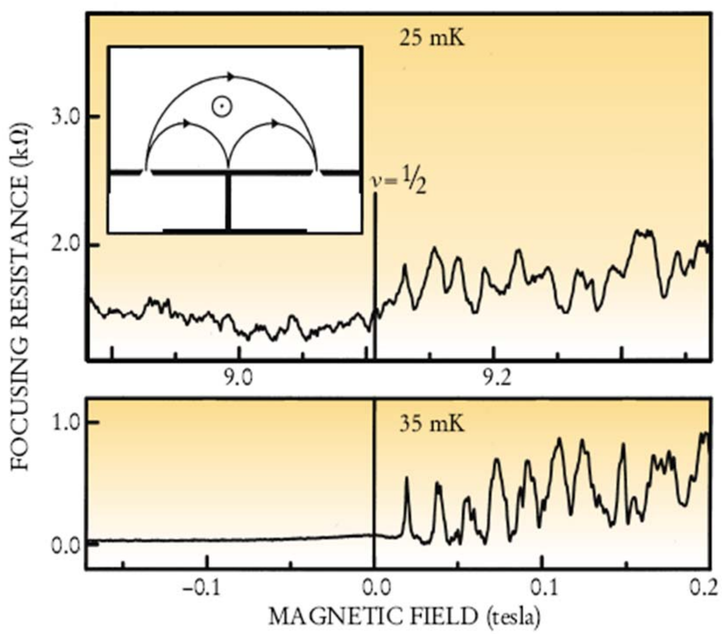
J. Jain (1989)

Transport properties of “Composite Fermions”

Electrons in disordered potential of doping ions

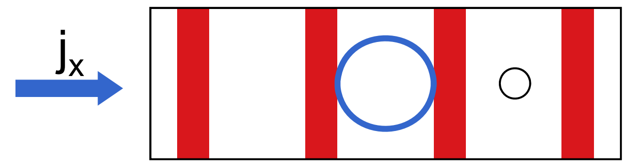
→ “Composite fermions” in a disordered magnetic field

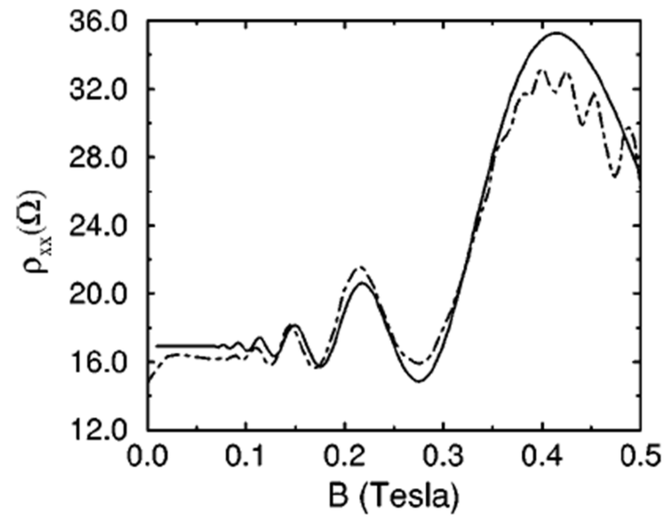
Focussing of CFs



Smet et al. (1996)

QH-sample with stripe pattern





Weiss, von Klitzing, et al. (1989)

Mirlin, Wölfle (1998)

