Matching Squark Pair Production at NLO with parton showers

GK Workshop Bad Liebenzell 2013

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Outline

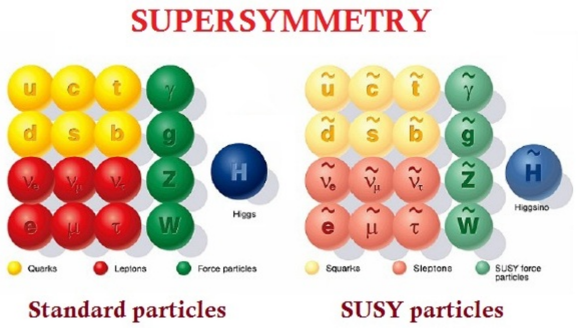
1. Supersymmetry
2. Squark Pair Production
3. Matching NLO calculations with parton showers
4. Applying the POWHEG method to $\tilde{q}\tilde{q}$
5. Summary

Why Susy?

- only possible extension of spacetime symmetries
- in R-parity conserving SUSY: (often) LSP is $\tilde{\chi}_0 \rightarrow$ candidate for Dark Matter
- Unification of 3 forces at the GUT scale is possible
- local SUSY enforces gravity
- solution to the hierarchy 'problem'

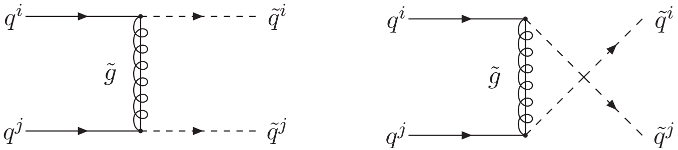
The Minimal Superymmetric Standard Model

- simplest form of a supersymmetric theory ($N = 1$, i.e. one SUSY generator)
- (roughly) doubles particle content of the SM:



- in the following: only squarks ($\tilde{q}$) and gluinos ($\tilde{g}$) relevant

Squark Pair Production



- for large $m_{\tilde{q}}$, $m_{\tilde{g}}$: $\tilde{q}\tilde{q}$ (often) dominant sparticle production channel
- large K-factors ($K = \sigma_{NLO}/\sigma_{LO}$)
- (SUSY)-QCD-corrections at NLO only available from PROSPINO [Beenakker et.al. 1997]:
  - mass-degenerate squarks
  - all individual channels summed up
  - only total K-factors, no distributions
- (re)calculate the SQCD-corrections fully differentially [Popenda 2012]
- realistic simulation for LHC physics requires combination with parton shower, hadronization, ...

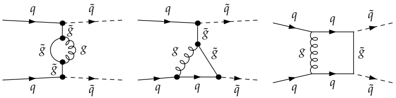
Elements of an NLO calculation

$$d\sigma_{\mathrm{NLO}} = \Bigg[\mathcal{B}(\Phi_2) + \mathcal{V}(\Phi_2) + \overbrace{\int \Big[\underbrace{\mathcal{R}(\Phi_2, \Phi_{rad})}_{\text{IR divergent}} - \underbrace{\mathcal{C}(\Phi_2, \Phi_{rad})}_{\text{IR divergent}}\Big] d\Phi_{rad}}^{\text{IR finite}}\Bigg] d\Phi_2$$

- Virtual contribution:

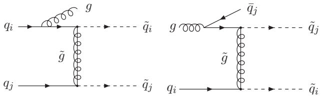
$$\mathcal{V} = \overbrace{\mathcal{V}_b(\Phi_2) + \int \mathcal{C}(\Phi_2, \Phi_{rad}) d\Phi_{rad}}^{\text{IR finite}},$$

$$\mathcal{V}_b = 2Re(\mathcal{M}_B \cdot \mathcal{M}_V^*)$$



- Real radiation:

$$d\Phi_3 = d\Phi_2 d\Phi_{rad}$$



- Subtraction terms:

$\mathcal{C} \to \mathcal{R}$ in soft/collinear limit, e.g. Catani-Seymour, FKS

Squark Pair Production at NLO - some results

Is the K-factor similar for different subchannels?

