Quantum corrections at the LHC

Ansgar Denner, Würzburg

Workshop des Graduiertenkollegs
Elementarteilchenphysik bei höchster Energie und höchster Präzision
Bad Liebenzell, 30.09. - 02.10.2013

- Lecture 1: Precision tests of the Standard Model
- Lecture 2: NLO Calculations for the LHC

NLO calculations for the LHC

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- Physics at the LHC
- Relevance of NLO (QCD) corrections
- Calculation of NLO corrections
- Relevance of Electroweak Radiative Corrections
- Example processes

Physics at the LHC

Standard Model (SM) physics at the LHC

Some important SM measurements and relevant processes:

- Measurement of Higgs-boson properties
  various production and decay processes
- improved measurement of W-boson mass M_{W}
  main process: $\mathrm{pp} \to \mathrm{W} \to \mathrm{l}\nu_l + X$
- improved measurement of effective weak mixing angle $\sin^2\theta_{\mathrm{eff}}$
  main process: $\mathrm{pp} \to \mathrm{Z} \to \mathrm{ll} + X$
- improved measurement of non-Abelian gauge couplings
  main processes:
  - $\mathrm{pp} \to \mathrm{W}\gamma, \mathrm{Z}\gamma + X$
  - $\mathrm{pp} \to \mathrm{WW}, \mathrm{WZ}, \mathrm{ZZ} + X$
- improved measurement of top-quark mass
  main process $\mathrm{pp} \to \mathrm{tt} + X$

New Physics at LHC

New physics may reveal itself by

- spectacular new signatures that are easily distinguishable from the Standard Model
  example: new resonance in $\mu^+\mu^-$ like a Z'
  so far nothing of this sort found
- less spectacular signatures with Standard Model background
  (e.g. excess)
  example: missing energy in production of supersymmetric particles
  $\Rightarrow$ need SM prediction
- (small) deviations from Standard Model predictions
  examples: anomalous couplings
  contributions of heavy degrees of freedom via loop processes
  $\Rightarrow$ need precise SM prediction

In the absence of striking new signatures,
to distinguish new physics from SM effects
precise predictions of SM processes are necessary!

Relevance of NLO (QCD) corrections

Importance of NLO calculations for LHC

QCD corrections: substantial part of predictions

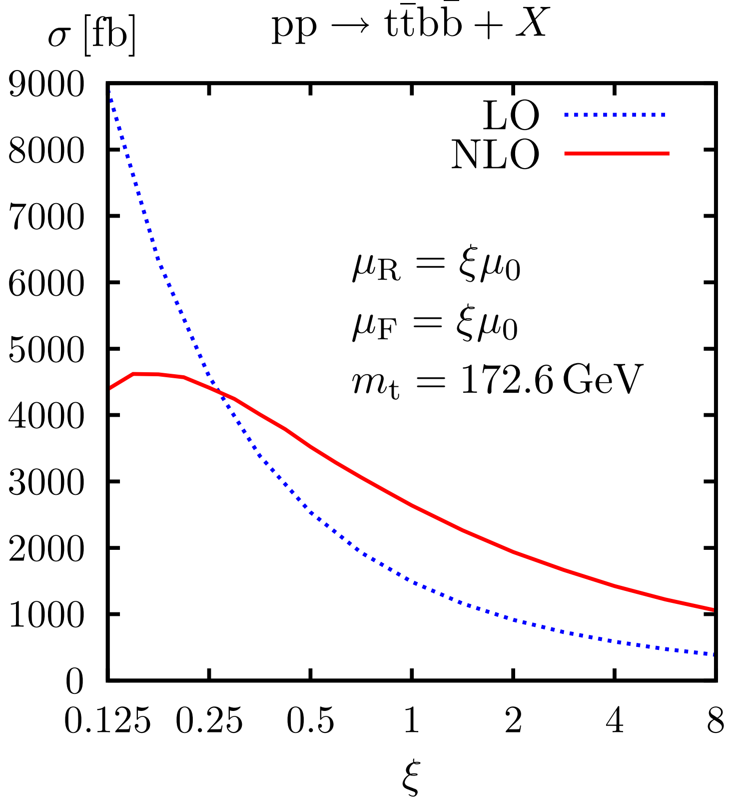
- LO predictions depend on $\alpha_s = \alpha_s(\mu)$
  renormalization scale μ free parameter
  $\Rightarrow$ large scale uncertainty (up to factor 2)
  $\Rightarrow$ often no quantitative prediction possible
  μ dependence due to missing higher orders
- NLO predictions: reduced scale uncertainty
  first real prediction
  $\Rightarrow$ needed for all scattering processes at the LHC
  $\mathcal{O}(\alpha_s) \times \log(\ldots) \sim 10\% - 100\%, \qquad \alpha_s(M_Z) \approx 0.12$
- NNLO predictions: scale uncertainty further reduced
  first real uncertainty estimate
  $\Rightarrow$ needed for selected processes like
  single W/Z production, $t\bar{t}$ production
  $\mathcal{O}(\alpha_s^2) \times \log^2(\ldots) \sim \text{few}\% - 20\%$

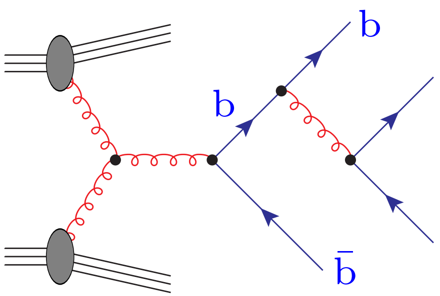
$\hookrightarrow$ NLO (NNLO) corrections important for reliable predictions

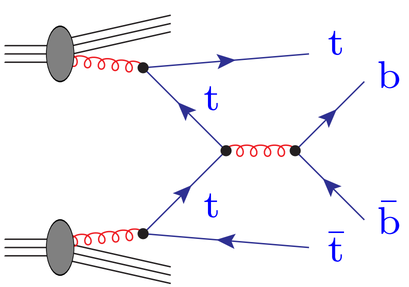
Example: scale dependence for $\mathrm{pp} \to \mathrm{t\bar{t}b\bar{b}} + X$

Background process to
$\mathrm{pp} \to \mathrm{t\bar{t}H} + X \to \mathrm{t\bar{t}b\bar{b}} + X$

Bredenstein, Denner, Dittmaier, Pozzorini '10



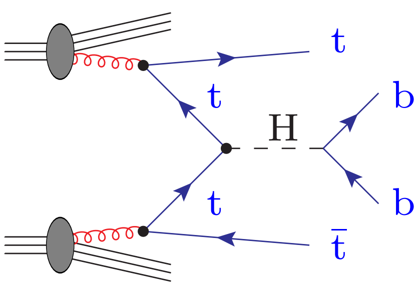


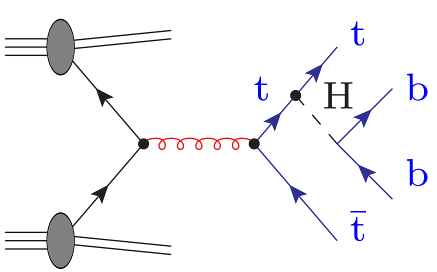


LO $\propto \alpha_\mathrm{S}(\mu_\mathrm{R})^4 \Rightarrow$ large scale uncertainty

