Karlsruhe Institute of Technology

Higgs boson pair production at the LHC:

corrections to top quark mass dependence at NLO

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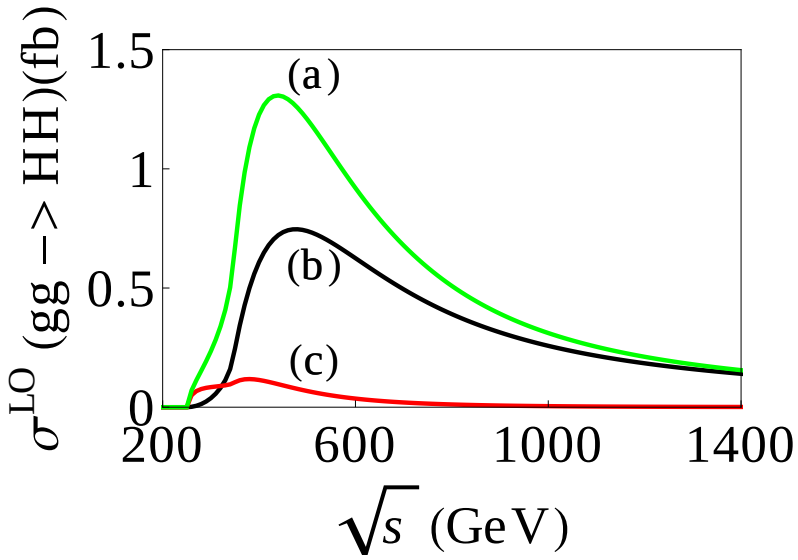
1. Introduction Motivation

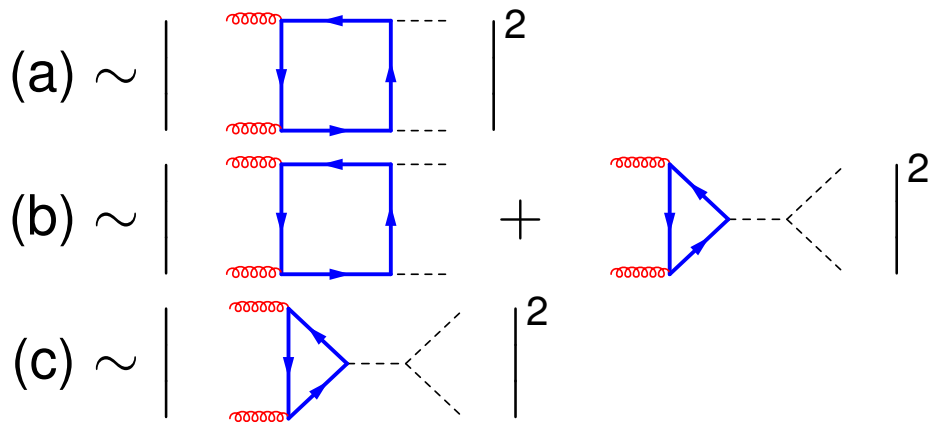
Higgs discovered, now measure & verify:

- couplings to particles ($\sim$ masses?)
- self-couplings

Higgs Potential in the Standard Model:

$$V(H) = \frac{1}{2} m_H^2 H^2 + \lambda v H^3 + \frac{1}{4}\lambda H^4, \quad \lambda^{\mathrm{SM}} = \frac{m_H^2}{2v^2} \approx 0.13, \quad v\text{: Higgs vev.}$$





1. Introduction Measurement

Prospects for the LHC @ 14 TeV:

- $b\bar{b}\gamma\gamma$ channel, 600 fb^{-1}: $\lambda \neq 0$

  [Baur, Plehn, Rainwater; '04]

- $b\bar{b}\gamma\gamma$, $b\bar{b}\tau^+\tau^-$ channels: **"promising"**;
  $b\bar{b}W^+W^-$ channel: **"not promising"**

  [Baglio, Djouadi, Gröber, Mühlleitner, Quevillon, Spira; '13]

- 600 fb^{-1}: $\lambda > 0$;
  3000 fb^{-1}: $\lambda^{+30\%}_{-20\%}$

  [Goertz, Papaefstathiou, Yang, Zurita; '13]

- and many others, e.g.:

  [Dolan, Englert, Spannowsky; '12],

  [Papaefstathiou, Yang, Zurita; '13],

  [Barr, Dolan, Englert, Spannowsky; '13],

  ...

1. Introduction Predictions

What was already known?

- LO result with exact M_t dependence

  [Glover, van der Bij; '88], [Plehn, Spira, Zerwas; '98]

- NLO result in $M_t \to \infty$ limit [Dawson, Dittmaier, Spira; '98]

  $\sigma_{\text{tot., hadr.}} \approx \left(20^{\text{LO}} + 20^{\text{NLO, } M_t \to \infty}\right)$ fb

  if not stated otherwise: $\sqrt{s_{\text{hadr.}}} = 14$ TeV and $\mu = 2m_H$

Just recently:

- NLO + NNLL ($M_t \to \infty$) $\approx$ NLO + 20%

  [Shao, Li, Li, Wang; '13]