Karlsruhe Institute of Technology

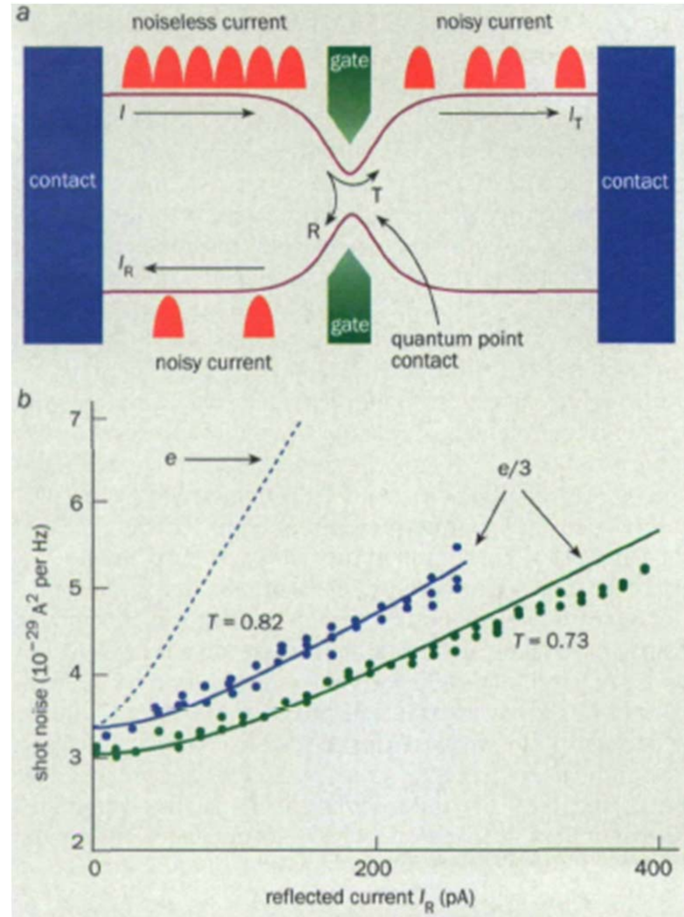
Shot noise and fractional quasi particles: QHE

Laughlin quasi particles:

At ν=1/3 FQHE state have
→ 3 flux quanta per electron

Fractional quasi particle: **vortex excitation carrying**

1 flux quant → 1/3 electron



Detectable via shot noise measurement

$S=2e^* I_R$

effective fractional charge $e^*=e/3$

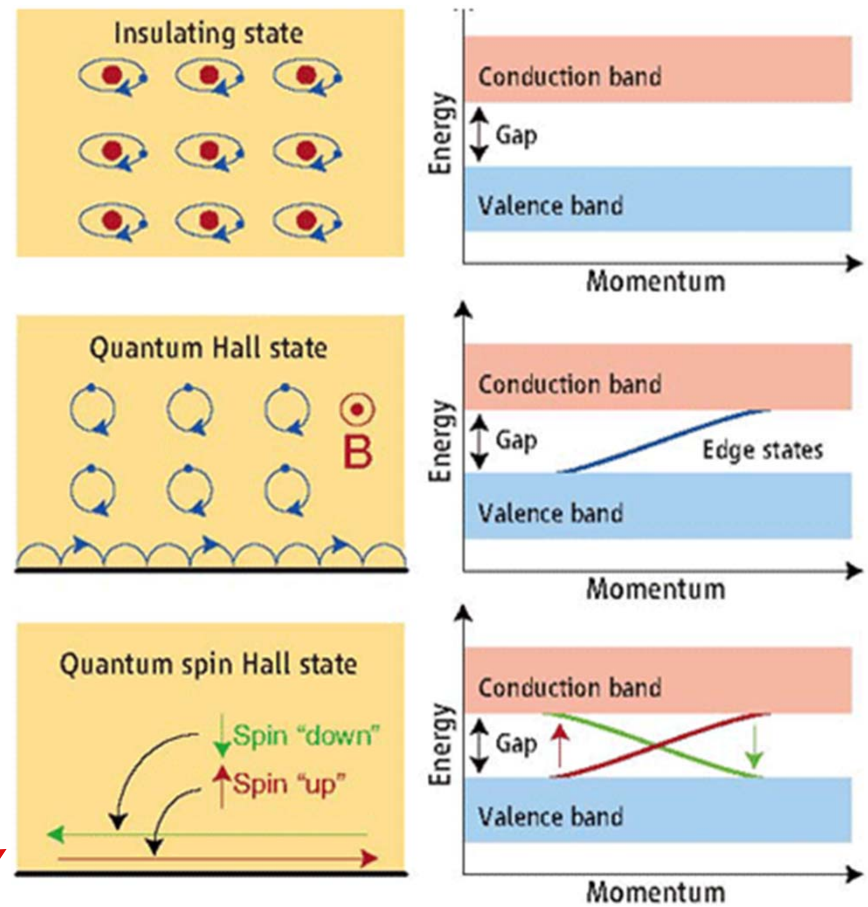
Fractional statistics ?