μ_R	$m_\mathrm{t}/8$	$m_\mathrm{t}/4$	$m_\mathrm{t}/2$	m_t	$2m_\mathrm{t}$	$4m_\mathrm{t}$	$8m_\mathrm{t}$
$\alpha(\mu_\mathrm{R})$	0.151	0.133	0.119	0.108	0.098	0.091	0.084
$\frac{\alpha(\mu_\mathrm{R})}{\alpha(m_\mathrm{t})}$	1.40	1.24	1.11	1.00	0.91	0.84	0.78
$\left(\frac{\alpha(\mu_\mathrm{R})}{\alpha(m_\mathrm{t})}\right)^4$	3.88	2.34	1.49	1.00	0.70	0.50	0.37

original (ATLAS) scale choice based on $\mathrm{t\bar{t}H}$



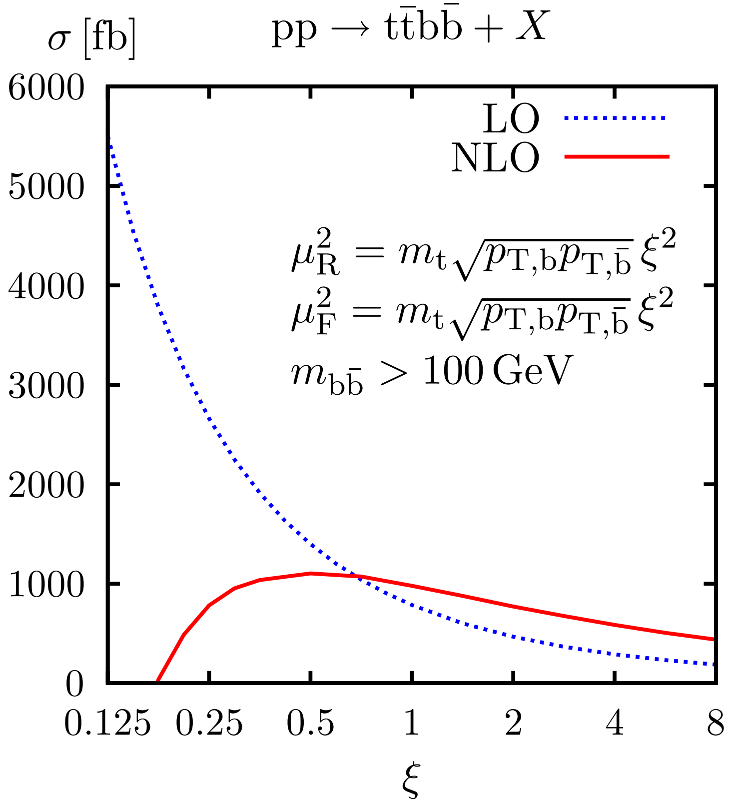


$$\mu_0 = E_\mathrm{thr}/2 = m_\mathrm{t} + m_{\mathrm{b\bar{b}}}/2$$

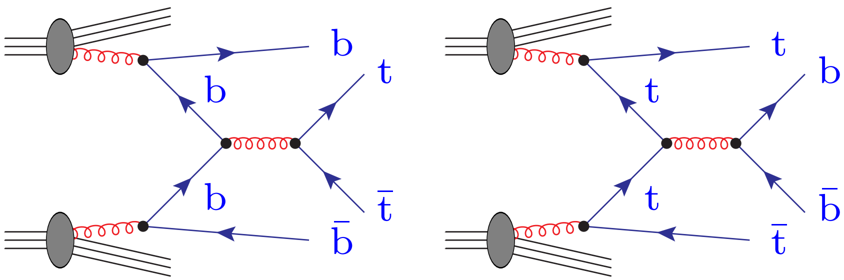
$\Rightarrow$ large K factor (1.8) and scale dependence (34%)

Example: scale dependence for $\mathrm{pp} \to \mathrm{t\bar{t}b\bar{b}} + X$

Bredenstein, Denner, Dittmaier, Pozzorini '10



QCD dynamics of $\mathrm{t\bar{t}H}$ and $\mathrm{t\bar{t}b\bar{b}}$ different



various different channels for $\mathrm{t\bar{t}b\bar{b}}$

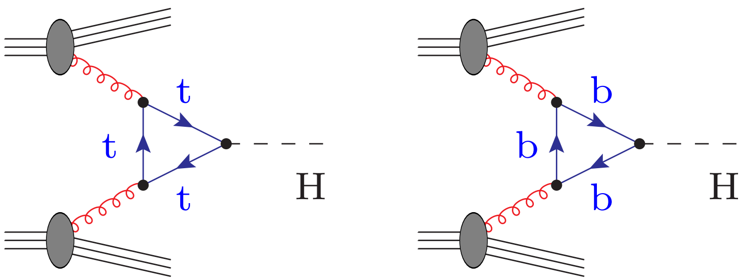
good central scale

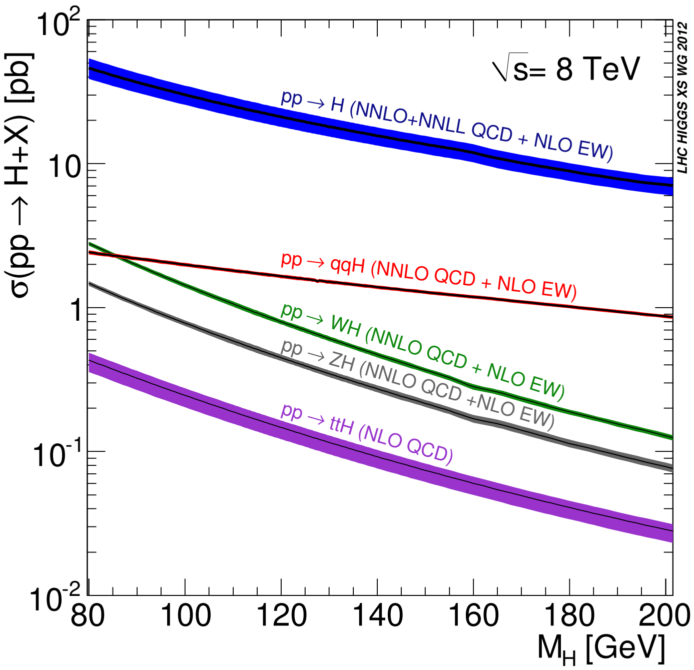
$$\mu_0^2 = m_\mathrm{t}\sqrt{p_{\mathrm{T,b}}p_{\mathrm{T,\bar{b}}}}$$

one α_s at scale m_t
one α_s at scale of p_T of b quarks

- small correction and uncertainty: $K = 1.24 \pm 21\%$
- central scale close to a maximum

