Quantum corrections at the LHC

Ansgar Denner, Würzburg

Workshop des Graduiertenkollegs

Elementarteilchenphysik bei höchster Energie und höchster Präzision

Bad Liebenzell, 30.09. - 02.10.2013

- Lecture 1: Precision tests of the Standard Model
- Lecture 2: NLO Calculations for the LHC

Precision tests of the Standard Model

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- The Standard Model
- Relevance of radiative corrections
- Evaluation of radiative corrections
- Radiative corrections to muon decay
- Precision tests of the Standard Model

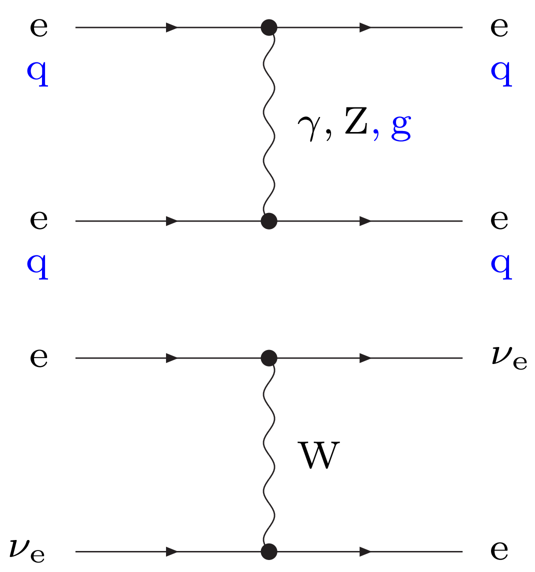
Standard Model

Particle content of Standard Model

Matter particles: fermions (spin 1/2)

generation	leptons	quarks		
1	$\mathrm{e}, \nu_\mathrm{e}$	d,u	d,u	d,u
2	μ, ν_μ	s,c	s,c	s,c
3	τ, ν_τ	b,t	b,t	b,t

interactions: via exchange of spin 1 bosons



photon	electromagnetic interaction
Z boson	neutral weak interaction
W boson	charged weak interaction
gluons	strong interaction

mass generation: via Higgs doublet field

$\Rightarrow$ physical scalar Higgs boson H (spin 0)

Gauge symmetry and field content

SM is a gauge theory: gauge group $\mathrm{SU(3)_c} \times \mathrm{SU(2)_w} \times \mathrm{U(1)_Y}$

field content:

		$\mathrm{SU(3)_c}$	$\mathrm{SU(2)_w}$	I^3_w	Y	Q
$\mathrm{U(1)}_Y$ gauge field:	B_μ	singlet	singlet	0	0	0
$\mathrm{SU(2)_w}$ gauge field:	W^α_μ	singlet	triplet	$0,\pm1$	0	$0,\pm1$
$\mathrm{SU(3)_c}$ gauge field:	G^a_μ	octet	singlet	0	0	0
left-handed leptons:	$\Psi^\mathrm{L}_{L,i} = \begin{pmatrix}\nu^\mathrm{L}_\mathrm{e}\\ \mathrm{e}^\mathrm{L}\end{pmatrix}, \begin{pmatrix}\nu^\mathrm{L}_\mu\\ \mu^\mathrm{L}\end{pmatrix}, \begin{pmatrix}\nu^\mathrm{L}_\tau\\ \tau^\mathrm{L}\end{pmatrix}$	singlet	doublet	$+\frac{1}{2}$ $-\frac{1}{2}$	-1	0 -1
right-handed leptons:	$\psi^\mathrm{R}_{l,i} = \mathrm{e}^\mathrm{R}, \quad \mu^\mathrm{R}, \quad \tau^\mathrm{R}$	singlet	singlet	0	-2	-1
left-handed quarks:	$\Psi^\mathrm{L}_{Q,i} = \begin{pmatrix}\mathrm{u}^\mathrm{L}\\ \mathrm{d}^\mathrm{L}\end{pmatrix}, \begin{pmatrix}\mathrm{c}^\mathrm{L}\\ \mathrm{s}^\mathrm{L}\end{pmatrix}, \begin{pmatrix}\mathrm{t}^\mathrm{L}\\ \mathrm{b}^\mathrm{L}\end{pmatrix}$	triplet	doublet	$+\frac{1}{2}$ $-\frac{1}{2}$	$+\frac{1}{3}$	$+\frac{2}{3}$ $-\frac{1}{3}$
right-handed quarks:	$\psi^\mathrm{R}_{u,i} = \mathrm{u}^\mathrm{R}, \quad \mathrm{c}^\mathrm{R}, \quad \mathrm{t}^\mathrm{R}$	triplet	singlet	0	$+\frac{4}{3}$	$+\frac{2}{3}$
	$\psi^\mathrm{R}_{d,i} = \mathrm{d}^\mathrm{R}, \quad \mathrm{s}^\mathrm{R}, \quad \mathrm{b}^\mathrm{R}$	triplet	singlet	0	$-\frac{2}{3}$	$-\frac{1}{3}$
Higgs doublet:	$\Phi(x) = \begin{pmatrix}\phi^+\\ \frac{1}{\sqrt{2}}(v+H+\mathrm{i}\chi)\end{pmatrix}$	singlet	doublet	$+\frac{1}{2}$ $-\frac{1}{2}$	1	$+1$ 0

right-handed neutrinos ignored

$$Q = I_3 + \frac{1}{2} Y_\mathrm{w}$$

Lagrangian

$$\mathcal{L} = -\frac{1}{4}F_{1,\mu\nu}(x)F_1^{\mu\nu}(x) - \frac{1}{4}F^{\alpha}_{2,\mu\nu}(x)F_2^{\alpha,\mu\nu}(x) - \frac{1}{4}F^{a}_{3,\mu\nu}(x)F_3^{a,\mu\nu}(x)$$

$$+ \sum_i (\mathrm{i}\overline{\Psi^{\mathrm{L}}_{L,i}}\not{D}\Psi^{\mathrm{L}}_{L,i} + \mathrm{i}\overline{\Psi^{\mathrm{L}}_{Q,i}}\not{D}\Psi^{\mathrm{L}}_{Q,i} + \mathrm{i}\overline{\psi^{\mathrm{R}}_{l,i}}\not{D}\psi^{\mathrm{R}}_{l,i} + \mathrm{i}\overline{\psi^{\mathrm{R}}_{u,i}}\not{D}\psi^{\mathrm{R}}_{u,i} + \mathrm{i}\overline{\psi^{\mathrm{R}}_{d,i}}\not{D}\psi^{\mathrm{R}}_{d,i})$$

$$+ (D_\mu\Phi)^\dagger (D^\mu\Phi) - \frac{\lambda}{4}\left(\Phi^\dagger\Phi\right)^2 + \mu^2\Phi^\dagger\Phi$$

$$- \sum_{i,j}\left(\overline{\Psi^{\mathrm{L}}_{L,i}}Y^l_{ij}\psi^{\mathrm{R}}_{l,j}\Phi + \overline{\Psi^{\mathrm{L}}_{Q,i}}Y^u_{ij}\psi^{\mathrm{R}}_{u,j}\tilde{\Phi} + \overline{\Psi^{\mathrm{L}}_{Q,i}}Y^d_{ij}\psi^{\mathrm{R}}_{d,j}\Phi + \mathrm{h.c.}\right)$$

$$F_{1,\mu\nu}(x) = \partial_\mu B_\nu(x) - \partial_\nu B_\mu(x)$$

$$F^{\alpha}_{2,\mu\nu}(x) = \partial_\mu W^{\alpha}_\nu(x) - \partial_\nu W^{\alpha}_\mu(x) + g_2\varepsilon^{\alpha\beta\gamma}W^{\beta}_\mu(x)W^{\gamma}_\nu(x)$$

$$F^{a}_{3,\mu\nu}(x) = \partial_\mu G^{a}_\nu(x) - \partial_\nu G^{a}_\mu(x) - g_3 f^{abc}G^{b}_\mu(x)G^{c}_\nu(x)$$

$$D_\mu = \left(\partial_\mu + \mathrm{i}g_3 T^a G^a_\mu - \mathrm{i}g_2 I^{\alpha}_{\mathrm{w}} W^{\alpha}_\mu + \mathrm{i}g_1\frac{Y_{\mathrm{w}}}{2}B_\mu\right)$$

parameters: g_1, g_2, g_3 gauge couplings
λ, μ parameters of Higgs Potential, Y^f Yukawa coupling matrices

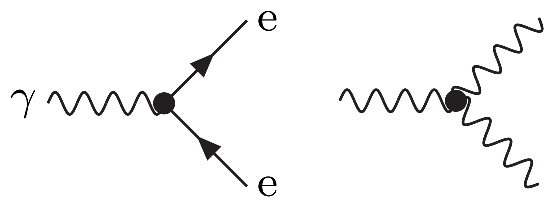
Perturbation theory and Feynman diagrams

Standard Model is defined via its Lagrangian density
predictions are obtained within perturbation theory
Lagrangian is equivalent to Feynman rules

propagators for free fields

etc.

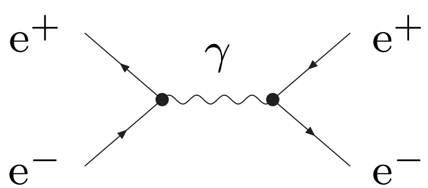
vertices for interactions



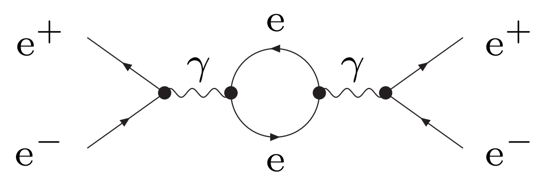
etc.