Assume $m_{\tilde{q}} = 1800\,\text{GeV},\ m_{\tilde{g}} = 1600\,\text{GeV}$ for $\sqrt{s} = 8\,\text{GeV}$

$$\sigma_{\mathrm{LO}}^{\mathrm{Prospino}} = 2.57 \cdot 10^{-1}\mathrm{fb}, \quad \sigma_{\mathrm{NLO}}^{\mathrm{Prospino}} = 2.99 \cdot 10^{-1}\mathrm{fb}$$

for all possible 36 channels ($\tilde{u}, \tilde{d}, \tilde{c}, \tilde{s}$ production) summed up, using

$$\mu_R = \mu_F = m_{\tilde{q}}$$

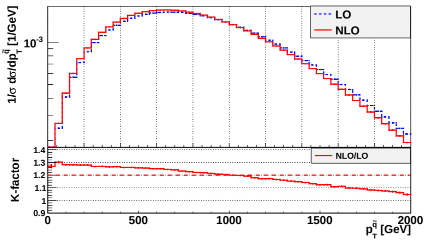
channel	σ_{LO} [fb]	σ_{NLO} [fb]	K
$\tilde{u}_L\tilde{u}_L$	$1.29 \cdot 10^{-1}$	$1.43 \cdot 10^{-1}$	1.11
$\tilde{u}_L\tilde{d}_L$	$8.00 \cdot 10^{-2}$	$9.92 \cdot 10^{-2}$	1.23
$\tilde{u}_L\tilde{u}_R$	$3.40 \cdot 10^{-2}$	$4.00 \cdot 10^{-2}$	1.18
$\tilde{u}_L\tilde{d}_R$	$1.39 \cdot 10^{-2}$	$1.74 \cdot 10^{-2}$	1.26
Sum	$2.57 \cdot 10^{-1}$	$3.00 \cdot 10^{-1}$	1.16

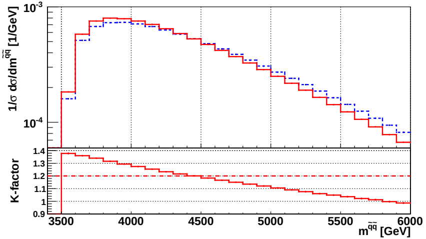
$\rightarrow$ important to treat channels separately if $\tilde{q}$ have different decay widths

Are differential K-factors flat?

consider cMSSM benchmark point, first two generations are degenerate in mass:

$m_{\tilde{u}_L}$	$m_{\tilde{u}_R}$	$m_{\tilde{d}_L}$	$m_{\tilde{d}_R}$	$m_{\tilde{g}}$
1799.53	1769.21	1801.08	1756.40	1602.91

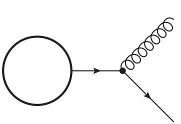




$\longrightarrow$ differential K-factors are not necessarily flat

Parton showers and why we need them

- soft/collinear emission enhanced:



$$\frac{1}{(p_q+p_g)^2} = \frac{1}{2E_g E_q(1-\cos\theta_{gq})}, \; m_q = 0$$

soft divergence: $E_g \to 0$

collinear divergence: $\theta_{gq} \to 0$

- **parton-shower:** recursive calculation of these contributions to all orders

Advantages/Disadvantages

- correct shape for soft/collinear region (fixed order divergent)
- realistic 'events' after including hadronization effects, UE, ... (fixed order has only low multiplicity)
- BUT: only LO, description not sensible beyond soft/collinear region

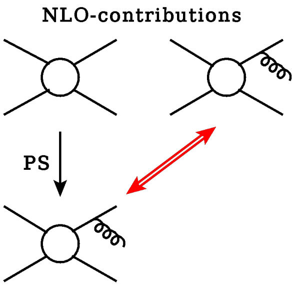
$\Rightarrow$ try to combine advantages of fixed order calculations and parton shower:

