Electron contribution to the muon anomalous magnetic moment at four-loop order

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Outline

Anomalous magnetic moment

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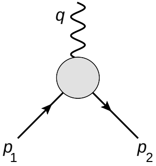

$$= \ -ie\ \bar{\psi}(p_2)\ \left(\gamma^\mu F_E(q^2) + i\frac{\sigma^{\mu\nu} q_\nu}{2m} F_M(q^2)\right)\ \psi(p_1)$$

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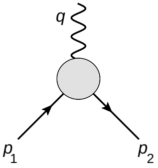

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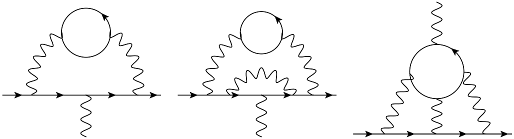
[PDG] $a_\mu^{\text{exp}} = 116592089(63) \cdot 10^{-11}$

$a_\mu^{\text{th}} = 116591803(49) \cdot 10^{-11}$

$\Rightarrow \; \Delta a_\mu = 286(80) \cdot 10^{-11}$

$= 3 - 4 \; \sigma$

Leptonic correction



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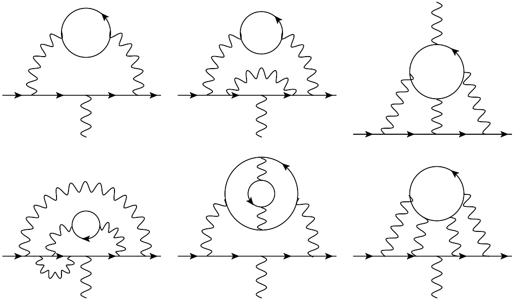

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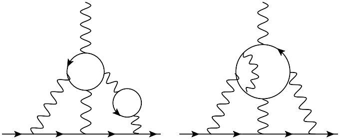

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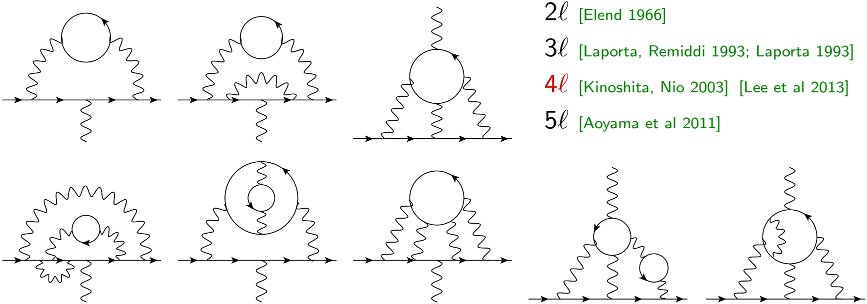

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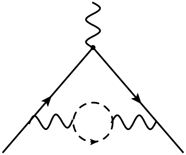
$$a_\mu^{4\ell}(e) = 132.685 \left(\frac{\alpha}{\pi}\right)^4 \approx 386 \cdot 10^{-11} \; > \; \Delta a_\mu = 286 \cdot 10^{-11}$$

[Aoyama, Hayakawa, Kinoshita, Nio 2012]

Asymptotic expansion in m_e/m_μ

- 

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expansion in $m_e \to 0$

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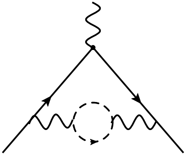
- 

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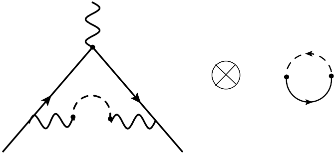
- 

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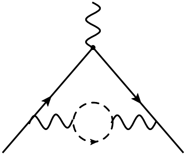
- 

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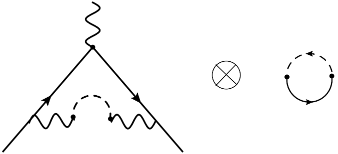
- 

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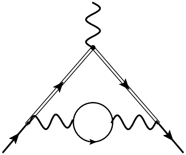
- 

  $\ell_1^2 \approx \ell_2^2 \approx m_e^2$

  $$\frac{1}{(\ell+p)^2 - m_\mu^2} = \frac{1}{\ell^2 + 2\ell p} = \frac{1}{2\ell p} \sum_n \left(\frac{-\ell^2}{2\ell p} \right)^n$$

Calculation of integrals

- treatment of tensor structure and γ-matrices in FORM
  $\Rightarrow$ only scalar integrals left

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$$\int dk \, \frac{\partial}{\partial k_{\mathrm{i}}^{\nu}} k_{\mathrm{j}}^{\nu} f(k^2) = 0 \quad \Rightarrow \quad \text{arb.Integral} = \sum_n c_n \, \text{MasterIntegral}_n$$

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- Evaluation of master integrals
  - Sector decomposition (`FIESTA`)
  - Mellin-Barnes

Results

$$a_\mu^{4\ell}(e) = \sum_i A^i(x) \cdot \left(\frac{\alpha}{\pi}\right)^4, \qquad x = \frac{m_e}{m_\mu}, \quad \ell_x = \log(x)$$

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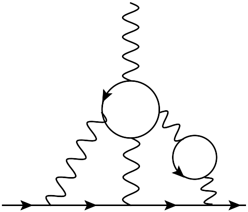

$$\begin{aligned}
A^{\mathrm{IV(a0)}} =& \\
& 7.5018 \pm 0.0026 + 14.8808\ell_x + 6.5797\ell_x^2 \\
& +x[6.29 \pm 0.46 - 14.6216\ell_x + 8.7729\ell_x^2] \\
& +x^2[-16.81 \pm 0.43 + 30.0172\ell_x - 6.5069\ell_x^2 \\
& \qquad +7.6489\ell_x^3 - 0.8889\ell_x^4] \\
& +x^3[-48.31 \pm 0.24 - 4.8739\ell_x + 13.1595\ell_x^2]
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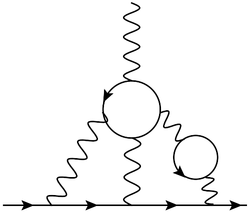

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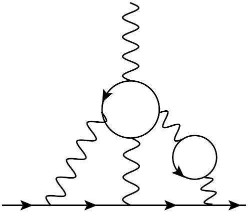

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$$\Rightarrow A^{\mathrm{IV(a0)}} = 116.76 \pm 0.02$$

111.1 ± 8.1 [Calmet, Peterman 1975]

117.4 ± 0.5 [Chlouber, Samuel 1977]

Results

$A(x)$	our work$^{(*)}$	literature$^{(+)}$
IV(a0)	116.76 ± 0.02	116.759183 ± 0.000292
IV(a1)	2.69 ± 0.14	2.697443 ± 0.000142
IV(a2)	4.33 ± 0.17	4.328885 ± 0.000293
IV(b)	-0.38 ± 0.08	-0.4170 ± 0.0037
IV(c)	2.94 ± 0.30	2.9072 ± 0.0044

(*) [Kurz, Liu, Marquard, Smirnov, Smirnov, Steinhauser 2015]

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- uncertainty: $0.4\ (\alpha/\pi)^4 \approx 1.2 \cdot 10^{-11} \ll \Delta a_\mu^{\text{unc.}} = 80 \cdot 10^{-11}$
- other diagram classes will be published soon $\Rightarrow$ full $a_\mu^{4\ell}(e)$

Conclusions

- calculation of electron contribution to a_μ at $\mathcal{O}\left(\alpha^4\right)$
- very good agreement with the known result
- dominant part already published (light-by-light)
- other parts will be published soon