Feynman diagrams provide an exact formulation of perturbation theory

leading order (LO):
tree diagrams



next-to-leading order (NLO):
loop diagrams



matrix element $= \mathcal{M}_{if} = \langle f|\mathcal{M}|i\rangle \;=\; \sum$ all Feynman diagrams for $|i\rangle \to |f\rangle$

perturbative series = series in coupling constant e or g_s or both
= expansion in # of loops of Feynman diagrams
= expansion in powers of $\hbar$ (quantum corrections)

Feynman rules for Standard Model

Transformation to physical fields (mass eigenstates)

$\hookrightarrow$ Lagrangian in terms of physical fields G^a_μ, A_μ, Z_μ, $W^\pm_\mu$, H, f

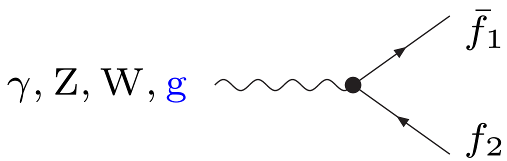
$$A_\mu = \cos\theta_\mathrm{w} B_\mu - \sin\theta_\mathrm{w} W^3_\mu, \qquad Z_\mu = \sin\theta_\mathrm{w} B_\mu + \cos\theta_\mathrm{w} W^3_\mu$$

and parameters g_s, e, M_W, M_Z, M_H, m_f, V_CKM

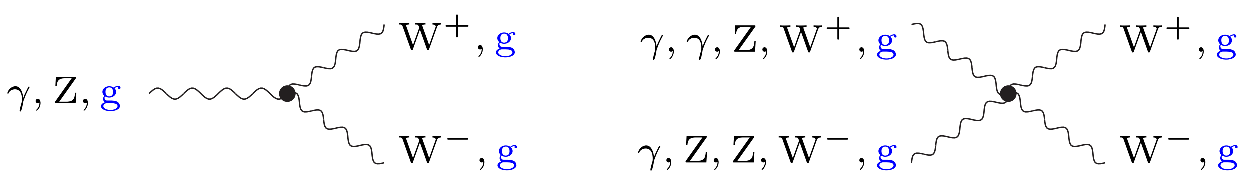
$$\cos\theta_\mathrm{w} = c_\mathrm{w} = M_\mathrm{W}/M_\mathrm{Z}, \qquad \theta_\mathrm{w}\text{: weak mixing angle}$$

$\hookrightarrow$ Feynman rules for physical fields

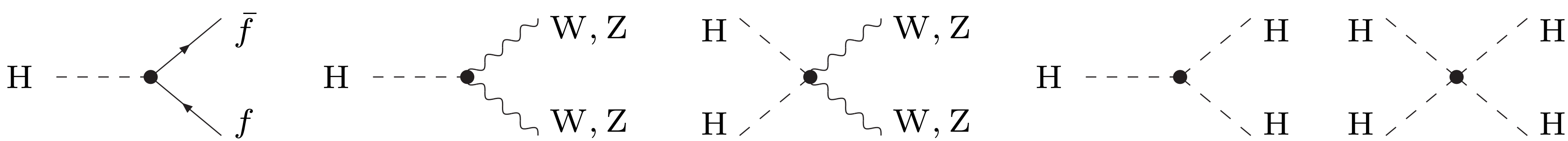
gauge-boson–fermion couplings



gauge-boson self-interactions



Higgs couplings



Relevance of radiative corrections

Situation at LEP

e^+e^- collider 1989–2000

events

1.5×10^7 $e^+e^-\to Z\to$ hadrons, 1.7×10^6 $e^+e^-\to Z\to$ leptons
4.8×10^4 $e^+e^-\to W^+W^-$

typical accuracies $\lesssim 0.1\%$

$e^+e^-\to Z\to$ hadrons: $\Delta\sigma/\sigma\sim 0.09\%$
$\Delta\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}\sim 0.03\%$
$e^+e^-\to W^+W^-$: $\Delta\sigma/\sigma\sim 1\%$
$\Delta M_W/M_W\sim 0.05\%$

predictions are calculated within perturbation theory

expansion parameter: $\alpha/\pi\sim 0.23\%$, $\alpha/(\pi\sin^2\theta_w)\sim 1\%$
generic one-loop corrections: $\sim 2\%$
generic two-loop corrections: $\sim 0.05\%$

implications

complete one-loop electroweak radiative corrections (EWRC) mandatory
for $e^+e^-\to Z$ (at least leading) two-loop corrections needed

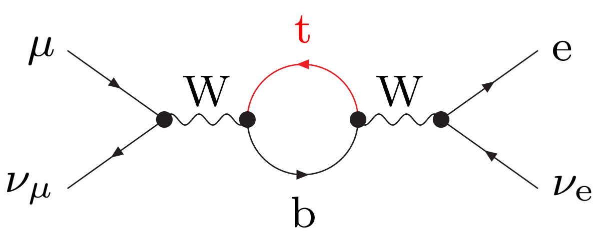
Relevance of electroweak radiative corrections

Mandatory for precision tests of the electroweak (EW) Standard Model (SM)

$$\sim \left[\frac{\alpha}{\pi} \ldots \frac{\alpha}{\pi} \ln \frac{E^2}{m_\mathrm{e}^2}\right] \times \mathcal{O}(1) \sim [0.2\% \ldots 6\%] \times \mathcal{O}(1) \text{ for } E = 100\,\mathrm{GeV}$$

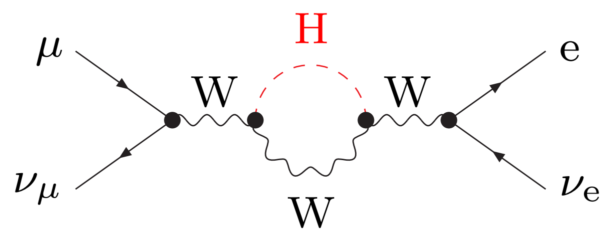
depend on all details of the theory

- top quark



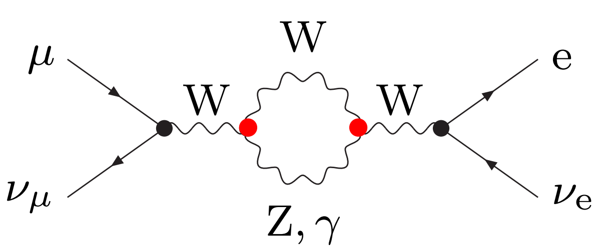
$$\sim \frac{\alpha}{\pi} \frac{m_\mathrm{t}^2}{s_\mathrm{w}^2 M_\mathrm{W}^2} \approx 0.05$$

- Higgs boson

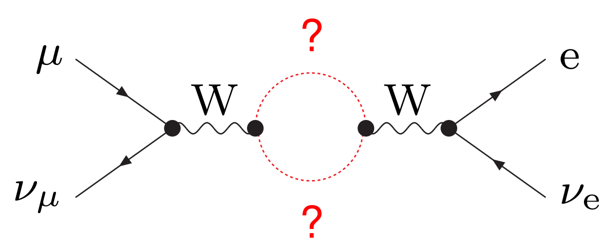


$$\sim \frac{\alpha}{\pi} \ln \frac{M_\mathrm{H}}{M_\mathrm{W}}$$

- gauge-boson self couplings



- New physics (supersymmetry) ?



$\Rightarrow$ allow for indirect experimental tests of not directly accessible quantities

Evaluation of radiative corrections

Perturbative evaluation of quantum field theories

Formulate theory: Lagrangian

$\Downarrow$

quantization $\rightarrow$ gauge fixing, Faddeev–Popov ghosts

$\Downarrow$

perturbative evaluation: Feynman rules

$\Downarrow$

Feynman graphs

$\Downarrow$

loop integrals $\rightarrow$ technical problem: divergences (UV, IR)

$\Downarrow$

regularization $\rightarrow$ divergences mathematically meaningful

$\Downarrow$

define input parameters: renormalization $\rightarrow$ absorbs UV divergences

$\Downarrow$

theoretical predictions: calculation of observables (cross sections, decay widths, etc.)

$\hookrightarrow$ IR divergences cancel for sufficiently inclusive quantities (e.g. inclusion of photon bremsstrahlung)

Loop integrals and regularization

Observation: loop integrals involve divergences

- ultraviolet (UV) divergences for $q \to \infty$ (large momenta), e.g.:

$$\int \mathrm{d}^4 q \, \frac{1}{(q^2 - m_0^2)((q+p)^2 - m_1^2)} \sim \int \frac{\mathrm{d}q}{q} \text{ for } q \to \infty \qquad \to \text{logarithmic divergence}$$

- infrared (IR) divergences for $q \to q_0$ (small/collinear momenta), e.g.:

$$\int \mathrm{d}^4 q \, \frac{1}{q^2(q^2 + 2qp_1)(q^2 + 2qp_2)} \sim \int \frac{\mathrm{d}q}{q} \text{ for } q \to 0 \qquad \to \text{logarithmic divergence}$$

regularization: extension of theory by free parameter δ (cut-off) such that

- original theory is obtained as limiting case $\delta \to \delta_0$
- integrals (and thus the theory) become finite, i.e. well defined for $\delta \neq \delta_0$

$\hookrightarrow$ fix input parameters x_i of regularised theory ($\delta \neq \delta_0$) by experiment

$\Rightarrow$ observables must have finite limit $\delta \to \delta_0$ as functions of x_i

(independent of regularization scheme)

relations between physical quantities should be finite and independent of cut-off

if true: theory is renormalizable

Convenient regularization schemes

Cut-off regularization: require $q_0^2 + \mathbf{q}^2 < \Lambda^2$ in momentum space

- UV divergences appear as $\log \Lambda^2, \Lambda^2, \ldots$
- breaks Lorentz invariance and gauge invariance $\Rightarrow$ not used

dimensional regularization: switch to $D \neq 4$ space-time dimensions

- regularises UV and IR divergences, respects gauge invariance, easy use
- prescription: (μ = arbitrary reference mass, drops out in observables)

$$\int \mathrm{d}^4 q \;\to\; (2\pi\mu)^{4-D} \int \mathrm{d}^D q$$ and D-dim. momenta, metric, Dirac algebra

and analytic continuation to continuous complex D !