Example: Higgs production in gluon–gluon fusion





LHC HIGGS XS WG '12

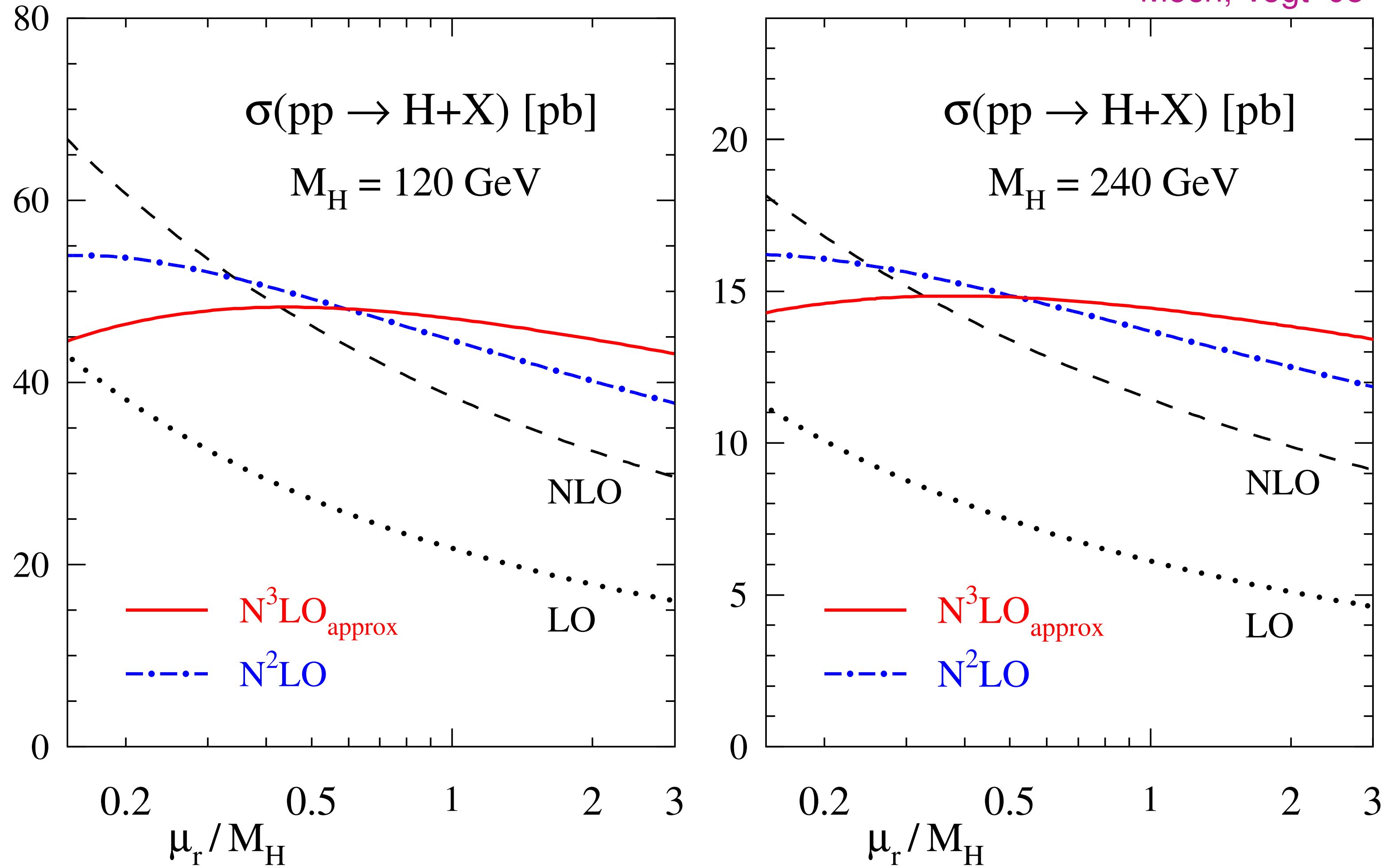
- most important production channel at LHC
- $\sigma_{\mathrm{LO}} \propto \alpha_{\mathrm{s}}^2$
  strong dependence on factorization and renormalization scales (100%)
  $\Rightarrow$ higher-order corrections very important
- complete NLO: $80{-}100\%$
  virtual contribution $\pi\alpha_{\mathrm{s}} \sim 35\%$ $[\pi^2(\alpha_{\mathrm{s}}/\pi)]$
  real contribution $\sim 50\%$

  Graudenz, Spira, Zerwas '93
  Djouadi, Graudenz, Spira, Zerwas '95
- NNLO: $\sim 25\%$

  Harlander, Kilgore '01, '02; Catani, de Florian, Grazzini '01
  Anastasiou, Melnikov '02; Ravindran, Smith, van Neerven '03, '04
  Ahrens, Becher, Neubert, Yang '08

QCD scale dependence for $gg \to H$

Moch, Vogt ’05



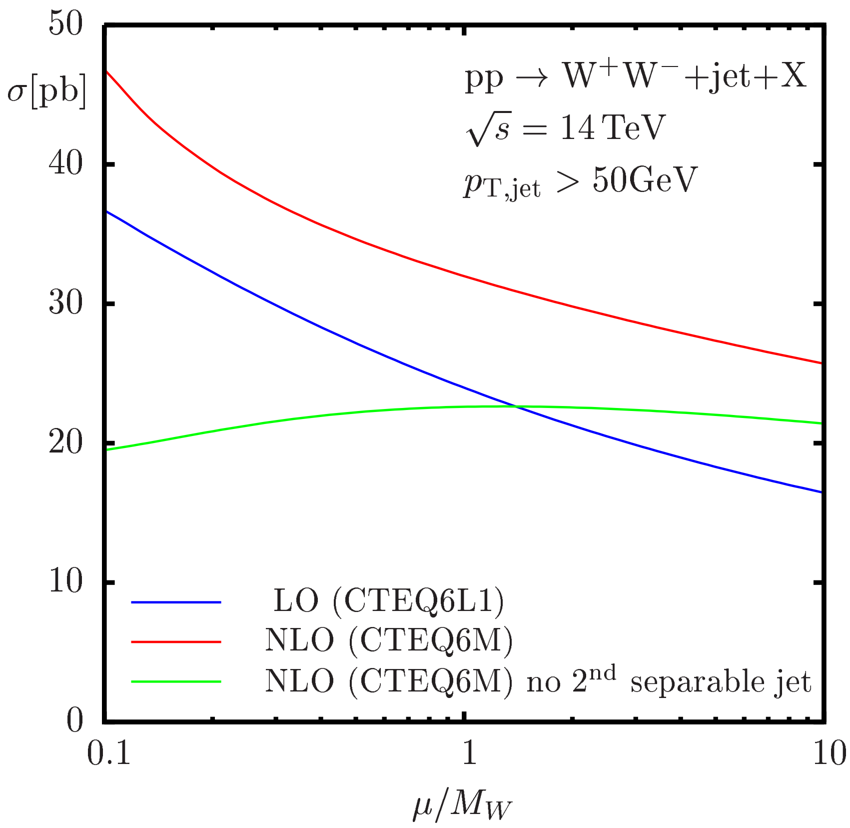
Reduction of renormalization scale dependence with increasing orders!

$\Rightarrow$ residual scale uncertainty $\lesssim 5$–10%

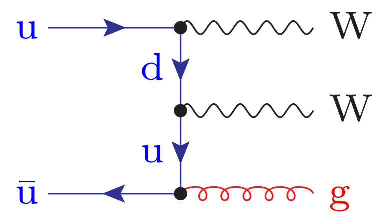
Example: scale dependence for $\mathrm{pp} \to \mathrm{WWj} + X$

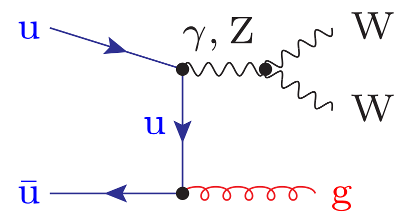
Appearance of new channels:

Dittmaier, Kallweit, Uwer '07



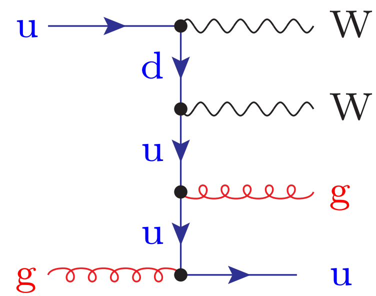
- $\sigma_{\mathrm{LO}} \propto \alpha_{\mathrm{s}}$