- NNLO soft-virtual approx. ($M_t \to \infty$) $\approx$ NLO + 20%

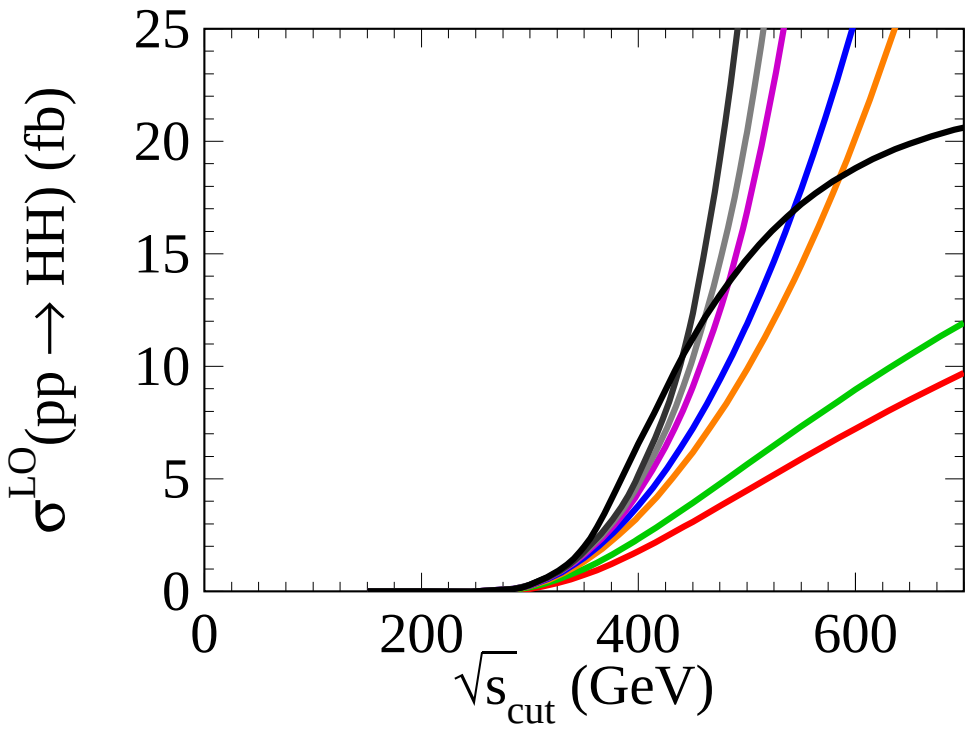
  [de Florian, Mazzitelli; '13]

- NNLO ($M_t \to \infty$) $\approx$ NLO + 20%

  [de Florian, Mazzitelli; '13]

1. Introduction Predictions

Why $\mathcal{O}(1/M_t)$ corrections?



$$\rho = \frac{m_H^2}{M_t^2}$$

ρ^0, ρ^1, ρ^2, ρ^3, ρ^4, ρ^5, ρ^6

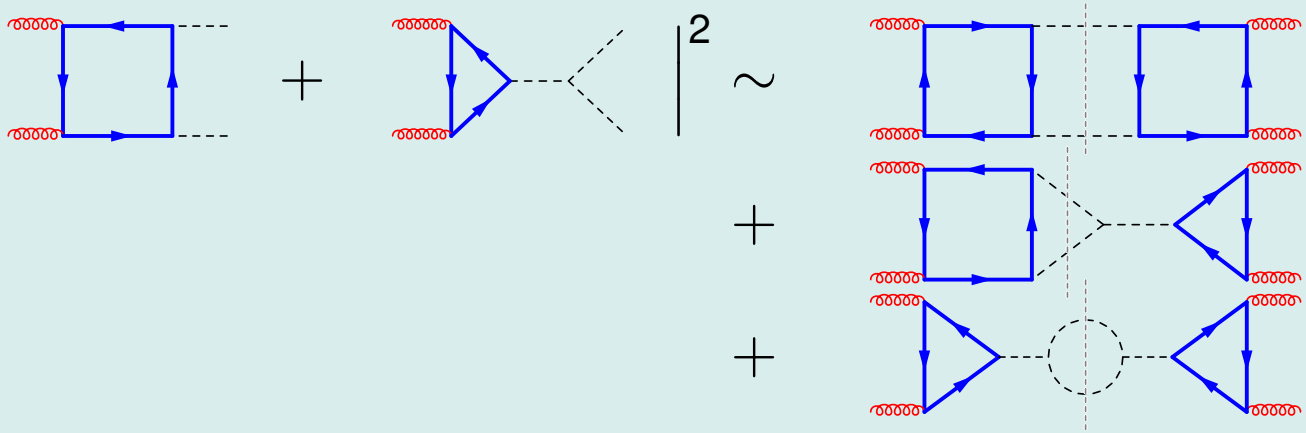
- s_{cut} = cut on invariant mass of Higgs pair
- black: exact in M_t, colored lines: expansions in ρ

2. Calculation Method

Forward Scattering & Optical Theorem:

$$\sigma_{\text{tot.}}(gg \to HH) \sim \text{Disc.}(\mathcal{M}(gg \to gg))$$

$$\int d\text{PS} \left| \quad + \quad \right|^2 \sim \quad + \quad + \quad$$



pro
- forward scattering $\Rightarrow$ simplified kinematics
- loop and phase space integration at once
- calculating Disc.(. . .) just for master integrals

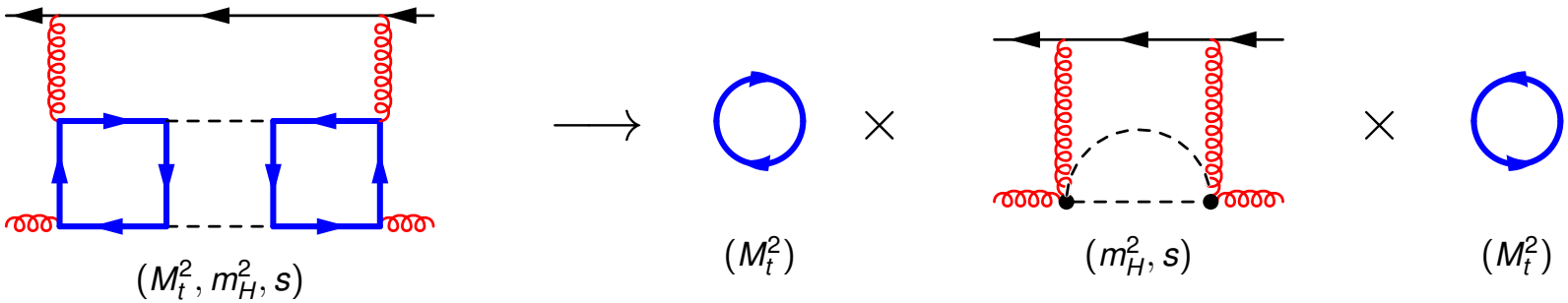
con
- more loops & diagrams
- only total cross section

2. Calculation Method

Asymptotic Expansion:

- expand at diagram level $\equiv$ series expansion in analytic result
- hierarchy: $M_t^2 \gg s, m_H^2 \Rightarrow$ series in $\rho = m_H^2/M_t^2$
- effectively reduce number of loops & scales

E.g.: NLO 4-loop 3-scale diagrams (real)



2. Calculation Tools

Chain of Programs:

- create diagrams: `QGRAF` [Nogueira; '93]
- select appropriate cuts [Hoff, Pak; (unpublished)]
- asymptotic expansion: `q2e` and `exp` [Harlander, Seidensticker, Steinhauser; '98]
- reduction to scalar integrals: `(T)FORM`
  [Vermaseren; '90], [Tentyukov, Vermaseren; '10], [Kuipers, Ueda, Vermaseren, Vollinga; '13]
- reduction to master integrals: `FIRE` [Smirnov; '08], [Smirnov^2; '13]

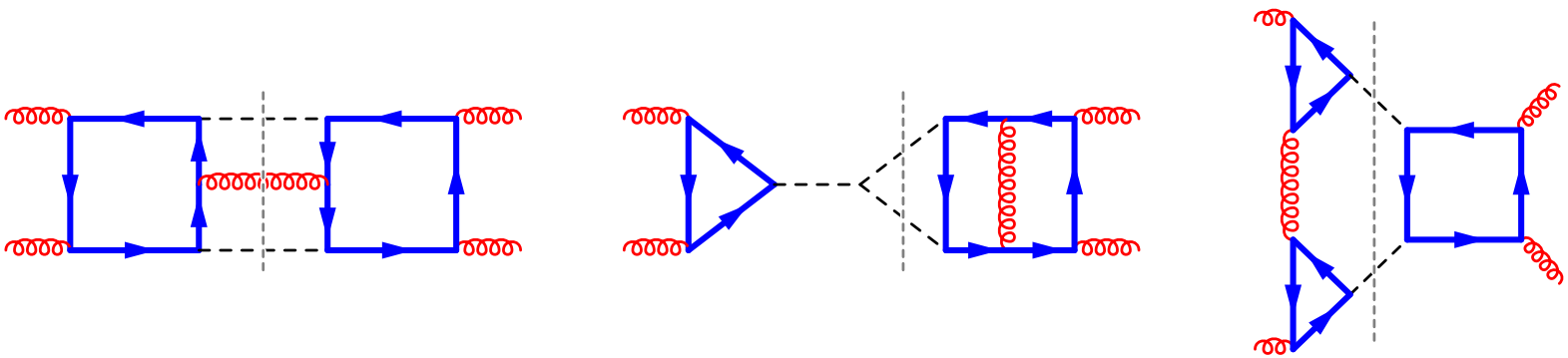
Bottleneck: reduction to scalar integrals

- `FORM` $\rightarrow$ `TFORM` $\rightarrow$ ✖ $\rightarrow$ `(T)FORM` bugfix $\rightarrow$ ✔
- limited in the gluon-gluon channel $\mathcal{O}\left(\rho^{n\geq 6}\right)$:
  $\mathcal{O}(4\text{ weeks})$ runtime with $\mathcal{O}(5\text{ TB})$ used disk space

2. Calculation Summary

Gluon-Gluon Channel:

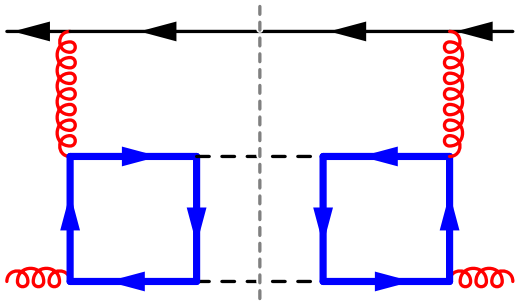
- virtual corrections:
  - $gg \to HH$: 126 two-loop diagrams
  - $gg \to gg$: 1052 four-loop diagrams (cross check)
- real corrections:
  - $gg \to gg$: 1530 four-loop diagrams (2 indep. calcs.)
    $\Rightarrow$ 64 “effective” two-loop diagrams (after expansion)



2. Calculation Summary

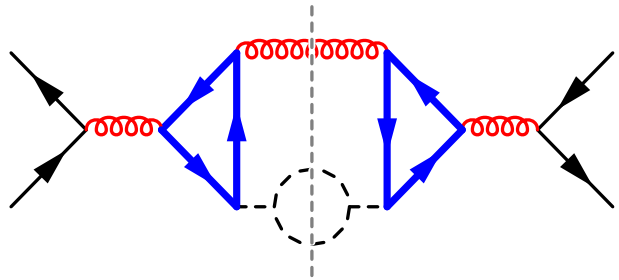
Quark-Gluon Channel:

- $qg \to qg$:
  34 four-loop diagrams
  $\Rightarrow$ 4 “effective” diagrams



Quark-Anti-Quark Channel:

- $q\bar{q} \to q\bar{q}$:
  34 four-loop diagrams
  $\Rightarrow$ 4 “effective” diagrams