- divergences appear as poles $\dfrac{1}{4-D}$

$\hookrightarrow$ define $\Delta \equiv \dfrac{2}{4-D} - \gamma_{\mathrm{E}} + \ln(4\pi) = \dfrac{2}{4-D} +$ const.

IR regularization by infinitesimal photon mass m_γ

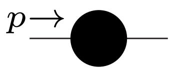
and (if relevant) by small fermion mass m_f

- prescription: photon propagator pole $\dfrac{1}{q^2} \to \dfrac{1}{q^2 - m_\gamma^2}$
- divergences appear as $\ln(m_\gamma)$ and $\ln(m_f)$ terms

Need for renormalization

Propagators and 2-point functions:

structure of one-loop self-energies (scalar case as example):



$$= \Sigma(p^2) = C_1\, p^2\, \Delta + C_2\, \Delta + \Sigma_{\mathrm{finite}}(p^2) = \text{UV divergent}$$

behaviour of propagator near pole for free propagation:

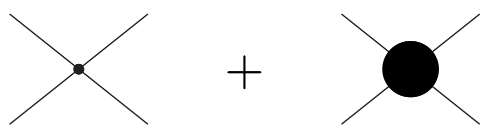
$$G^{\phi\phi}(p^2) = \frac{\mathrm{i}}{p^2 - m^2 + \Sigma(p^2)} \underset{p^2 \to m^2}{\widetilde{\quad}} \frac{1}{1 + \Sigma'(m^2)} \frac{\mathrm{i}}{p^2 - m^2 + \Sigma(m^2)}$$

$\hookrightarrow$ higher-order corrections change location and residue of propagator pole !

interaction vertices:

example: scalar 4-point interaction $\mathcal{L}_{\phi^4} = -\lambda\phi^4/4!$

$$\Gamma^{\phi\phi\phi\phi}(p_1, p_2, p_3) = -\mathrm{i}\lambda + \mathrm{i}\Lambda\,(p_1, p_2, p_3)$$



momentum-dependent one-loop correction:

$$\Lambda\,(p_1, p_2, p_3) = C_3\Delta + \Lambda_{\mathrm{finite}}\,(p_1, p_2, p_3) = \text{UV divergent}$$

$\hookrightarrow$ higher-order corrections change coupling strength !

Structure of UV divergences

- Renormalizable field theories:

  UV divergences in vertex functions have analytical form of elementary vertex structures (directly related to $\mathcal{L}$)

  $\hookrightarrow$ idea: absorb divergences in free parameters

  $\Rightarrow$ reparametrization of theory = renormalization

  different types of renormalizable theories:

  - ► theories with unrelated couplings of non-negative mass dimensions

    $\hookrightarrow$ renormalizability proven by power counting and “BPHZ procedure”

  - ► gauge theories (couplings unified by gauge invariance)

    $\hookrightarrow$ renormalizability non-trivial consequence of gauge symmetry ‘t Hooft ’71

- non-renormalizable field theories:

  e.g. theories with couplings of negative mass dimensions
  (cf. Fermi model, effective field theories)

  operators of higher and higher mass dimensions needed to absorb UV divergences in higher orders

  $\hookrightarrow$ infinitely many free parameters, much less predictive power

Practical procedure for renormalization

Consider original ("bare") parameters and fields as preliminary
(denoted with subscripts "0" in the following)

$\hookrightarrow$ introduce new "renormalized" parameters and fields that obey certain conditions

propagators and 2-point functions:

- mass renormalization: $m_0^2 = m^2 + \delta m^2$,
  $m^2 \stackrel{!}{=}$ location of propagator pole $=$ "physical mass" $\quad \to \; \delta m^2 = \Sigma(m^2)$
- field renormalization: rescale fields $\phi_0 = \sqrt{Z_\phi}\phi$, $\quad G^{\phi\phi} = Z_\phi^{-1} G^{\phi_0\phi_0}$
  fix $Z_\phi = 1 + \delta Z_\phi$ such that residue of $G^{\phi\phi}$ at $p^2 = m^2$ equals 1
  $\hookrightarrow \; \delta Z_\phi = -\Sigma'(m^2)$

$\Rightarrow$ renormalized propagator $G^{\phi\phi}$ is UV finite:

$$G^{\phi\phi}(p^2) = \frac{\mathrm{i}}{p^2 - m^2 + \Sigma_{\mathrm{ren}}(p^2)} = \frac{\mathrm{i}}{Z_\phi[p^2 - m_0^2 + \Sigma(p^2)]}$$

renormalized self energy

$$\Sigma_{\mathrm{ren}}(p^2) = \Sigma(p^2) - \delta m^2 + (p^2 - m^2)\delta Z_\phi \; = \; \Sigma(p^2) - \Sigma(m^2) - (p^2 - m^2)\Sigma'(m^2)$$
$$= \Sigma_{\mathrm{finite}}(p^2) - \Sigma_{\mathrm{finite}}(m^2) - (p^2 - m^2)\Sigma'_{\mathrm{finite}}(m^2) \; = \; \text{UV finite}$$

Practical procedure for renormalization

Vertex functions for interactions:

- coupling-constant renormalization: $\lambda_0 \;=\; \lambda + \delta\lambda$

  fix $\delta\lambda$ such that λ assumes a measured value for special kinematics p_i^{exp} (renormalization point)

  note: $\Gamma^{\phi\phi\phi\phi} \;=\; Z_\phi^2 \Gamma^{\phi_0\phi_0\phi_0\phi_0}$

  $\hookrightarrow\; \delta\lambda = -2\delta Z_\phi \lambda - \Lambda(p_1^{\mathrm{exp}}, p_2^{\mathrm{exp}}, p_3^{\mathrm{exp}})$

$\Rightarrow$ renormalized vertex function is UV finite:

$$\Gamma^{\phi\phi\phi\phi}(p_1, p_2, p_3) = -\mathrm{i}\lambda + \mathrm{i}\Lambda_{\mathrm{ren}}(p_1, p_2, p_3),$$

$$\begin{aligned}\Lambda_{\mathrm{ren}}(p_1, p_2, p_3) &= \Lambda(p_1, p_2, p_3) - \delta\lambda - 2\delta Z_\phi \lambda \\ &= \Lambda_{\mathrm{finite}}(p_1, p_2, p_3) \;-\; \Lambda_{\mathrm{finite}}(p_1^{\mathrm{exp}}, p_2^{\mathrm{exp}}, p_3^{\mathrm{exp}}) \;=\; \text{UV finite}\end{aligned}$$

$\hookrightarrow$ all divergences of ϕ^4 theory can be absorbed by renormalization of parameters (masses and coupling constants) and the fields (counterterms: $\delta m, \delta\lambda, \delta Z_\phi$)

Any physical observable calculated in terms of renormalized parameters is finite and well-defined (although affected by a systematic perturbative uncertainty).

Renormalization of the Standard Model

Bare input parameters: $e_0, M_{\mathrm{W},0}, M_{\mathrm{Z},0}, M_{\mathrm{H},0}, m_{f,0}, V_{ij,0}, g_{\mathrm{s},0}$

renormalization transformation:

- parameter renormalization:

$$e_0 = (1+\delta Z_e)e, \qquad g_{\mathrm{s},0} = (1+\delta Z_{g_\mathrm{s}})g_\mathrm{s},$$

$$M^2_{\mathrm{W},0} = M^2_\mathrm{W} + \delta M^2_\mathrm{W}, \quad M^2_{\mathrm{Z},0} = M^2_\mathrm{Z} + \delta M^2_\mathrm{Z}, \quad M^2_{\mathrm{H},0} = M^2_\mathrm{H} + \delta M^2_\mathrm{H},$$

$$m_{f,0} = m_f + \delta m_f, \qquad V_{ij,0} = V_{ij} + \delta V_{ij}, \quad \text{(both } V_{ij,0}, V_{ij} \text{ unitary)}$$

note: renormalization of $c_\mathrm{w}, s_\mathrm{w}$ fixed by on-shell condition $c_\mathrm{w} = M_\mathrm{W}/M_\mathrm{Z}$
(s_w is not a free parameter if M_W, M_Z are used as input parameters)

- field renormalization: (physical fields)

$$W_0^\pm = \sqrt{Z_W}\,W^\pm, \quad \begin{pmatrix} Z_0 \\ A_0 \end{pmatrix} = \begin{pmatrix} \sqrt{Z_{ZZ}} & \sqrt{Z_{ZA}} \\ \sqrt{Z_{AZ}} & \sqrt{Z_{AA}} \end{pmatrix} \begin{pmatrix} Z \\ A \end{pmatrix}, \quad H_0 = \sqrt{Z_H}\,H,$$

$$\psi^\mathrm{L}_{f,0} = \sqrt{Z^\mathrm{L}_{ff'}}\,\psi^\mathrm{L}_{f'}, \qquad \psi^\mathrm{R}_{f,0} = \sqrt{Z^\mathrm{R}_{ff'}}\,\psi^\mathrm{R}_{f'}, \qquad G^a_0 = \sqrt{Z_G}\,G^a$$

note: matrix renormalization necessary to account for loop-induced mixing

Renormalization conditions

- Mass renormalization:

  on-shell definition: mass^2 is location of pole in propagator

  $\hookrightarrow \ \delta M_\mathrm{W}^2 = \mathrm{Re}\{\Sigma_\mathrm{T}^W(M_\mathrm{W}^2)\}$, similar expressions for $\delta M_\mathrm{Z}^2, \delta M_\mathrm{H}^2, \delta m_f$

  note: ◇ location of pole is complex for unstable particles

  $\hookrightarrow$ important for processes with unstable-particle production

  (gauge-invariant definition: mass^2 as real part of pole location)

  ◇ other definitions of quark masses often more appropriate

  (e.g. running masses)

- field renormalization: (bosons and fermions)

  ► residues of propagators (diagonal, transverse parts) normalized to 1

  $\hookrightarrow \ \delta Z_W = -\mathrm{Re}\{\Sigma_\mathrm{T}^{W\,\prime}(M_\mathrm{W}^2)\}$,

  similar expressions for $\delta Z_{AA}, \delta Z_{ZZ}, \delta Z_H, \delta Z_{ff}^{\mathrm{L/R}}$

  ► suppression of mixing propagators on particle poles

  physical on-shell particles do not mix

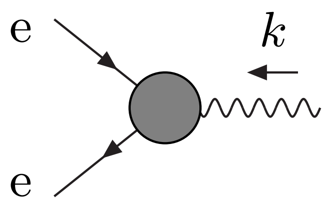
  $\hookrightarrow$ fixes non-diagonal constants $\delta Z_{AZ}, \delta Z_{ZA}, \delta Z_{ff'}^{\mathrm{L/R}} \ (f \neq f')$

  note: problems for unstable particles beyond one loop

  (field-renormalization constants become complex)

Renormalization conditions (cont.)