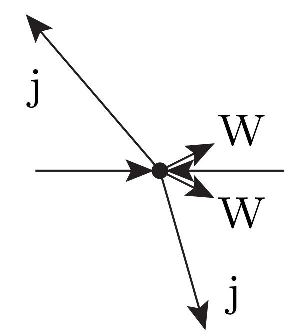


- scale dependence stabilises at NLO for genuine $\mathrm{WW} + \mathrm{j}$ production
- significant scale dependence is introduced by $\mathrm{WW} + 2\,\mathrm{j}$ (difference between green and red curves)

new channels



new kinematical configuration



Importance of multiparticle processes at the LHC

Most signal processes involve few final-state particles:

- $2 \to 2 \quad \mathrm{pp} \to \mathrm{ll}, \mathrm{W}\gamma, \mathrm{WW}, \mathrm{tt}, \ldots + X$
- $2 \to 3 \quad \mathrm{pp} \to \mathrm{Hjj}, \mathrm{WW}\gamma, \ldots + X$

however,

- heavy particles $(\mathrm{W}, \mathrm{Z}, \mathrm{t}, \ldots)$ decay into jets, leptons, photons
  $\mathrm{pp} \to \mathrm{WW} \to \mathrm{ll}\nu_l\nu_l + X$, $\mathrm{pp} \to \mathrm{tt} \to \mathrm{be}\nu_\mathrm{e}\mathrm{b}\mu\nu_\mu + X$
- irreducible backgrounds involve genuine multiparticle final states
  $\mathrm{pp} \to \mathrm{ll}\nu_l\nu_l + X$, $\mathrm{pp} \to \mathrm{be}\nu_\mathrm{e}\mathrm{b}\mu\nu_\mu + X$
  (backgrounds often not fully accessible to measurements)
- large fraction of final states contains additional jets
  $\mathrm{pp} \to \mathrm{WWj} + X$, $\mathrm{pp} \to \mathrm{WWjj} + X, \ldots$

$\Rightarrow$ Need reliable predictions for multiparticle processes!

Existing NLO calculations

NLO calculations

- $2 \to 2$ trivial (textbook)
- $2 \to 3$ standard (many groups)
- $2 \to 4$ state of the art (several groups)

  first $2 \to 4$ EW calculation:

  $\mathrm{e^+e^-} \to f_1 \bar{f}_2 f_3 \bar{f}_4$ Denner, Dittmaier, Roth, Wieders '05

  first $2 \to 4$ QCD calculations:

  $\mathrm{pp} \to \mathrm{t\bar{t}b\bar{b}}$ Bredenstein, Denner, Dittmaier, Pozzorini '09, Bevilacqua et al. '09

  $\mathrm{pp} \to \mathrm{W}jjj$ Berger et al. '09; R.K.Ellis et al. '09

  NLO QCD exists for $\gtrsim 10–20$ LHC $2 \to 4$ processes

- $2 \to \geq 5$ only very few (few groups)

  $\mathrm{pp} \to \mathrm{W/Z} + 4j$ Berger et al. '10/'11

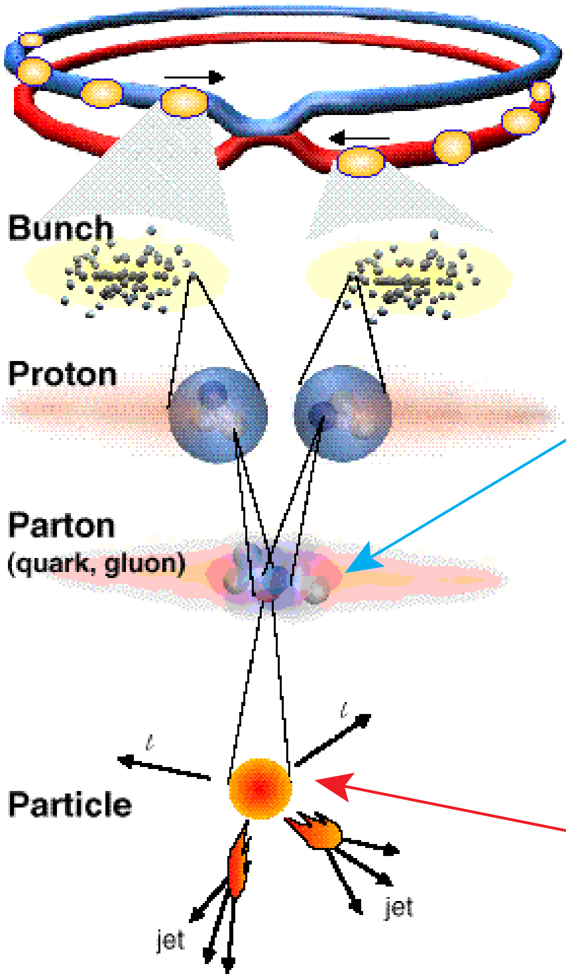
  $\mathrm{e^+e^-} \to 7j$ Becker et al. '11

  $\mathrm{pp} \to 5j$ Badger et al. '13

  $\mathrm{pp} \to \mathrm{W} + 5j$ Bern et al. '13

Calculation of NLO corrections

Hadronic cross sections



parton content of the proton:

- valence quarks uud,
- sea quarks $\mathrm{u, d, c, s, (+b,)}$ + antiquarks
- gluons g (+ photons γ)

"parton distribution functions" (PDFs) $f_{i/\mathrm{p}}(x, \mu_\mathrm{F})$

- probability for parton i to have fraction x
- of momentum p at "factorization scale" μ_F
- = non-perturbative input (from experiment)
  - process independent

hard interaction of partons

- $\hookrightarrow$ perturbative QFT applicable,
- model for hard interaction
- (except QCD/QED) only enters here

$$\sigma_{\mathrm{pp}\to F+X}(p_1, p_2) = \int_0^1 \mathrm{d}x_a \int_0^1 \mathrm{d}x_b \sum_{a,b} f_{a/\mathrm{p}}(x_a, \mu)\, f_{b/\mathrm{p}}(x_b, \mu_\mathrm{F})\, \hat{\sigma}_{ab\to F}(x_a p_1, x_b p_2, \mu_\mathrm{F})$$

LO partonic cross section

LO partonic cross section for a $2 \to n$ process can be written as

$$\mathrm{d}\sigma_{\mathrm{LO}} = \frac{1}{2s} \int \mathrm{d}\Phi_n |\mathcal{M}_{\mathrm{LO}}|^2$$

$\int \mathrm{d}\Phi_n = (2\pi)^4 \delta^{(4)} \left(P - \sum_{i=1}^{n} q_i \right) \prod_{i=1}^{n} \frac{\mathrm{d}^3 q_i}{(2\pi)^3 2E_i}$ n-particle phase space

$\mathcal{M}_{\mathrm{LO}}$: LO matrix element (contains model for hard interaction)

$s = P^2 = (\hat{p}_1 + \hat{p}_2)^2$ square of centre-of-mass energy of hard process ($\hat{p}_i = x_i p_i$)

Integration over phase space by Monte Carlo methods $\Rightarrow$

- any distribution can be determined simultaneously
- Monte Carlo events can be unweighted

Many generic codes exist at LO:

- MADGRAPH Alwall, Herquet, Maltoni, Mattelaer, Stelzer
- WHIZARD Kilian, Ohl, Reuter
- SHERPA Höche, Krauss, Schuhmann, Siegert, Winter
- HELAC Papadopoulos, Worek
- ...many more