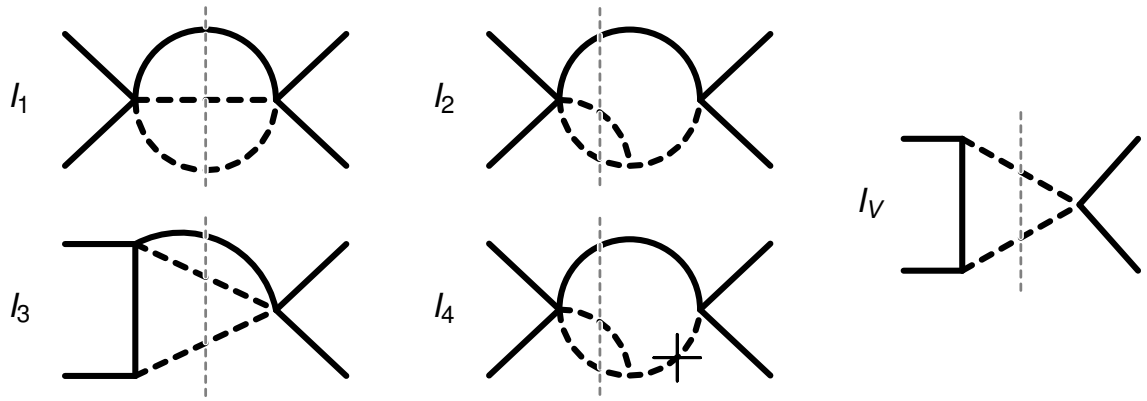


2. Calculation Master Integrals

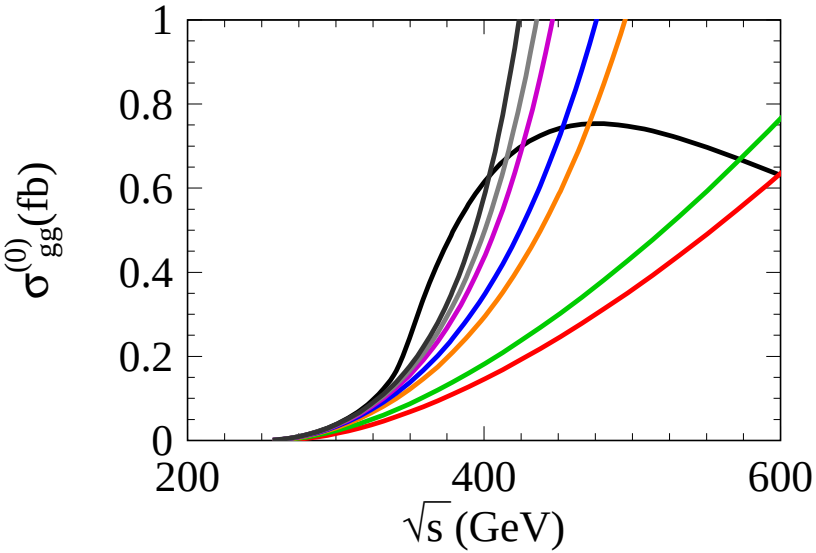
- NLO: 4 real and 1 virtual (+ 2-loop tadpoles)
- phase space integrals depend on $s = (q_1 + q_2)^2$ and m_H
- derive 1-dimensional integral representation: e.g.

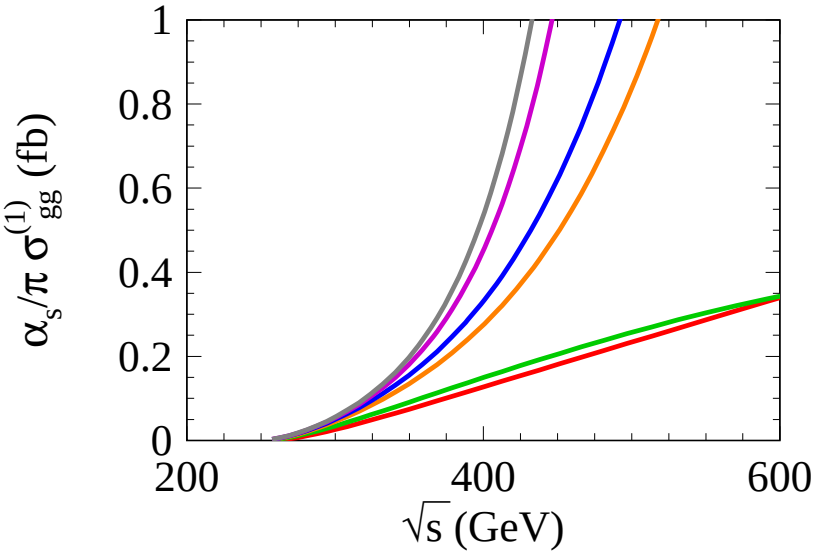
$$I_1 = \mathcal{N} s^{1-2\epsilon} \delta^{5/2-3\epsilon} \int_0^1 \frac{d\mu}{\sqrt{1-\mu\delta}} (1-\mu)^{1/2-\epsilon} \mu^{1-2\epsilon}, \quad \delta = 1 - \frac{4m_H^2}{s}$$

- simplification: expand up to $\mathcal{O}\left(\delta^{100}\right)$
  $\Rightarrow$ very good convergence, small impact on numerics
  $\Rightarrow$ analytic results for partonic cross sections



3. Results Partonic Cross Sections



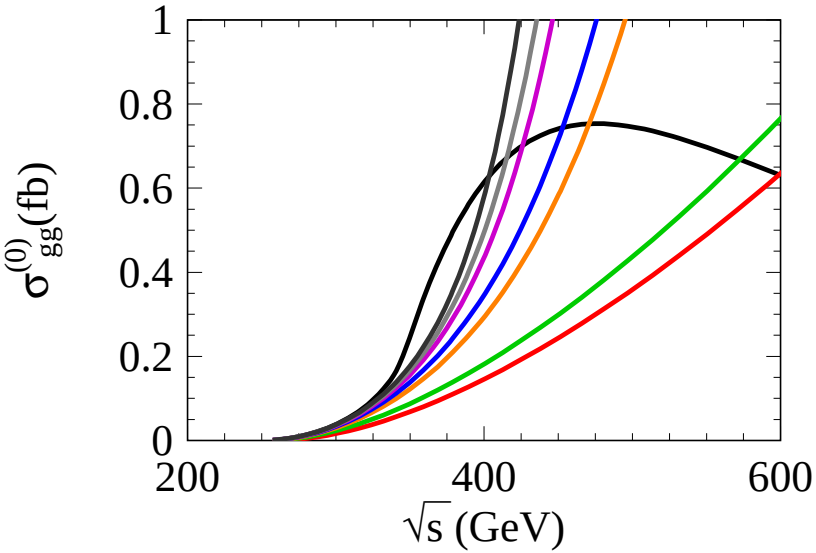


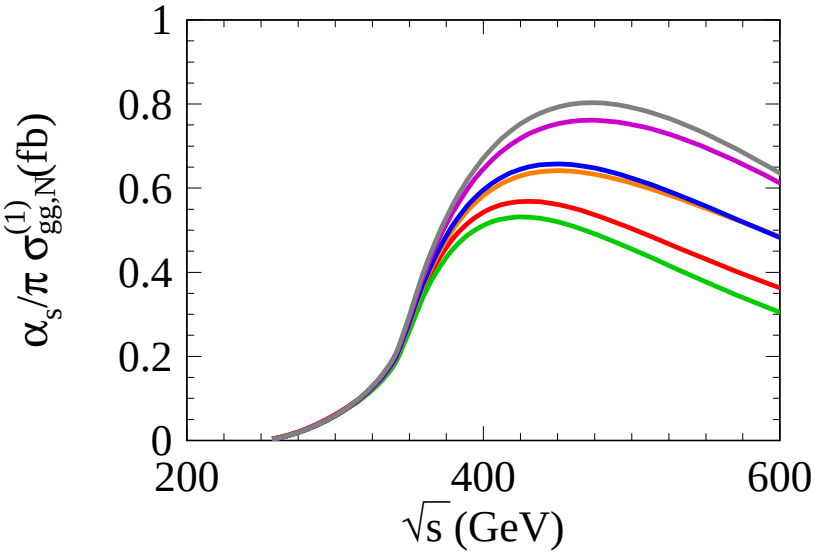
$\rho^0, \rho^1, \rho^2, \rho^3, \rho^4, \rho^5, \rho^6$