- Charge renormalization: define e in Thomson limit (low-energy-elastic scattering)



$A_\mu \xrightarrow{k\to 0} \mathrm{i}e\gamma_\mu$ for on-shell electrons

$\hookrightarrow e =$ elementary charge of classical electrodynamics

fine-structure constant $\alpha(0) = \dfrac{e^2}{4\pi} = 1/137.03599976$

gauge invariance relates δZ_e to photon field renormalization:

$$\delta Z_e = -\frac{1}{2}\delta Z_{AA} - \frac{s_\mathrm{w}}{2c_\mathrm{w}}\delta Z_{ZA} \qquad \text{(at one loop)}$$

- CKM-matrix renormalization $\to$ fixes δV_{ij}

  rotation to mass eigenstates;
  CKM part requires a careful (non-trivial) investigation
  of mixing energies, mass eigenstates, LSZ reduction, etc.

general result: all renormalization constants can be obtained from self-energies.

different renormalization schemes are possible.

Renormalization of strong coupling

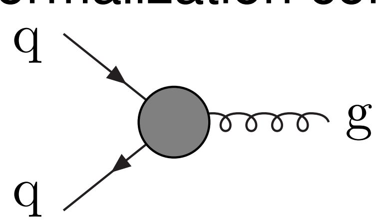
- QCD becomes strongly interacting for small momentum transfers
- quarks and gluons are confined in hadrons

$\hookrightarrow$ on-shell renormalization makes no sense
$\hookrightarrow$ no obvious renormalization point for strong coupling

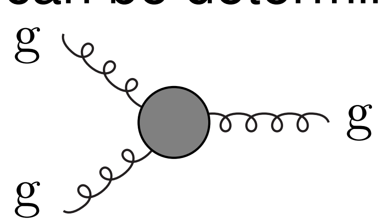
renormalization schemes for QCD

- momentum subtraction scheme (MOM): choose symmetric Euclidean renormalization point, $p_i^2 = -M^2$, for vertex functions
- minimal subtraction scheme (MS):
  subtract only UV divergences $1/(D-4)$
- modified minimal subtraction scheme ($\overline{\mathrm{MS}}$): commonly used
  subtract Δ (UV-divergences plus some universal constants)
  renormalized results depend on scale μ of dimensional regularization
  $\Rightarrow$ renormalization-scale dependence of observables

renormalization constant δg_{s} can be determined from any one vertex function



or

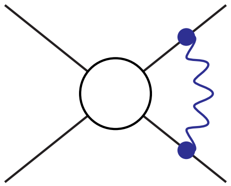


or ...

IR divergences and photon bremsstrahlung

Consider processes with charged external particles, e.g., $\mathrm{e}^+\mathrm{e}^- \to \mu^+\mu^-$

- virtual corrections: loop diagrams

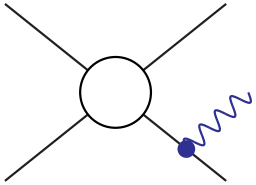


IR divergences from soft virtual photons ($q \to 0$)

$$\int \frac{\mathrm{d}^4 q}{(q^2 - m_\gamma^2)(q^2 + 2qp_1)(q^2 + 2qp_2)} \to C \ln(m_\gamma)$$

- “real” corrections: photon bremsstrahlung = “real” radiative corrections

$$\int \frac{\mathrm{d}^3\mathbf{q}}{2q_0} \left| \quad \right|^2$$



IR divergences from soft real photons ($\mathbf{q} \to 0$)

$$\int \frac{\mathrm{d}^3\mathbf{q}}{\sqrt{\mathbf{q}^2 + m_\gamma^2}(2qp_1)(2qp_2)} \to -C \ln(m_\gamma)$$

Bloch–Nordsieck theorem:

IR divergences of virtual and real corrections cancel in the sum

$\hookrightarrow$ virtual and soft-photonic corrections cannot be discussed separately
are related due to limited experimental resolution of soft photons

$\hookrightarrow$ predictions depend on treatment of photon emission (energy and angular cuts)

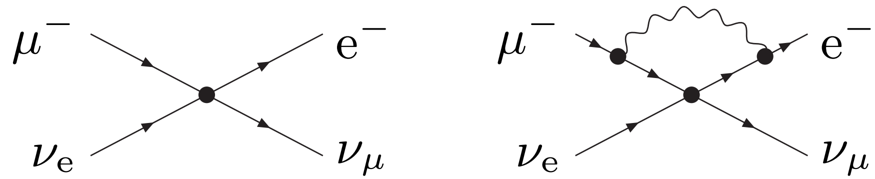
for massless charged particles (QCD) $\Rightarrow$ additional collinear singularities

Radiative corrections to muon decay

Gauge-boson-mass relation

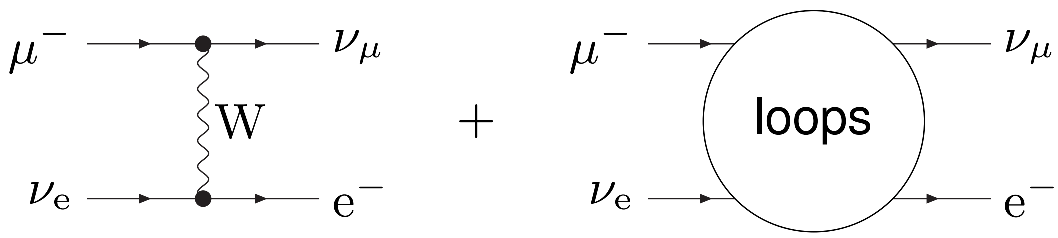
M_W and M_Z correlated via muon lifetime $\leftrightarrow$ Fermi constant G_μ

Fermi model



$$\tau_\mu^{-1} = \frac{G_\mu^2 m_\mu^5}{192\pi^3}\left(1 - 8\frac{m_\mathrm{e}^2}{m_\mu^2}\right)(1+\delta_\mathrm{QED})$$

Standard Model



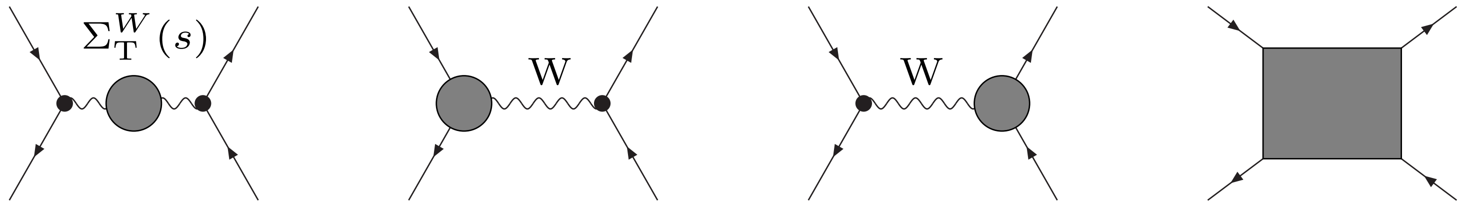
$$G_\mu = \frac{\pi\alpha}{\sqrt{2}}\frac{1}{M_\mathrm{W}^2(1-M_\mathrm{W}^2/M_\mathrm{Z}^2)}\frac{1}{1-\Delta r}$$

Δr: calculable quantum corrections

(beyond electromagnetic corrections to Fermi model)

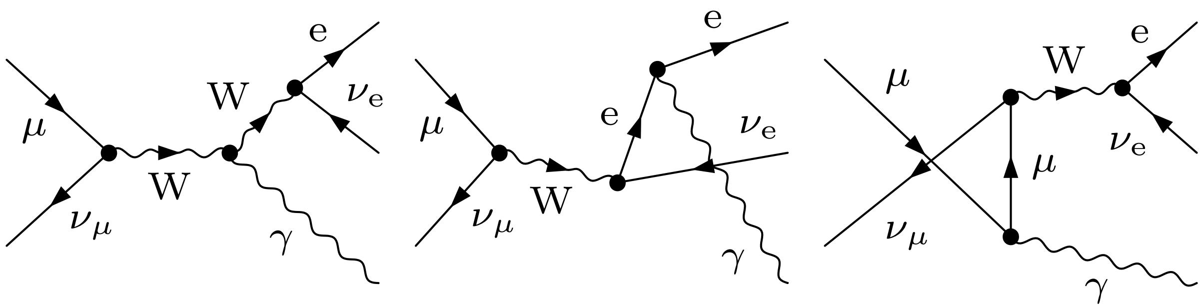
Electroweak corrections to μ decay

Virtual correction – one-loop diagrams:



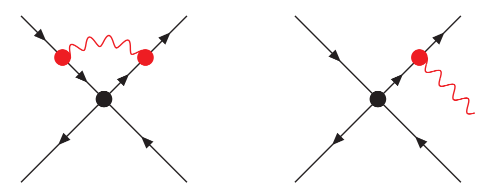
W self-energy $\qquad$ $\mathrm{W}l\nu_l$ vertex correction $\qquad$ box diagrams

real correction – one-photon bremsstrahlung:



consistent use of G_μ:

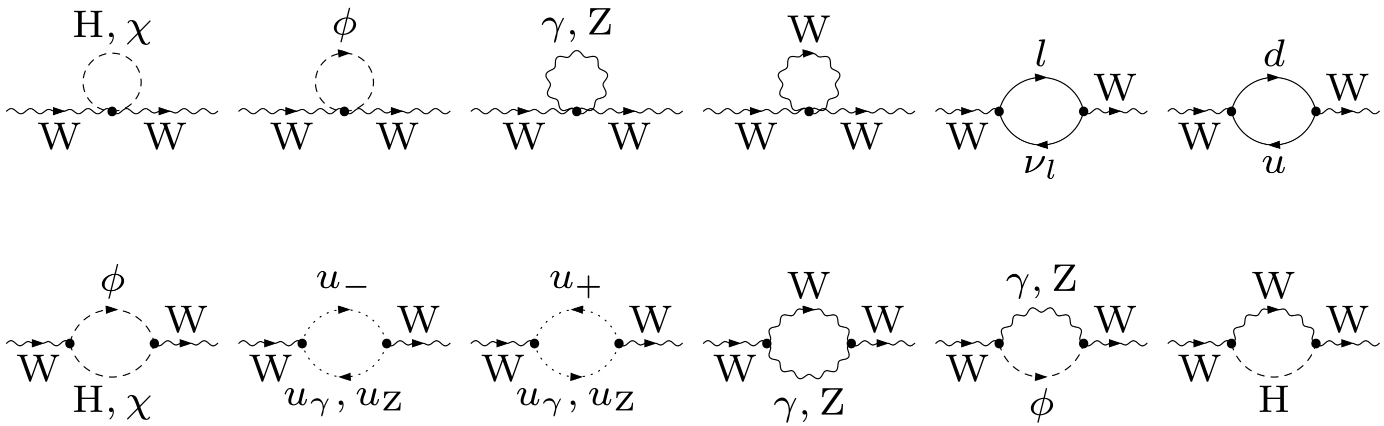
photonic QED corrections are treated in the Fermi model and subtracted from Δr



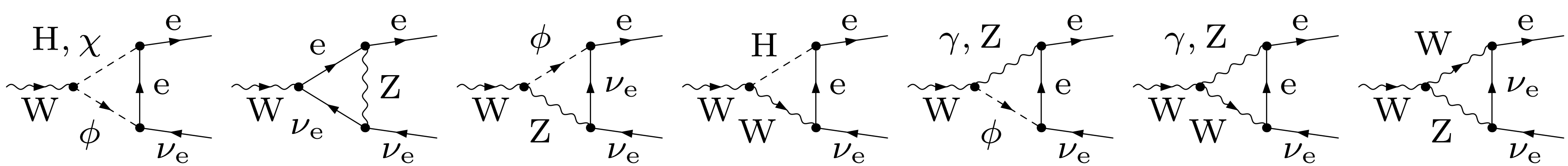
etc.

Feynman diagrams for vertex functions

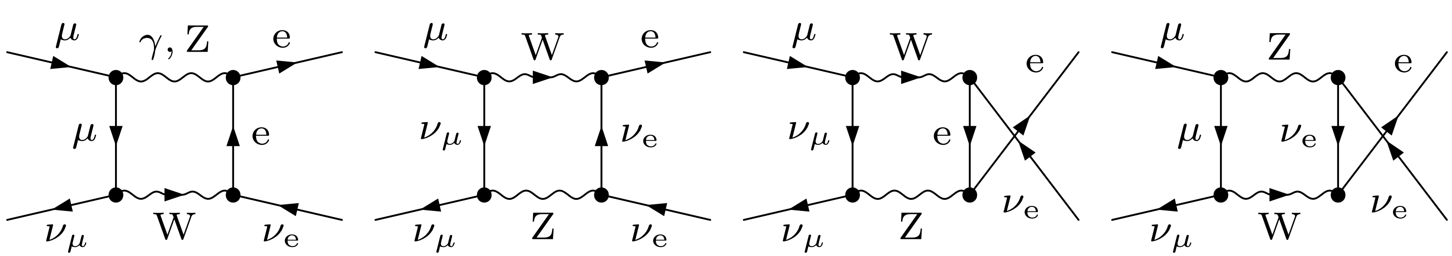
W-boson self-energy:



$\mathrm{We}\nu_\mathrm{e}$ vertex correction:



box diagrams:



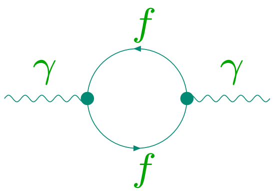
Structure of electroweak corrections to μ decay

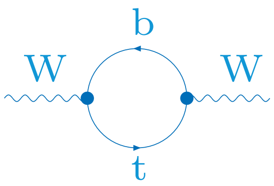
$\mathcal{O}(\alpha)$ corrections: Sirlin '80, Marciano, Sirlin '80

$$\Delta r_{\text{1-loop}} = \Delta\alpha(M_Z^2) - \frac{c_w^2}{s_w^2}\Delta\rho + \Delta r_{\text{rem}}(M_H)$$

$$\sim 6\% \qquad \sim 3\% \qquad \sim 1\%$$

$$\alpha\ln(m_f/M_Z) \qquad G_\mu m_t^2 \qquad \alpha\ln(M_H/M_Z)$$





- $\Delta\alpha(M_Z^2)$: contribution of running electromagnetic coupling

  $\Delta\alpha(M_Z^2) \sim \frac{\alpha}{3\pi}\sum_f Q_f^2 \ln\frac{M_Z^2}{m_f^2}$

  $\hookrightarrow$ large effects from small fermion masses

- $\Delta\rho$: leading corrections to the ρ-parameter

$$\Delta\rho_{\text{top}} \sim \left(\frac{\Sigma_T^{ZZ}(0)}{M_Z^2} - \frac{\Sigma_T^{WW}(0)}{M_W^2}\right)_{\text{top}} \sim \frac{3G_\mu m_t^2}{8\sqrt{2}\pi^2}$$

$\hookrightarrow$ large effects from top–bottom loops in W self-energy Veltman '77

Hadronic contributions

$$\Delta\alpha(s) = -\,\mathrm{Re}\{\Sigma^{AA}_{\mathrm{T,ren}}(s)/s\} = -\,\mathrm{Re}\{\Sigma^{AA}_{\mathrm{T}}(s)/s\} + (\Sigma^{AA}_{\mathrm{T}})'(0)$$

$$= \Delta\alpha_{\mathrm{lept}}(s) + \Delta\alpha^{(5)}_{\mathrm{had}}(s) + \Delta\alpha_{\mathrm{top}}(s) \sim \frac{\alpha}{3\pi}\sum_f Q_f^2 \ln\frac{M_\mathrm{Z}^2}{m_f^2}$$

$\Delta\alpha^{(5)}_{\mathrm{had}}$ becomes sensitive to unphysical quark masses m_q

$\hookrightarrow$ $\Delta\alpha^{(5)}_{\mathrm{had}}$ not calculable in perturbation theory

solution: $\Delta\alpha^{(5)}_{\mathrm{had}}$ obtainable from fit to experimental data with subtracted dispersion relation

$$\Delta\alpha^{(5)}_{\mathrm{had}} = -\frac{\alpha}{3\pi} M_\mathrm{Z}^2 \,\mathrm{Re}\left\{\int_{4m_\pi^2}^{\infty} \mathrm{d}s' \,\frac{R(s')}{s'(s'-M_\mathrm{Z}^2-\mathrm{i}\varepsilon)}\right\}$$

$$R(s) = \frac{\sigma(\mathrm{e}^+\mathrm{e}^- \to \gamma^* \to \text{hadrons})}{\sigma(\mathrm{e}^+\mathrm{e}^- \to \gamma^* \to \mu^+\mu^-)}$$

$R(s)$ is taken from perturbative QCD for high energies ($\sqrt{s} \gtrsim 13\,\mathrm{GeV}$)

$\Rightarrow$ $\Delta\alpha^{(5)}_{\mathrm{had}} = 0.027498 \pm 0.000135$

Jegerlehner; Burkhardt, Pietrzyk; Eidelman; Davier, Höcker,. . .