Calculation of leading-order matrix elements

- Feynman diagrams: double factorial complexity $[2n!! = 2n(2n-2)(2n-4)...2]$

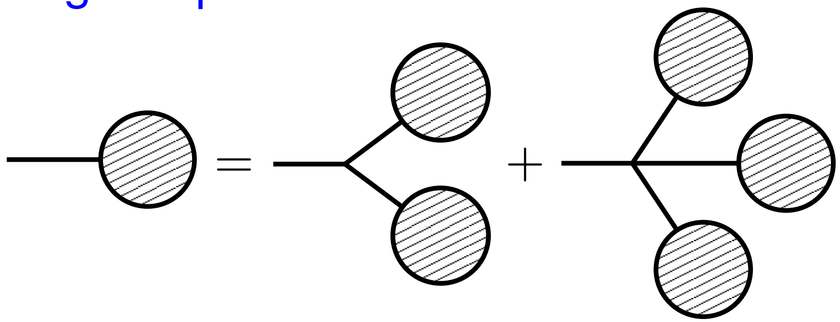
  # diagrams for pure gluon (scalar) processes

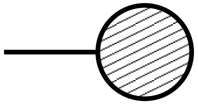
external gluons	4	5	6	7	8	9	...	n
# diags w/ only 3-g vertices	3	15	105	945	10395	135135		$(2n-5)!!$
# diags w/ 3-g and 4-g vert.	4	25	220	2485	34300	559405		

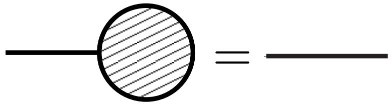
- recursion-relation technique: polynomial complexity of rank 4: $\mathcal{O}(n^4)$

  Berends, Giele '88; Kleiss, Kuijf '89, Caravaglios, Moretti '95, Draggiotis, Kleiss, Papadopoulos '98

  based on Dyson–Schwinger equations



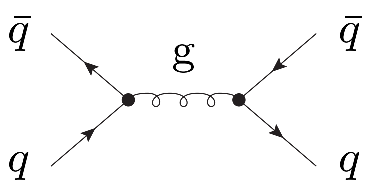
 : off-shell current with arbitrary number of on-shell lines
sub-amplitude with one off-shell and arbitrary many on-shell lines

starting point of iteration:  wave function (polarization vector)

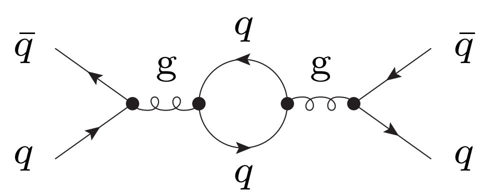
IR singularities in NLO corrections

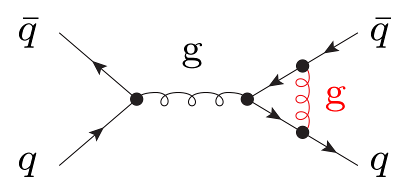
NLO corrections consist of Feynman diagrams of higher order to the same process:

tree diagrams



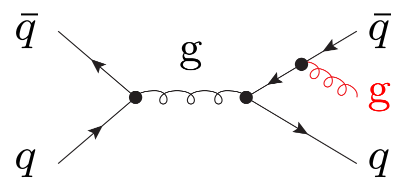
loop diagrams





loop diagrams contain infrared singularities
from internal gluons soft or collinear to quarks

Process with an additional gluon has to be added to cancel IR singularities
(soft or collinear photons cannot be separated experimentally!)



Square of bremsstrahlung matrix element of the same order as interference between LO diagrams and loop diagrams!

Some collinear singularities from the initial state do not cancel but can be absorbed by a renormalization of the PDFs $\Rightarrow$ collinear counter term

Structure of NLO computation

NLO partonic cross section can be written as

$$\mathrm{d}\sigma_{\mathrm{NLO}} = \int \mathrm{d}\Phi_n \left[|\mathcal{M}_{\mathrm{LO}}|^2 + 2\,\mathrm{Re}\{\mathcal{M}^*_{\mathrm{LO}}\mathcal{M}_{\mathrm{NLO,V}}\} + C \right] + \int \mathrm{d}\Phi_{n+1} \left[|\mathcal{M}_{\mathrm{NLO,R}}|^2 \right]$$

$\int \mathrm{d}\Phi_{n(+1)}$: n or $n+1$ particle phase space

$\mathcal{M}_{\mathrm{LO}}$, $\mathcal{M}_{\mathrm{NLO,V}}$, $\mathcal{M}_{\mathrm{NLO,R}}$: matrix elements for LO, virtual and real NLO

C collinear counter term from renormalization of PDFs
needed to cancel left-over collinear singularities from initial state

infrared singularities cancel only after phase-space integration
numerical phase-space integration impossible or inaccurate
$\Rightarrow$ use dedicated treatment of infrared singularities

Structure of NLO computation

NLO partonic cross section can be written as in subtraction method

$$\mathrm{d}\sigma_{\mathrm{NLO}} = \int \mathrm{d}\Phi_n \left[|\mathcal{M}_{\mathrm{LO}}|^2 + 2\,\mathrm{Re}\{\mathcal{M}^*_{\mathrm{LO}}\mathcal{M}_{\mathrm{NLO,V}}\} + C + \int \mathrm{d}\Phi_1 \sum_j S_j \right]$$

$$+ \int \mathrm{d}\Phi_{n+1} \left[|\mathcal{M}_{\mathrm{NLO,R}}|^2 - \sum_j S_j \right]$$

$\int \mathrm{d}\Phi_{n(+1)}$: n or $n+1$ particle phase space

$\mathcal{M}_{\mathrm{LO}}$, $\mathcal{M}_{\mathrm{NLO,V}}$, $\mathcal{M}_{\mathrm{NLO,R}}$: matrix elements for LO, virtual and real NLO

C collinear counter term from renormalization of PDFs

$\sum S_j$: subtraction terms for real corrections

$\int \mathrm{d}\Phi_1 \sum S_j$: (analytically) integrated subtraction terms

subtraction terms cancel but render individual integrals finite $\Rightarrow$ stable numerical integration

$\mathcal{M}_{\mathrm{NLO,R}}$: tree-level matrix elements
subtraction terms S_j: colour-weighted tree-level matrix elements
virtual corrections $\mathrm{Re}\{\mathcal{M}^*_{\mathrm{LO}}\mathcal{M}_{\mathrm{NLO,V}}\}$ (loop diagrams): require new methods

Methods for virtual corrections

Until ∼2005: Virtual corrections were the bottleneck of NLO calculations.

- Feynman diagrams: worse than factorial complexity
- relied on process-specific algebraic calculations, no full automation

NLO revolution: Ossola, Papadopoulos, Pittau '07, Bern, Dixon, Kosower, Britto, Cachazo, Feng, Ellis, Giele, Melnikov, . . .