Poor Convergence $\Rightarrow$ factorize exact LO cross section

$$\sigma^{\text{NLO}}_{\text{expanded}} \rightarrow \sigma^{\text{LO}}_{\text{exact}} \frac{\sigma^{\text{NLO}}_{\text{expanded}}}{\sigma^{\text{LO}}_{\text{expanded}}}$$

3. Results Partonic Cross Sections





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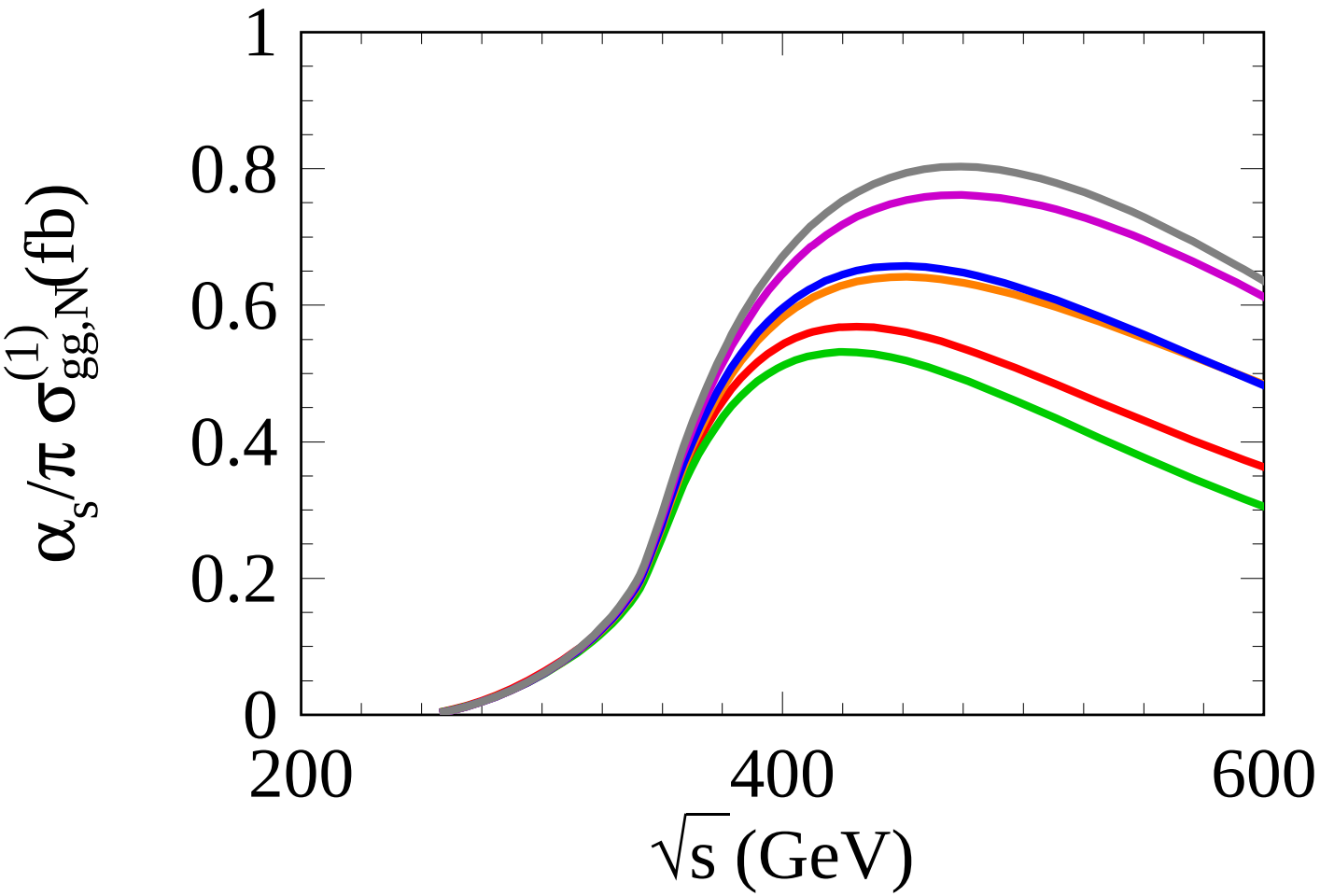
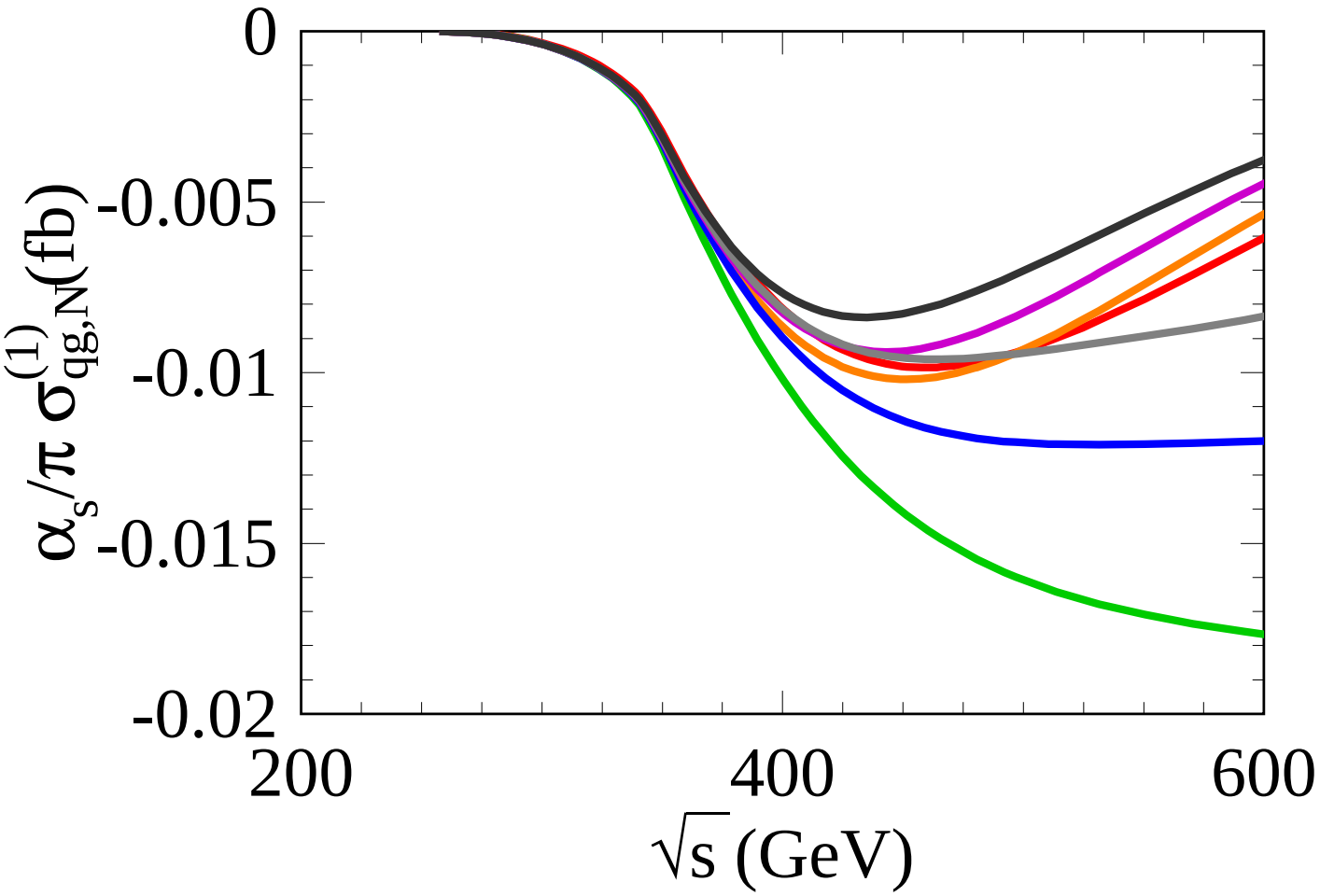
Gluon-Gluon Channel:



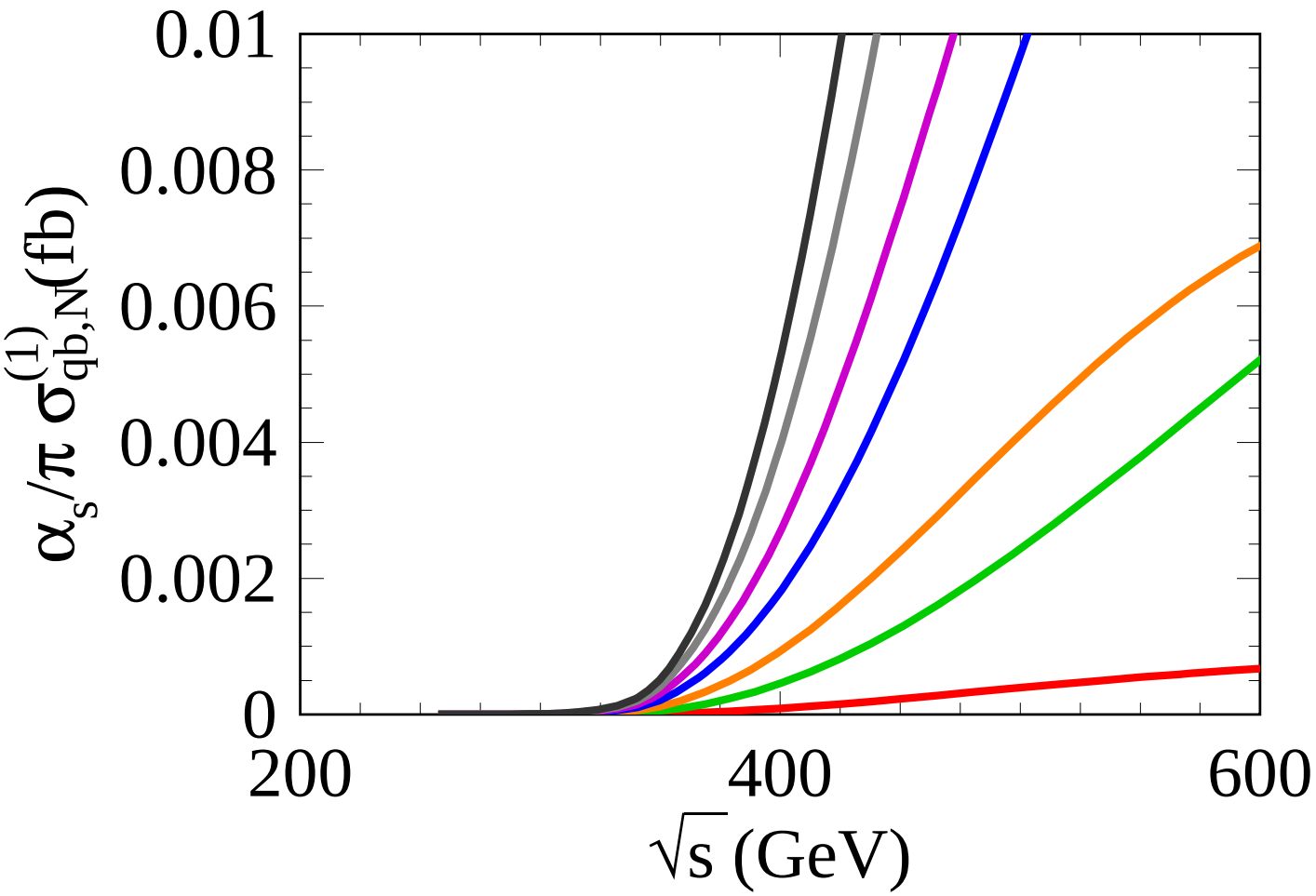
3. Results Partonic Cross Sections

Quark-Gluon Channel:



3. Results Partonic Cross Sections

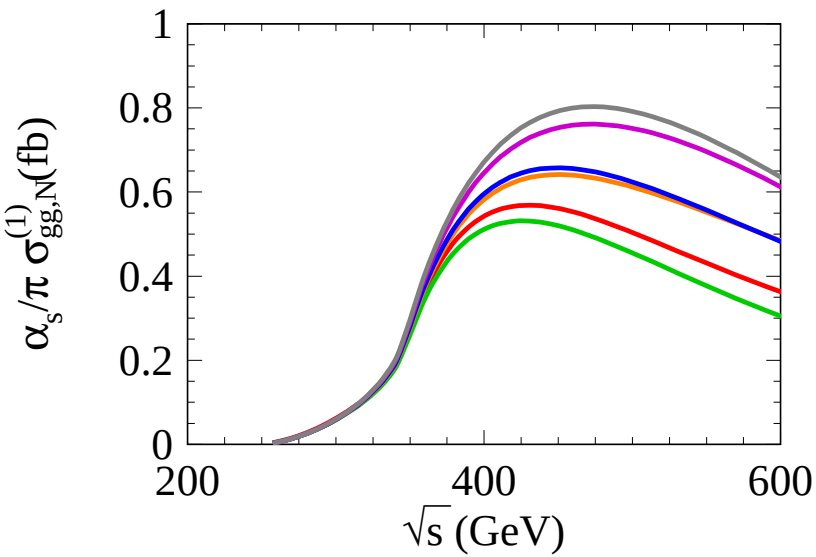
Quark-Anti-Quark Channel:



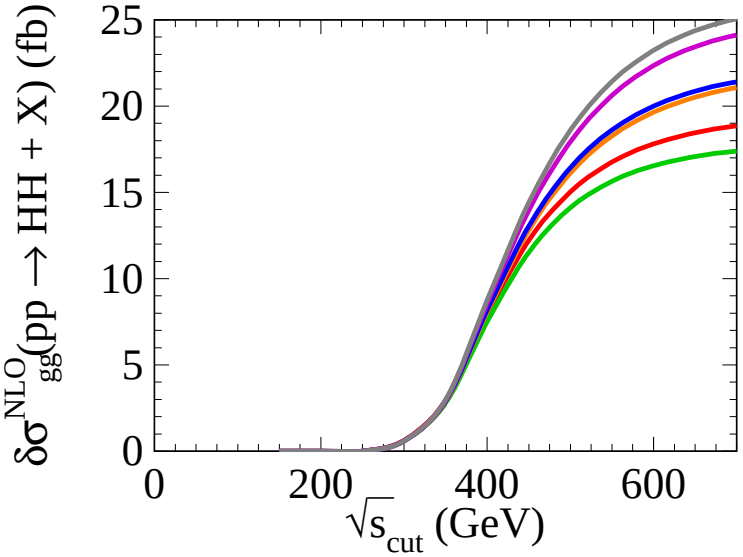
3. Results Hadronic Cross Sections

- MSTW2008 PDFs
- s_{cut} = cut on partonic $s \approx$ invariant mass of Higgs pair

partonic



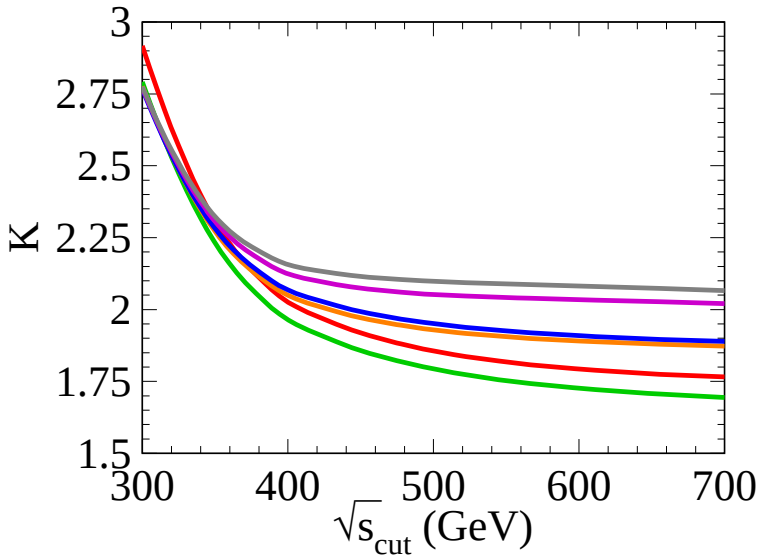
hadronic



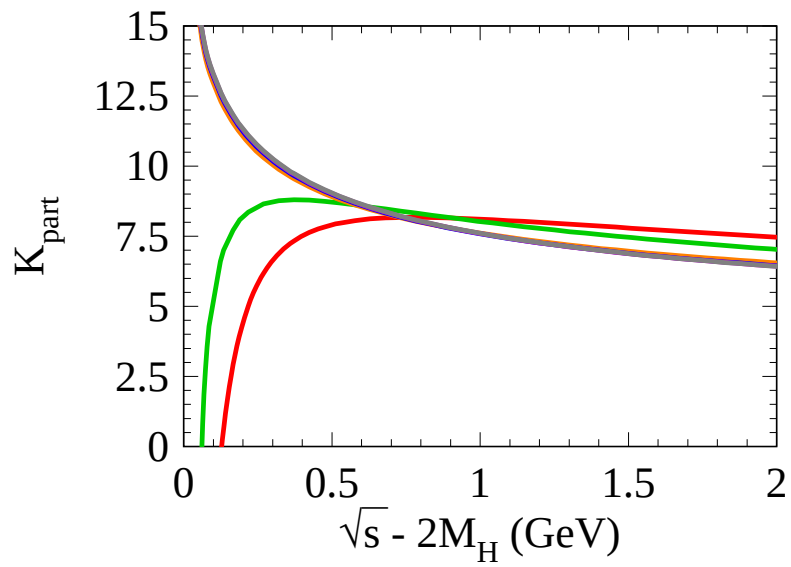
- check stability of $1/M_t$ expansion above threshold
- enhancement of low-s contributions by gluon luminosity