1. higher multiplicity in fixed order calculation $\to$ **Merging**
2. higher order in perturbation theory $\to$ **Matching**

The double counting problem

combination of fixed-order NLO calculation with parton shower non-trivial: avoid **double-counting**

events with Real configuration (i.e. $n+1$ final state particles) also obtained via splitting of Born configuration in parton shower



two NLO-matching-schemes:

- MC@NLO [Frixione,Webber 2002]
- POWHEG [Nason 2004]

IR-safe observable $\mathcal{O}_i \equiv \mathcal{O}(\Phi_i)$ after first branching in shower, starting from Born process:

$$\langle\mathcal{O}\rangle^{PS}_{LO} = \int d\Phi_n \mathcal{B}(\Phi_n)\Big[\overbrace{\mathcal{O}_n\Delta(Q_{IR})}^{\text{no emission}} + \overbrace{\int_{Q>Q_{IR}} d\Phi_{rad}\mathcal{O}_{n+1}\Delta(Q)\frac{\alpha_s(Q)}{2\pi}\frac{P(z)}{Q}}^{\text{one emission with } Q>Q_{IR}}\Big]$$

with the **Sudakov** (probability of NOT emitting between Q and Q_{max})

$$\Delta(Q) = \exp\left[-\int d\Phi'_{rad}\frac{\alpha_s(Q')}{2\pi}\frac{P(z')}{Q'}\Theta(Q'-Q)\right]$$

$$= 1 - \int d\Phi'_{rad}\frac{\alpha_s(Q')}{2\pi}\frac{P(z')}{Q'}\Theta(Q'-Q) + \mathcal{O}(\alpha_s^2)$$

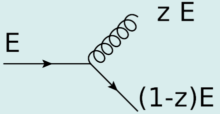
Q: ordering variable (e.g. p_T)

$\mathbf{Q_{IR}}$: IR cutoff

z: energy fraction

$\mathbf{d\Phi_{rad}}$: $dQ\, dz\, \frac{d\varphi}{2\pi}$

P(z): AP splitting kernel



$$\Rightarrow \langle\mathcal{O}\rangle^{PS}_{LO} = \int d\Phi_n\Big\{\mathcal{O}_n\mathcal{B}(\Phi_n)$$

$$\text{terms formally NLO} + \int_{Q>Q_{IR}} d\Phi_{rad}\left[\mathcal{O}_{n+1}\frac{\alpha_s(Q)}{2\pi}\frac{P(z)}{Q} - \mathcal{O}_n\frac{\alpha_s(Q)}{2\pi}\frac{P(z)}{Q}\right]\Big\} + \mathcal{O}(\alpha_s^2)$$

The POWHEG method

Observable $\mathcal{O}$ at NLO reads

$$\langle\mathcal{O}\rangle_{NLO} = \int d\Phi_n \left[\mathcal{O}_n[\mathcal{B}(\Phi_n)+\mathcal{V}(\Phi_n)] + \int [\mathcal{O}_{n+1}\mathcal{R}(\Phi_{n+1}) - \mathcal{C}(\Phi_{n+1})\mathcal{O}_n] d\Phi_{rad}\right]$$

Matching NLO and parton shower $\rightarrow$ subtract these terms consistently:

$$\langle\mathcal{O}\rangle^{sub}_{NLO} = \int d\Phi_n \left\{\mathcal{O}_n[\mathcal{B}+\mathcal{V}] + \int d\Phi_{rad}\left[\left(\frac{\alpha_s(Q)}{2\pi}\mathcal{B}\frac{P(z)}{Q} - \mathcal{C}\right)\mathcal{O}_n + \left(\mathcal{R} - \frac{\alpha_s(Q)}{2\pi}\mathcal{B}\frac{P(z)}{Q}\right)\mathcal{O}_{n+1}\right]\right\}$$