- unitarity-cut technique: polynomial complexity of rank 9: $\mathcal{O}(n^9)$ Giele, Zanderighi '08
- automation immediately performed by different groups

recursion-relation technique for NLO:

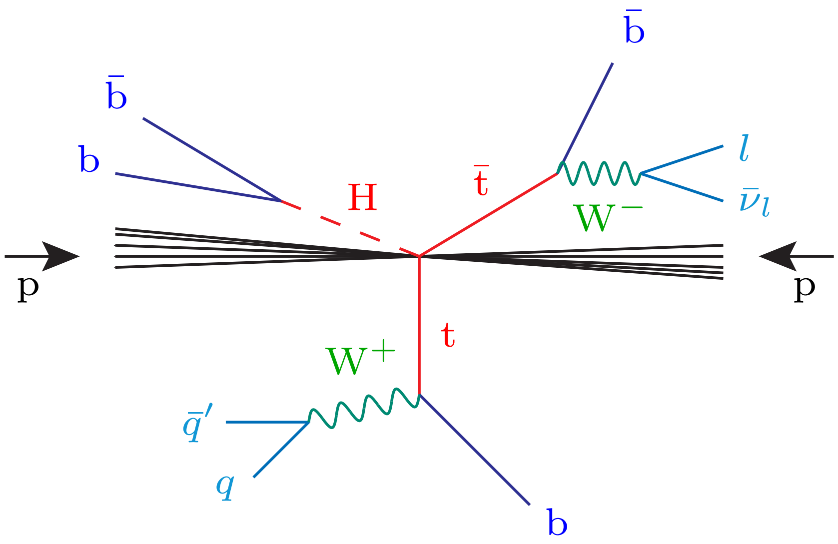
- exponential complexity: $\mathcal{O}(n^4 2^n)$ van Hameren '09 (pure gluon amplitudes)
- asymptotic behaviour not necessarily relevant for practical purposes
- basis for automation of EW corrections with RECOLA Actis et al. '12

combination of Feynman diagrams and recursion relations:

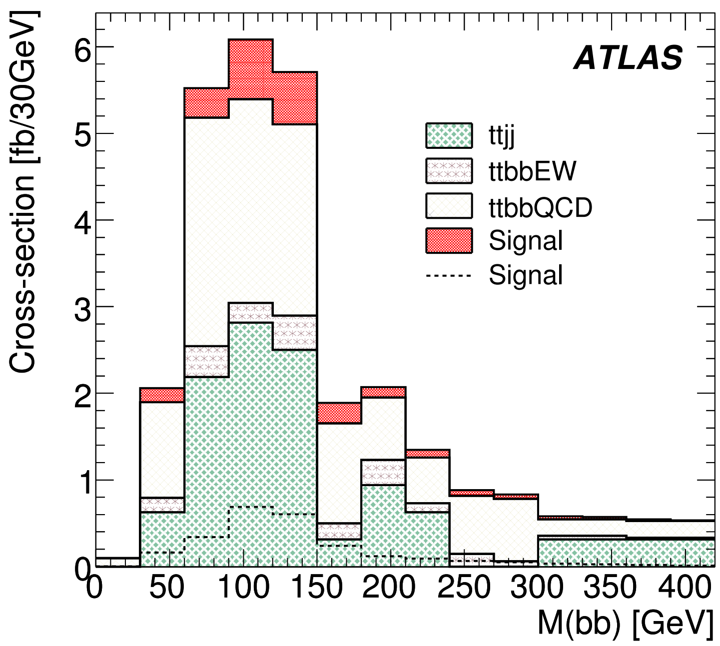
- OPENLOOPS Cascioli, Maierhöfer, Pozzorini '12
  NLO-QCD matrix elements for many LHC processes, linked with SHERPA
- Feynman diagrams allow efficient summation over colours and helicities

Example: $\mathrm{pp} \to \mathrm{t\bar{t}b\bar{b}}$

Background to $\mathrm{pp} \to \mathrm{t\bar{t}H}(\to \mathrm{b\bar{b}})$



"CSC book", CERN-OPEN-2008-020



- relevance: direct experimental access to $\mathrm{t\bar{t}H}$-Yukawa-coupling
- problem: control of background via $\mathrm{pp} \to \mathrm{t\bar{t}b\bar{b}}, \mathrm{t\bar{t}} + \mathrm{jets}$

  need: ▶ improved analysis methods (fat jets, boosted Higgs)

  ▶ NLO predictions for background processes

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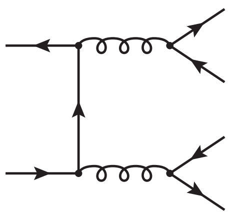
NLO QCD corrections for $\mathrm{pp} \to \mathrm{t\bar{t}b\bar{b}}$

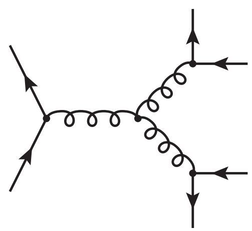
First complete NLO calculation for a $2 \to 4$ hadron-collider process

$q\bar{q} \to \mathrm{t\bar{t}b\bar{b}}$ 5% of cross section

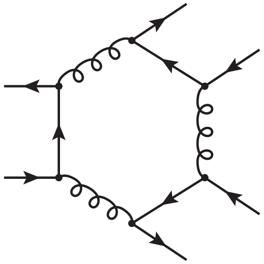
Bredenstein, Denner, Dittmaier, Pozzorini ’08

LO: 7 diagrams

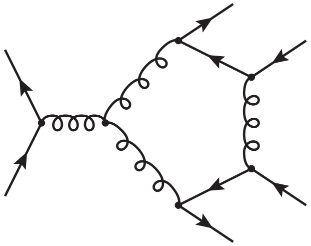




NLO: 188 diagrams



8 hexagons



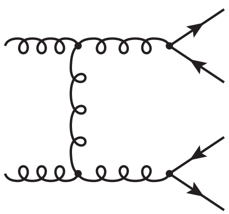
24 pentagons

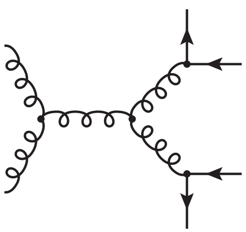
bremsstrahlung diagrams: 64

$\mathrm{gg} \to \mathrm{t\bar{t}b\bar{b}}$ 95% of cross section

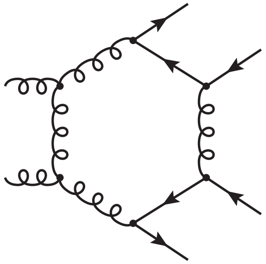
Bredenstein, Denner, Dittmaier, Pozzorini ’09