3. Results K-Factors

hadronic

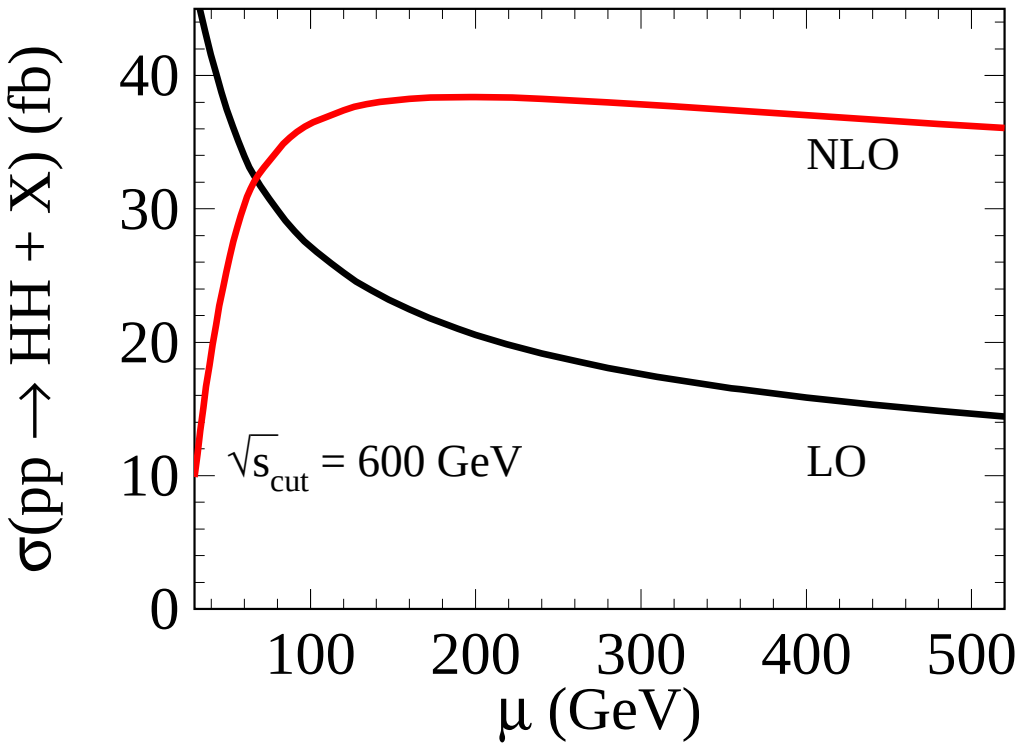


partonic



- large K-factors ($K = \sigma_{\text{NLO}}/\sigma_{\text{LO}} \approx 2 - 3$)
- strong dependence on $\sqrt{s_{\text{cut}}} \lesssim 400$ GeV
- close to threshold strong enhancement
- note: LO cross section suppressed at threshold

3. Results Scale Dependence

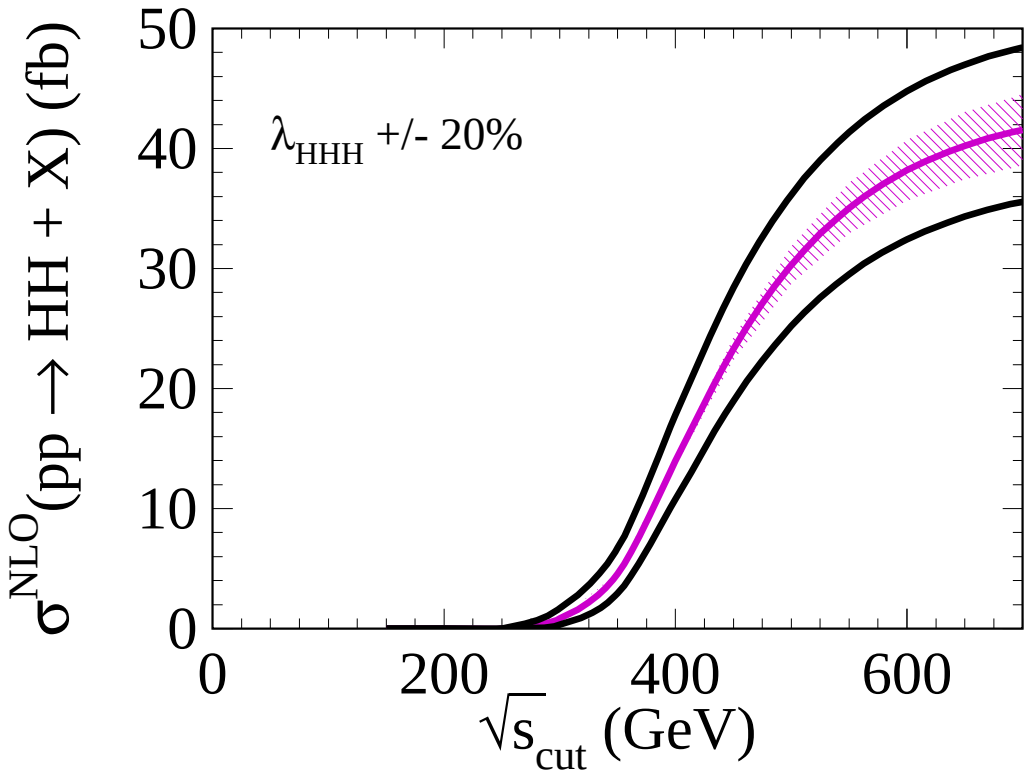


$$\mu = \mu_F = \mu_R$$
$$\mu_{\text{central}} = 2m_H$$
$$\sigma^{\text{LO}} = 18^{+6}_{-4}\ \text{fb}$$
$$\sigma^{\text{NLO}} = 38^{+0}_{-2}\ \text{fb}$$

- NLO curve almost μ independent
- NLO corrections of the same size as LO

$\Rightarrow$ weak μ dependence: misleading error estimate

3. Results Triple Higgs Coupling



violet line:
$\mathcal{O}(\rho^4)$ result

shaded violet area:
$\pm\mathcal{O}(\rho^3)$ result

solid black lines:
variation of λ_{HHH}

- with top mass corrections sufficient for $\mathcal{O}(10\%)$ deviations

4. Conclusion

$\sigma^{\text{NLO}}(pp \to HH)$: top mass corrected, 14 TeV, $\mu = 2m_H$, w/o cut

$$20^{\text{LO}} + 20^{\text{NLO, } M_t \to \infty} \quad \to \quad 20^{\text{LO}} + (26 \pm 8)^{\text{NLO, } 1/M_t^{10}} \quad \text{[fb]}$$

- 1st independent check of $M_t \to \infty$ result

  [Dawson, Dittmaier, Spira; '98]
- analytic results for partonic cross sections
- top mass corrections at NLO up to $\mathcal{O}\left(1/M_t^{10}\right)$
  $\Rightarrow$ reliable estimate for uncertainties
- based on

  [Nucl. Phys. B **875**, 1 (2013); arXiv:1305.7340 [hep-ph]]