Special case:

$$\frac{\mathcal{R}}{\mathcal{B}} = \frac{\alpha_s(Q)}{2\pi}\frac{P(z)}{Q}$$

$\rightarrow$ **POWHEG** (POsitive Weight Hardest Emission Generator):

- generate the hardest emission (w.r.t. p_T) before applying parton shower, using the exact real emission matrix element
- preserves NLO accuracy (for inclusive observables and for large p_T)
- use p_T-veto in shower, i.e. all subsequent radiation is softer (if shower is p_T-ordered; for angular ordered shower: truncated shower)

The POWHEG master formula [Frixione, Nason, Oleari 2007]

In full analogy to the first emission for 'LO+parton shower':

$$d\sigma_{PWG} = \overline{\mathcal{B}}(\Phi_n) d\Phi_n \Big[\Delta_{PWG}(\Phi_n, pT_{min}) + d\Phi_{rad}(pT) \Delta_{PWG}(\Phi_n, pT) \frac{\mathcal{R}(\Phi_{n+1})}{\mathcal{B}(\Phi_n)} \Theta(pT - pT_{min}) \Big]$$

with the POWHEG-Sudakov

$$\Delta_{PWG}(\Phi_n, pT) = \exp\Big[- \int d\Phi'_{rad} \frac{\mathcal{R}(\Phi_{n+1})}{\mathcal{B}(\Phi_n)} \Theta(pT'(\Phi_n, \Phi'_{rad}) - pT) \Big]$$

and

$$\overline{\mathcal{B}}(\Phi_n) = \mathcal{B} + \mathcal{V} + \int d\Phi_{rad} \left[\mathcal{R}(\Phi_{n+1}) - \mathcal{C}(\Phi_{n+1}) \right]$$

Properties of the POWHEG master formula

- NLO accurate for inclusive observables (by construction)
- NLO accuracy preserved in the hard region:

$$\Delta_{PWG}(\Phi_n, pT_{min}) \to 0, \quad \Delta_{PWG}(\Phi_n, p_T) \to 1$$

$$\Rightarrow \quad d\sigma_{PWG} \approx \frac{\overline{\mathcal{B}}(\Phi_n)}{\mathcal{B}(\Phi_n)} \mathcal{R}(\Phi_{n+1}) d\Phi_n d\Phi_{rad} = \mathcal{R}(\Phi_{n+1}) \left(1 + \mathcal{O}(\alpha_s)\right) \, d\Phi_n \, d\Phi_{rad}$$

- leading-log accuracy of a shower MonteCarlo in soft/collinear limit ($p_T \to 0$) is not destroyed:

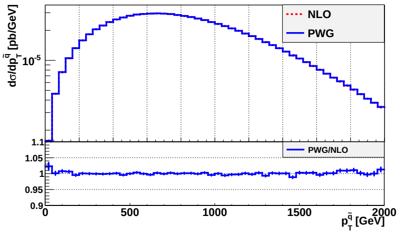
$$\frac{\mathcal{R}(\Phi_{n+1})}{\mathcal{B}(\Phi_n)} d\Phi_{rad} \approx \frac{\alpha_s}{2\pi} \frac{1}{Q} P(z) \, dQ \, dz \frac{d\varphi}{2\pi}, \quad \overline{\mathcal{B}} \approx \mathcal{B} \left(1 + \mathcal{O}(\alpha_s)\right)$$

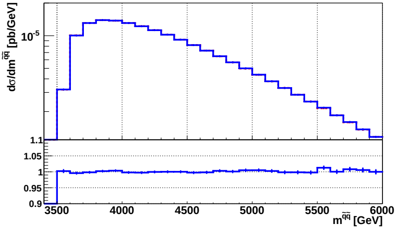
- Positive weights, as (usually) $\overline{\mathcal{B}} > 0$