LO: 36 diagrams

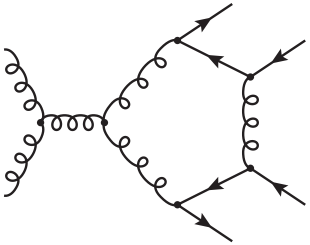




NLO: 1003 diagrams



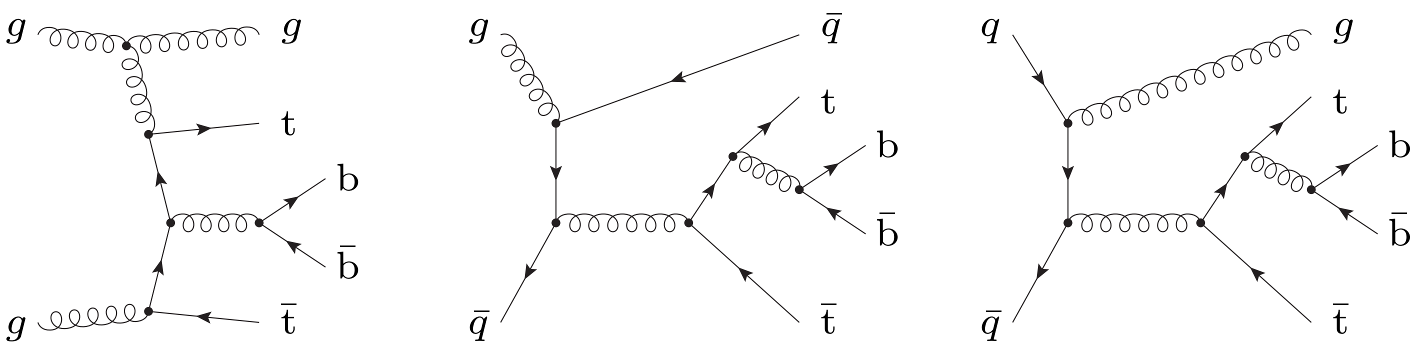
40 hexagons



114 pentagons

bremsstrahlung diagrams: 341

Corrections from real radiation: $\mathrm{pp} \to \mathrm{t\bar{t}b\bar{b}j}$

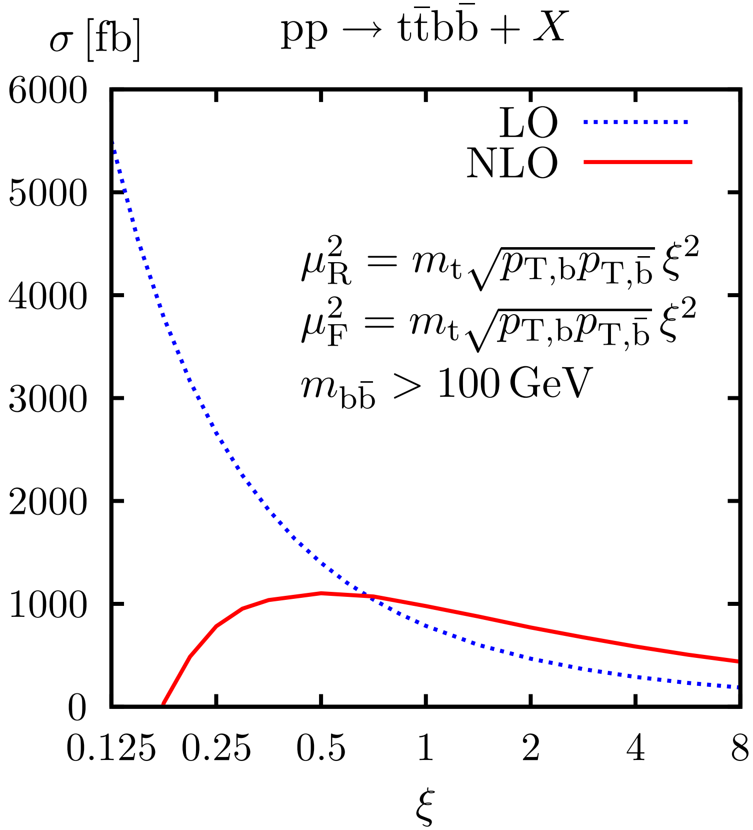


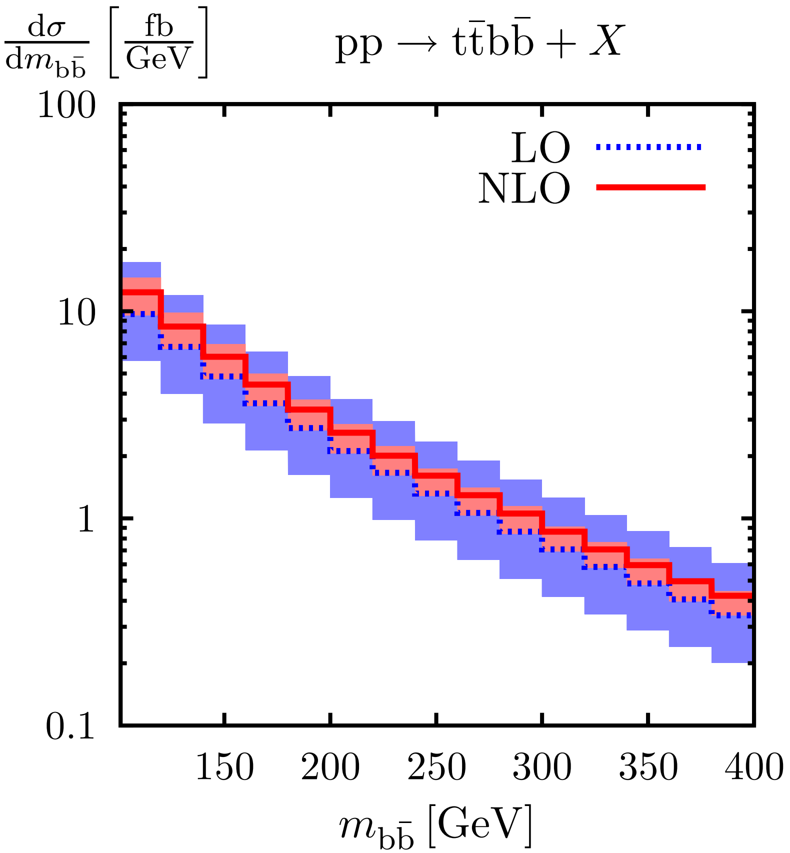
etc.

- channels: $\mathrm{gg} \to \mathrm{b\bar{b}t\bar{t}g}$, $q\mathrm{g} \to \mathrm{b\bar{b}t\bar{t}}q$, $\bar{q}\mathrm{g} \to \mathrm{b\bar{b}t\bar{t}}\bar{q}$, $q\bar{q} \to \mathrm{b\bar{b}t\bar{t}g}$
- numerical (Monte-Carlo-)integration over 11-dimensional phase space
- fast calculation of amplitudes (bremsstrahlung and LO) needed
- treatment of soft and collinear singularities via subtraction method
  $\Rightarrow$ $30\ (= 6 \times 5)$ dipole subtraction terms per channel
  $\Rightarrow$ LO matrix element has to be calculated 30 times for each event
- run times: $\sim 50\,$h on single CPU for 10^7 events
  $\Rightarrow$ 0.5% accuracy for total integrated cross section