Exp.: *Glattli et al. (1997)*
de Picciotto et al. (1997)

Karlsruhe Institute of Technology

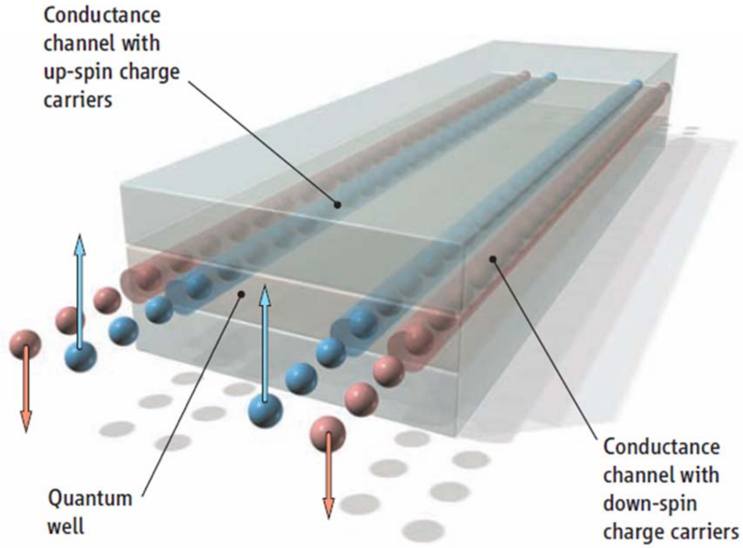
Topological insulators

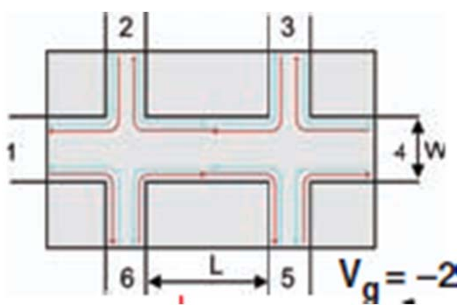
May the integer Hall effect be realized even without external applied magnetic field?
Yes! Following a proposal by C.J. Kane and E.J. Mele by way of
energy splitting of spin states via spin-orbit-coupling
(simulates the magnetic field).



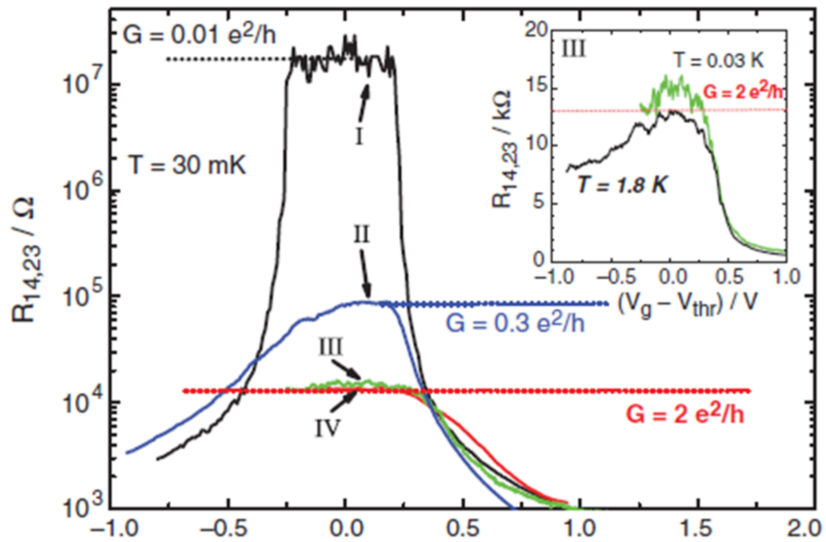
Dissipation free transport

Topological Insulators





Quantum Spin Hall Insulator State in HgTe Quantum Wells



L. Molenkamp et al., 2007

Karlsruhe Institute of Technology

Summary and Outlook

- Interacting quantum many-body systems (electrons, atoms, ..) condense into ordered states featuring spontaneous symmetry breaking and supporting a zoo of new "quasiparticles".

  The search for new types of order in new (artificially synthesized) materials with novel properties not encountered in nature goes on.

- More recently the search is focussing on "exotic" states of matter, characterized by more subtle types of order, sometimes with topological properties and/or with , "fractional quasiparticles" .

  The new concepts may be relevant for unraveling the puzzle of high temperature superconductivity (the holy grail!).
  Materials showing "topologically protected" quantum coherence properties are of interest in the context of quantum information processing.

"More is different" *P.W. Anderson*

Karlsruhe Institute of Technology