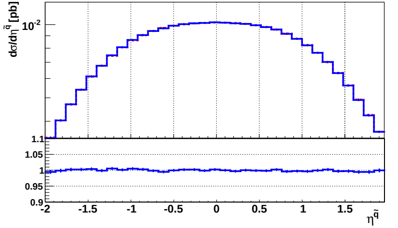
The POWHEG-BOX [Alioli,Nason,Oleari,Re 2010]

- POWHEG-BOX provides process-independent ingredients for a POWHEG-implementation of arbitrary processes:
  - automatized subtraction-scheme (FKS-scheme [Frixione, Kunszt, Signer 1996])
  - generation of radiation phasespace
  - hardest radiation according to POWHEG-Sudakov
  - NLO distributions as 'by-product'
  - LHE-output: unweighted events which can be interfaced to shower program
- user needs to implement the process specific parts
- So far: no processes with strongly interacting BSM particles implemented $\longrightarrow$ small changes in the main routines of the code concerning the FKS subtraction

POWHEG at work - inclusive observables



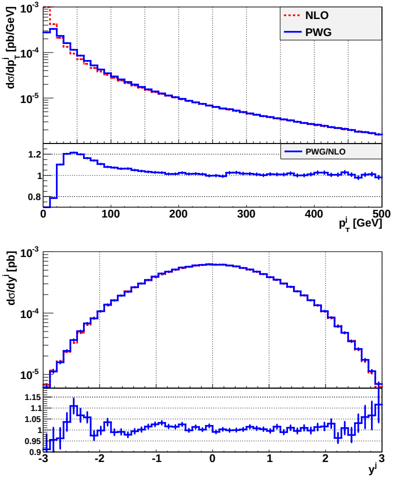




- PWG: results after first (hardest) emission
- $p_T^{\tilde{q}}$, $\eta^{\tilde{q}}$: sum of both $\tilde{q}$ distributions

$\Rightarrow$ perfect agreement, i.e. NLO accuracy preserved

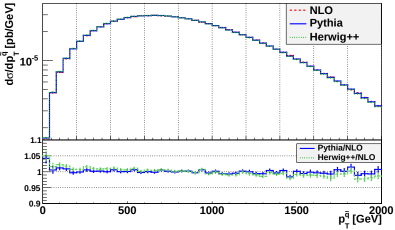
Exclusive observables

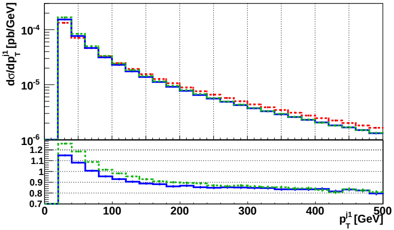


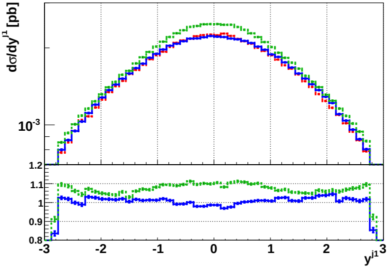
- NLO result diverges for small p_T, PWG result 'Sudakov damped'
- for y_j: demand $p_T^j > 200\,\mathrm{GeV}$

$\Rightarrow$ for large p_T the (N)LO result is reproduced

Parton shower effects - PYTHIA6 vs. HERWIG++







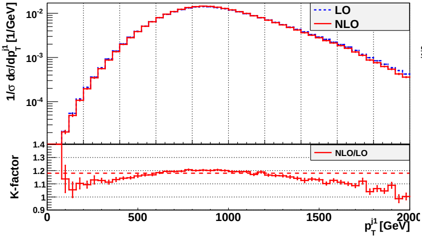
- hadronization, underlying event turned off
- partons clustered with anti-k_T ($R = 0.4$)
- only very basic cuts:
  $p_T^j > 20\text{GeV}$, $|\eta_j| < 2.8$
- inclusive quantities hardly affected
- p_T^{j1} softer than NLO, HERWIG++ slightly higher rates at low p_T^{j1}
- HERWIG++ predicts more central jets