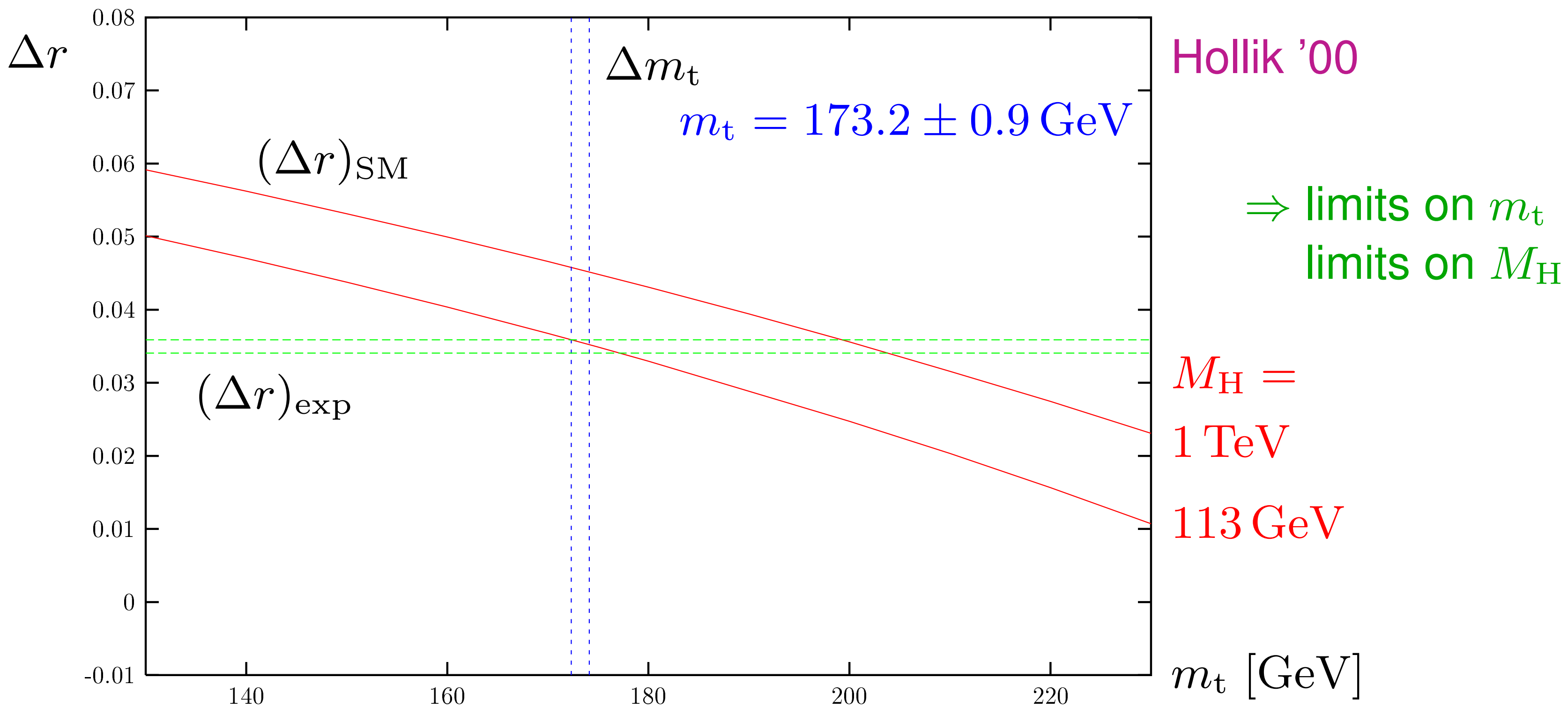
Experimental confirmation of quantum corrections

$$1 - \Delta r = \frac{\pi\alpha}{\sqrt{2}G_\mu} \frac{1}{M_{\mathrm{W}}^2(1 - M_{\mathrm{W}}^2/M_{\mathrm{Z}}^2)}$$

$\Rightarrow$ experimental determination of Δr $\Delta r = 0.0350 \pm 0.0009$

$\Delta r \neq 0$ with 39σ $\Rightarrow$ confirmation of quantum corrections

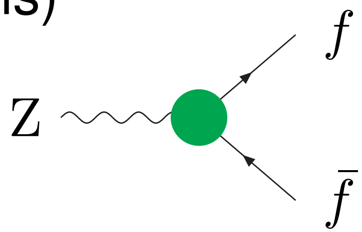
$\Delta r \neq \Delta\alpha \sim 0.0594$ wity 27σ $\Rightarrow$ confirmation of weak corrections



Precision tests of the Standard Model

Z-boson–fermion couplings

Z physics tests predominantly effective Z-boson–fermion couplings (on-shell, massless fermions)



$$= \mathrm{i}\frac{e}{2s_{\mathrm{w}}c_{\mathrm{w}}}\gamma_\mu\left(g_{v_f} - g_{a_f}\gamma_5\right)$$

effective couplings contain weak corrections to on-shell $Z\bar{f}f$ vertex

Z-boson width

$$\Gamma(\mathrm{Z}\to f\bar{f}) = \frac{G_\mu M_\mathrm{Z}^3}{6\pi\sqrt{2}}\left((g_{v_f})^2 + (g_{a_f})^2\right)\left(1+\delta_f^{\mathrm{QED}}\right)$$

forward–backward asymmetry

$$A_{\mathrm{FB}}^f = \frac{\sigma_f(\theta < 90^\circ) - \sigma_f(\theta > 90^\circ)}{\sigma_f(\theta < 90^\circ) + \sigma_f(\theta > 90^\circ)}$$

contribution of Z-boson exchange (pseudo observable: pole asymmetry)

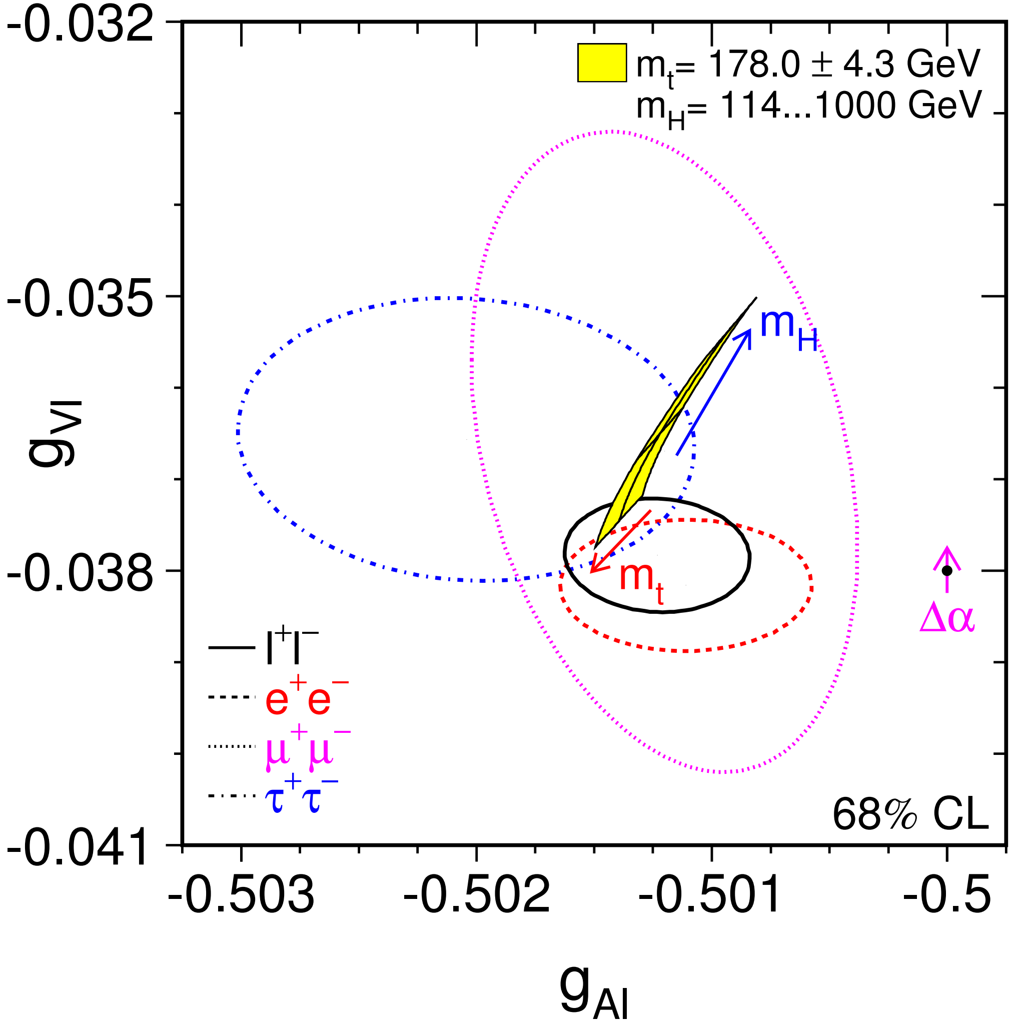
$$A_{\mathrm{FB}}^f(s = M_\mathrm{Z}^2) = \frac{3}{4}\,\frac{g_{v_\mathrm{e}}/g_{a_\mathrm{e}}}{(g_{v_\mathrm{e}}/g_{a_\mathrm{e}})^2+1}\,\frac{g_{v_f}/g_{a_f}}{(g_{v_f}/g_{a_f})^2+1}$$

$\Rightarrow g_{v_f},\ g_{a_f}$

Effective Z-boson couplings

LEPEWWG '05

before Higgs-boson discovery



parameters

$$m_t = 178.0 \pm 4.3\,\text{GeV}$$
$$M_H = 114\ldots1000\,\text{GeV}$$
$$\Delta\alpha_{\text{had}} = 0.02758 \pm 0.00035$$

confirmation of
lepton universality

constraints on m_t and M_H

experimental results
for couplings

$$g_{a_l} = -0.50123 \pm 0.00026$$
$$g_{v_l} = -0.03783 \pm 0.00041$$

Effective fermionic weak mixing angle

Lowest order:

$$g_{a_f,0} = I^3_{\mathrm{w},f}, \qquad g_{v_f,0} = (I^3_{\mathrm{w},f} - 2Q_f \sin^2\theta_{\mathrm{w}})$$

definition of effective fermionic mixing angle (pseudo observable)

$$g_{a_f} = \sqrt{\rho_{\mathrm{f}}} I^3_{\mathrm{w},f}, \qquad g_{v_f} = \sqrt{\rho_{\mathrm{f}}}(I^3_{\mathrm{w},f} - 2Q_f \sin^2\theta^{\mathrm{f}}_{\mathrm{eff}})$$

$$g_{v_f}, g_{a_f} \Rightarrow \sin^2\theta^{\mathrm{f}}_{\mathrm{eff}},\ \rho_{\mathrm{f}}, \qquad g_{v_f}/g_{a_f} \Rightarrow \sin^2\theta^{\mathrm{f}}_{\mathrm{eff}} = \frac{I^3_{\mathrm{w},f}}{2Q_f}\left(1 - \frac{g_{v_f}}{g_{a_f}}\right)$$

lowest-order perturbation theory:

$$\sin^2\theta^{\mathrm{f}}_{\mathrm{eff}} = \sin^2\theta_{\mathrm{w}} = 1 - \frac{M^2_{\mathrm{W}}}{M^2_{\mathrm{Z}}}, \qquad \rho_{\mathrm{f}} = 1$$

including weak corrections:

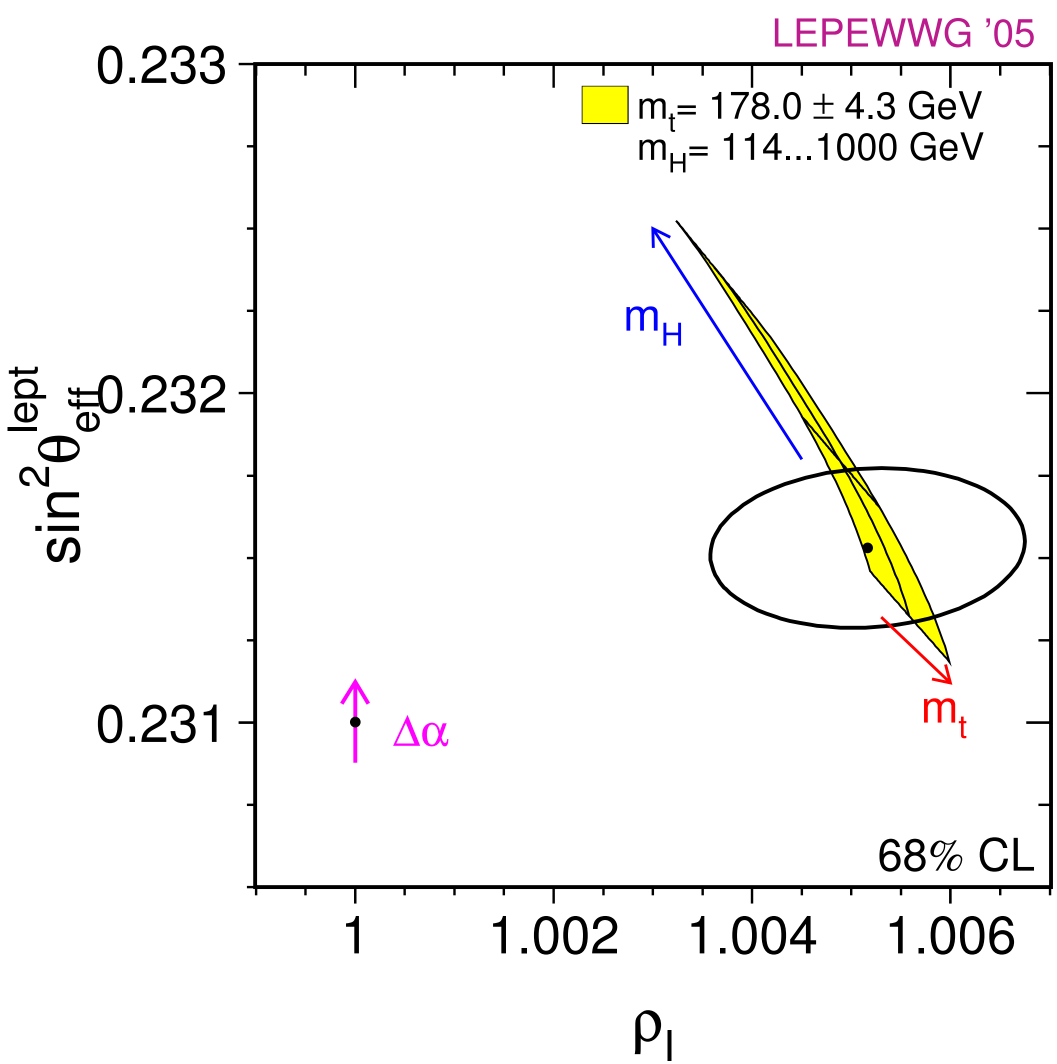
$$\sin^2\theta^{\mathrm{f}}_{\mathrm{eff}} \neq \sin^2\theta_{\mathrm{w}}, \qquad \rho_{\mathrm{f}} \neq 1$$

experiment:

$$\sin^2\theta^{\mathrm{lept}}_{\mathrm{eff}} \approx 0.2315, \qquad \sin^2\theta_{\mathrm{w}} \approx 0.2229, \qquad \sin^2\theta^{\mathrm{lept}}_{\mathrm{eff}}/\sin^2\theta_{\mathrm{w}} \approx 1.039$$

$$\rho_f \approx 1.005$$

Effective mixing angle versus leptonic ρ parameter



parameters

$$m_t = 178.0 \pm 4.3\,\mathrm{GeV}$$

$$M_H = 114 \ldots 1000\,\mathrm{GeV}$$

$$\Delta\alpha_{had} = 0.02758 \pm 0.00035$$

preference for light Higgs boson

experimental results

$$\sin^2\theta_{eff}^{lept} = 0.23153 \pm 0.00016$$

$$\rho_l = 1.0050 \pm 0.0010$$

Global electroweak fit

calculate all precision observables in SM including quantum corrections in terms of $\alpha(M_\mathrm{Z})$, G_μ, M_Z, $m_{f\neq t}$, m_t, M_H, $\alpha_s(M_\mathrm{Z})$

determine parameters by fit to all precision data

results

- good agreement between SM and data at the per-mille level (some exceptions)
- indirect determination of parameters

  all Z-pole data, M_W, Γ_W: $\Rightarrow m_\mathrm{t} = 179\,^{+12}_{-9}\,\mathrm{GeV}$

  1994: $m_\mathrm{t} = 169\,^{+24}_{-27}$ (top discovery 1995)

  present experimental value: $m_\mathrm{t} = 173.2 \pm 0.9\,\mathrm{GeV}$

  all Z-pole data, M_W, Γ_W, m_t

  $\Rightarrow M_\mathrm{H} = 94\,^{+29}_{-24}\,\mathrm{GeV}$

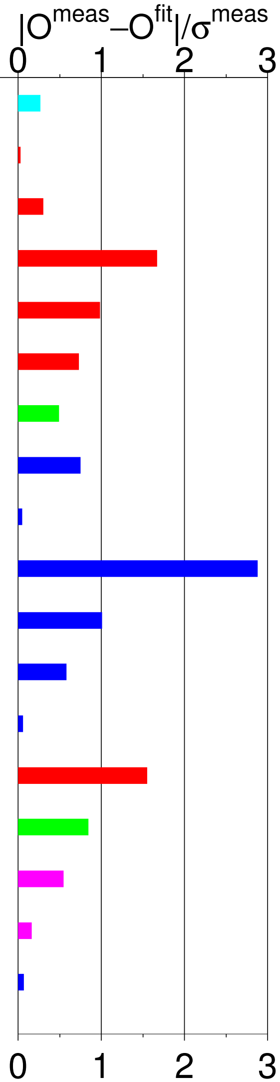
  $M_\mathrm{H} < 152\,\mathrm{GeV}$ (95% C.L.) preference for light Higgs boson

  $\alpha_s(M_\mathrm{Z}) = 0.1185 \pm 0.0026$

Global fit of LEP data

LEPEWWG '12 before Higgs discovery

	Measurement	Fit
$\Delta\alpha_{had}^{(5)}(m_Z)$	0.02750 ± 0.00033	0.02759
m_Z [GeV]	91.1875 ± 0.0021	91.1874
Γ_Z [GeV]	2.4952 ± 0.0023	2.4959
σ_{had}^0 [nb]	41.540 ± 0.037	41.478
R_l	20.767 ± 0.025	20.742
$A_{fb}^{0,l}$	0.01714 ± 0.00095	0.01645
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1481
R_b	0.21629 ± 0.00066	0.21579
R_c	0.1721 ± 0.0030	0.1723
$A_{fb}^{0,b}$	0.0992 ± 0.0016	0.1038
$A_{fb}^{0,c}$	0.0707 ± 0.0035	0.0742
A_b	0.923 ± 0.020	0.935
A_c	0.670 ± 0.027	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1481
$\sin^2\theta_{eff}^{lept}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.385 ± 0.015	80.377
Γ_W [GeV]	2.085 ± 0.042	2.092
m_t [GeV]	173.20 ± 0.90	173.26



March 2012

good agreement

best fit for Higgs-boson mass:

$M_H = 94^{+29}_{-24}$ GeV,

$M_H < 152$ GeV @ 95% CL

direct search at LEP2

$(\mathrm{e^+e^-} \not\to \mathrm{ZH})$ LEPHIGGS '03

$M_H > 114.4$ GeV

Higgs discovery at LHC

ATLAS/CMS July '12

$M_H \approx 126$ GeV

Input-parameter scheme

- Natural input parameters: α, M_{W}, M_{Z}, m_f, M_{H}, α_{s}
- weak mixing angle: on-shell definition $\sin\theta_{\mathrm{w}} = \sqrt{1 - M_{\mathrm{W}}^2/M_{\mathrm{Z}}^2}$
- alternative input parameter sets: G_μ instead of M_{W} or α
  G_μ no fundamental parameter, but precisely measured in μ decay
- definition of α
  - on-shell: $\alpha(0)$
    appropriate for external photons
  - $\alpha(M_{\mathrm{Z}})$, $\alpha(\sqrt{s})$: $\quad \frac{\alpha(M_{\mathrm{Z}})}{\alpha(0)} \approx 1.06$
    absorbs running of α from $Q = 0$ to EW scale
    appropriate for internal photons and weak bosons
  - G_μ scheme: $\quad \alpha_{G_\mu} = \sqrt{2} G_\mu M_{\mathrm{W}}^2 (1 - M_{\mathrm{W}}^2/M_{\mathrm{Z}}^2)/\pi$: $\quad \frac{\alpha_{G_\mu}}{\alpha(0)} \approx 1.03$
    absorbs running of α from $Q = 0$ to EW scale and $\Delta\rho$ in $Wf\bar{f}'$ coupling
    appropriate for processes with W bosons

  suitable choice of α reduces missing higher-order corrections
  effects can amount to several 10 per cent for high powers of α

gauge invariance demands unique input-parameter set!

Conclusions

Standard Model established as a quantum field theory

- in agreement with all experiments (accuracy $\gtrsim 0.1\%$)
- quantum corrections = radiative corrections are established
- indirect and direct determinations of m_{t} agree
- precision tests suggest light Higgs boson
- observed Higgs boson agrees well with SM predictions
- triple-gauge-boson self-interactions established at per-cent level

to be confirmed

- precise determination of Higgs-boson quantum numbers
- precise measurement of Higgs-boson couplings to gauge bosons and fermions
- Higgs-boson self-interaction $\Rightarrow$ Higgs potential

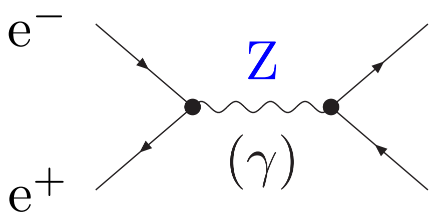
Literature

- Textbooks and Reviews:
  - Peskin/Schroeder, *An introduction to Quantum Field Theory*
  - Itzykson/Zuber, *Quantum Field Theory*
  - Weinberg, *The Quantum Theory of Fields, Vol. 1: Foundations*;
    The Quantum Theory of Fields, Vol. 2: Modern Applications
  - Böhm/Denner/Joos, *Gauge Theories of the Strong and Electroweak Interaction*
  - Collins, *Renormalization*
- Experimental results:
  - The ALEPH, DELPHI, L3, OPAL, SLD Collaborations, the LEP Electroweak Working Group, the SLD Electroweak and Heavy Flavour Groups, Phys. Rept. 427 (2006) 257
  - The ALEPH, CDF, D0, DELPHI, L3, OPAL, SLD Collaborations, the LEP Electroweak Working Group, the Tevatron Electroweak Working Group, and the SLD electroweak and heavy flavour groups, LEPEWWG/2010-01, http://lepewwg.web.cern.ch/LEPEWWG/
  - ATLAS collaboration, Phys. Lett. B716 (2012) 1, arXiv:1207.7214,
    ATLAS-CONF-2012-170;
    CMS collaboration, JHEP 06 (2013) 081, arXiv:1303.4571; CMS-HIG-12-036

Backup

Z physics
at LEP and SLC

Z-boson resonance



$f = \mathrm{e}^-, \mu^-, \tau^-, q$

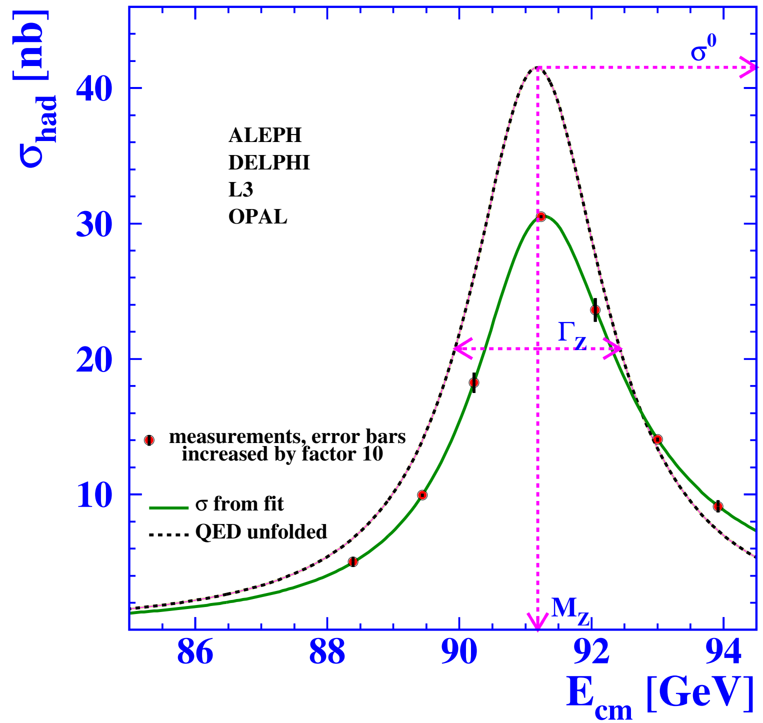
$\bar{f} = \mathrm{e}^+, \mu^+, \tau^+, \bar{q}$

LEP1: $\sim 16 \times 10^7$ events (1989–1995)

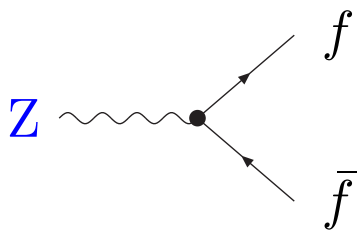
unfolded resonance cross-section $\qquad s = E_{\mathrm{CMS}}^2$

$$\sigma_f(s) = 12\pi \frac{s}{M_{\mathrm{Z}}^2} \frac{\Gamma(\mathrm{Z} \to \mathrm{e}^+\mathrm{e}^-)\,\Gamma(\mathrm{Z} \to f\bar{f})}{(s - M_{\mathrm{Z}}^2)^2 + s^2\Gamma_{\mathrm{Z}}^2/M_{\mathrm{Z}}^2} = \sigma^0 \frac{s\Gamma_{\mathrm{Z}}^2}{(s - M_{\mathrm{Z}}^2)^2 + s^2\Gamma_{\mathrm{Z}}^2/M_{\mathrm{Z}}^2}$$

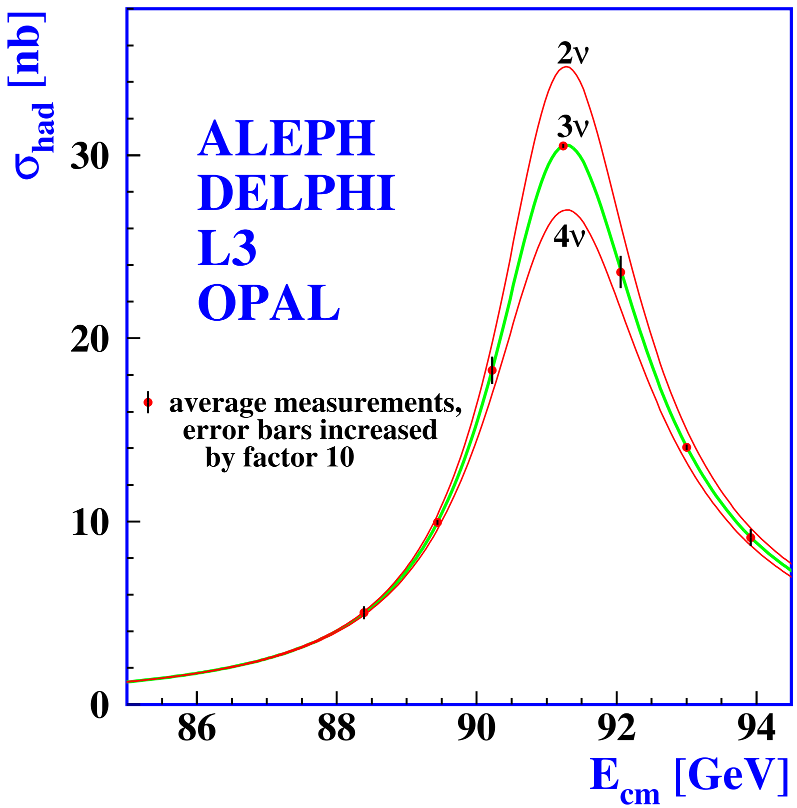
- line shape $\Rightarrow M_{\mathrm{Z}}, \Gamma_{\mathrm{Z}}$
- peak cross section σ^0
  $\Rightarrow \Gamma(\mathrm{Z} \to l^+l^-)/\Gamma_{\mathrm{Z}}$,
  $\Gamma(\mathrm{Z} \to \mathrm{hadrons})/\Gamma_{\mathrm{Z}}$
- angular distributions, polarization asymmetries
  $\Rightarrow$ effective $\mathrm{Z}f\bar{f}$ vector and axial-vector couplings g_{v_f}, g_{a_f}



Z-boson width and number of light neutrinos



$$\Gamma_Z = \underbrace{\Gamma(e^-e^+, \mu^-\mu^+, \tau^-\tau^+)}_{\text{leptonic}}$$
$$+ \underbrace{\sum_q \Gamma(q\bar{q})}_{\text{hadronic}} + \underbrace{N_\nu \Gamma(\nu\bar{\nu})}_{\text{invisible}}$$
$$= \Gamma_{had} + \Gamma_e + \Gamma_\mu + \Gamma_\tau + \Gamma_{inv}$$



- Γ_Z measured from Z line shape
- Γ_{had} and $\Gamma_{l=e,\mu,\tau}$ from $R_l = \dfrac{\Gamma_{had}}{\Gamma_l}$ and $\sigma^0_{had} = \dfrac{12\pi}{M_Z^2}\dfrac{\Gamma_e\Gamma_{had}}{\Gamma_Z^2}$

Fit of Γ_Z, R_l, and σ^0_{had} yields invisible Z-decay width: $\Gamma_{inv} = N_\nu \Gamma^{theory}_{Z\to\nu\bar{\nu}}$

$\hookrightarrow \quad N_\nu = 2.9840 \pm 0.0082$