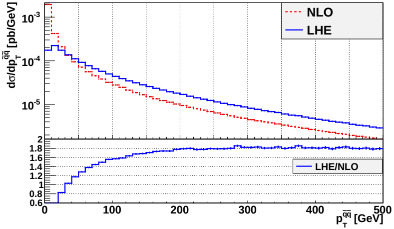
Summary

- Squark pair production is important sparticle production channel at the LHC
- NLO corrections are usually different for individual channels, K-factors are often large and not flat
- Matching NLO fixed order calculation with parton showers important for precise predictions for LHC physics ($\longrightarrow$ POWHEG method)
- Implementation of squark pair production in public program package POWHEG-BOX
- Parton shower effects for inclusive observables small, but important for radiated parton

Backup

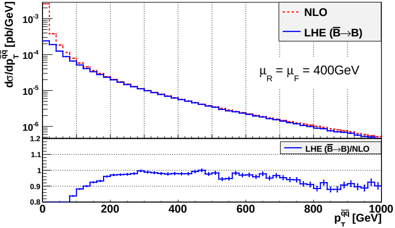
Differential K-factors after decay to $\tilde{\chi}_0 q$





- at NLO: $p_T^{\tilde{q}\tilde{q}} \leftrightarrow p_T^j$, the p_T of the radiated parton
- low $p_T^{\tilde{q}\tilde{q}}$: Sudakov damping (NLO result diverges here)
- high $p_T^{\tilde{q}\tilde{q}}$: LHE/NLO $\approx 1.8 \Rightarrow 80\%$ discrepancy!

- two reasons for this discrepancy:
  1. assumption $\overline{\mathcal{B}}/\mathcal{B} \approx 1$ is not valid here: sizeable K-factor ($K = 1.2$)
  2. different scales for $\overline{\mathcal{B}}$ ($\mu = \overline{m}_{\tilde{q}}$) and for $\mathcal{R}/\mathcal{B}$ (p_T of the radiated parton)
- check these two points: perform event generation with $\overline{\mathcal{B}} \to \mathcal{B}$ and $\mu_R = \mu_F = 400\text{GeV}$

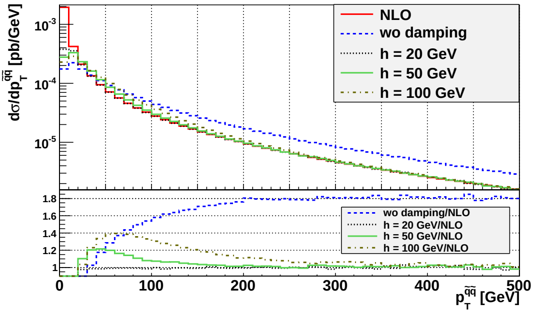


- idea [Alioli,Nason,Oleari,Re 2009]: 'split' the real contributions in the master-formula, use only IR-singular parts for radiation generation

$$\mathcal{R} = \mathcal{R}_s + \mathcal{R}_r = \mathcal{F}\mathcal{R} + (1-\mathcal{F})\mathcal{R}; \quad \mathcal{F} = \frac{h^2}{p_T^2 + h^2}$$

- 'new' master-formula:

$$d\sigma_{PWG} = \overline{\mathcal{B}_s}(\Phi_n)\, d\Phi_n \left[\Delta_s(\Phi_n, p_T^{min}) + \Delta_s(\Phi_n, k_T) \frac{\mathcal{R}_s(\Phi_n, \Phi_{rad})}{\mathcal{B}(\Phi_n)} \theta(k_T - p_T^{min}) d\Phi_{rad} \right] + \mathcal{R}_r d\Phi_n d\Phi_{rad}$$



Shower without ISR