Exemplary results for $\mathrm{pp} \to \mathrm{t\bar{t}b\bar{b}}$

Bredenstein, Denner, Dittmaier, Pozzorini '09





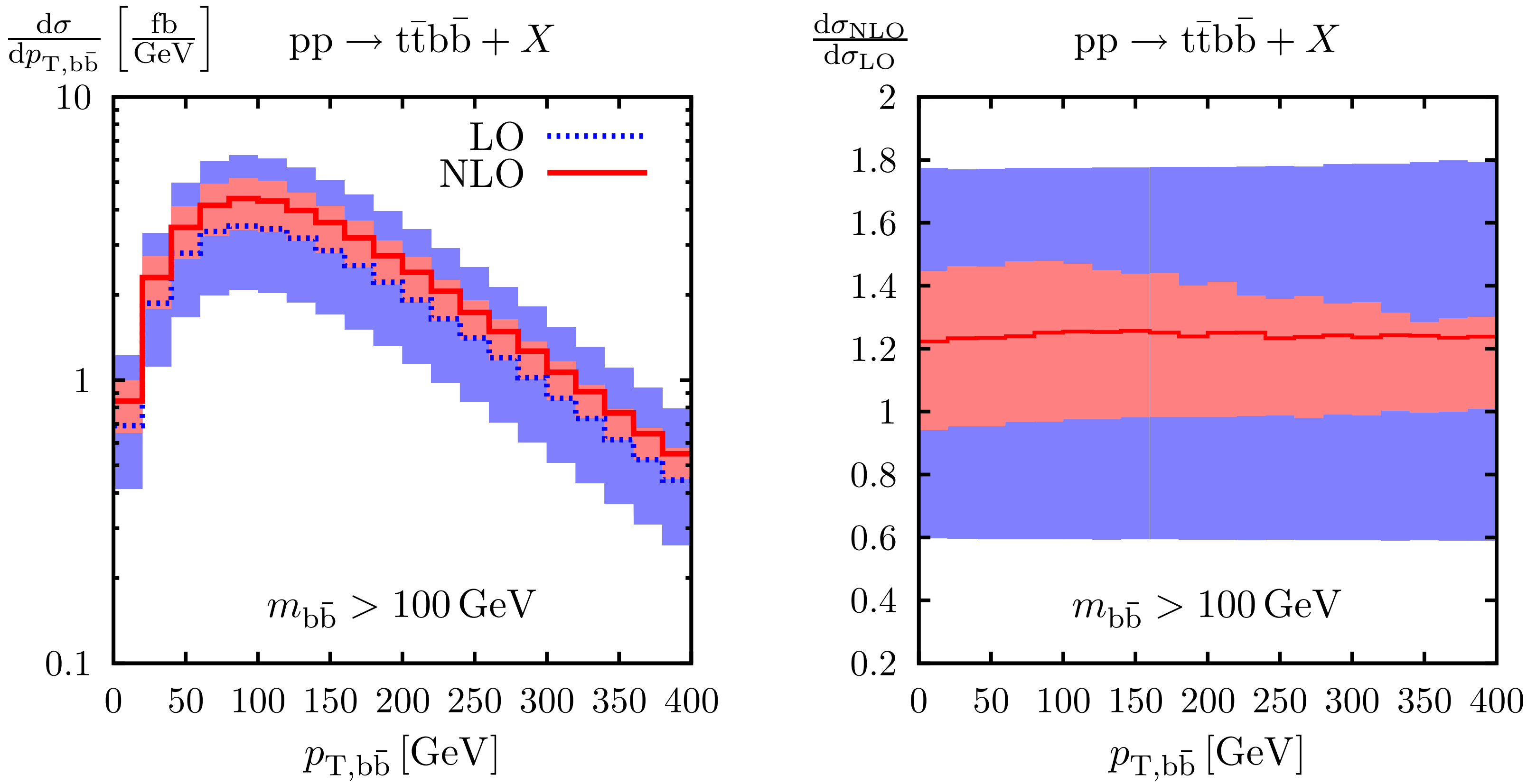
- small NLO correction $K \simeq 1.24$
- reduction of scale uncertainty

$$\Delta_{\mathrm{LO}} \sim 100\% \qquad \to \qquad \Delta_{\mathrm{NLO}} \sim 20\text{–}30\%$$

Exemplary results for $\mathrm{pp} \to \mathrm{t\bar{t}b\bar{b}}$

Distribution in transverse momentum of $\mathrm{b\bar{b}}$ pair

Bredenstein, Denner, Dittmaier, Pozzorini '09

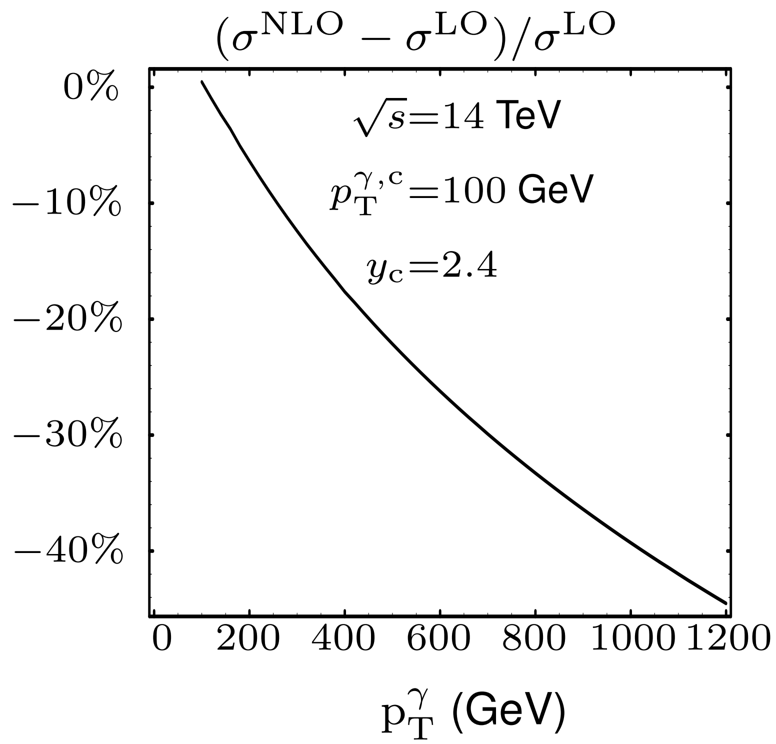


- K-factor almost constant over wide $p_{\mathrm{T,b\bar{b}}}$ range for dynamical scale
- NLO-analysis enables suitable dynamic scale choice
  $\Rightarrow$ improvement of LO prediction via rescaling

Relevance of EWRC

Relevance EW NLO corrections at LHC

- generically: $\mathcal{O}(\alpha) \sim \mathcal{O}(\alpha_\mathrm{s}^2) \sim$ few %
- EW corrections can be enhanced by high energy scales or kinematic effects
  example: electroweak corrections to $\mathrm{pp} \to \mathrm{Z}\gamma + X$ Hollik, Meier '04



small p_T^γ

- corrections of $\mathcal{O}(\alpha) \sim 1\%$

$p_\mathrm{T}^\gamma \gg 100$ GeV

- large negative corrections $\gg 1\%$
- increase with p_T^γ
- -40% at $p_\mathrm{T}^\gamma \sim 1\,\mathrm{TeV}$!

- leading NNLO EW corrections might be relevant for some processes
  (Drell–Yan, Z+jet, W+jet) $(40\%)^2 = 16\%$

Origin of large electroweak corrections

Energy scale $\gg$ characteristic scale of EW corrections:

e.g. $E \gg M_{\mathrm{W}} \approx 80\,\mathrm{GeV}$

$\Rightarrow$ large double logarithms

$$\ln^2\left(\frac{E^2}{M_{\mathrm{W}}^2}\right) \sim 25 \qquad \text{at} \quad E \sim 1\,\mathrm{TeV}$$

typical size of corrections:

$$\frac{\alpha}{\pi s_{\mathrm{w}}^2}\ln^2\left(\frac{E^2}{M_{\mathrm{W}}^2}\right) \sim 25\% \qquad \text{at} \quad E \sim 1\,\mathrm{TeV}$$

general feature of hard scattering processes!

Large EW logarithms can be related to mass singularities:

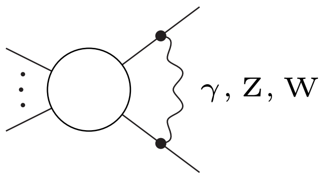
$$M_{\mathrm{W}}/E \ll 1 \quad \Rightarrow \quad E \to \infty \quad \text{or} \quad M_{\mathrm{W}} \to 0$$

EW logarithms can be calculated with process-independent methods.

Universal origin of leading EW logarithms

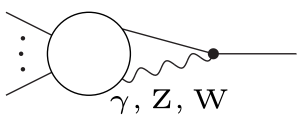
Large EW logarithms are of universal origin:

- infrared logarithms ⇔ external particles of the process
  - soft and collinear virtual gauge bosons

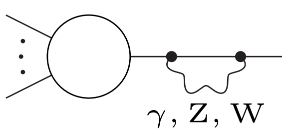


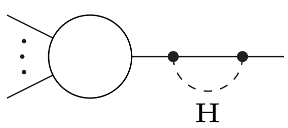
double logarithms $\alpha \ln^2 \frac{s}{M_\mathrm{W}^2}$

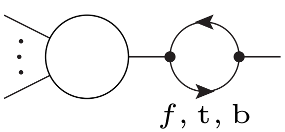
  - collinear or soft virtual gauge bosons, wave-function renormalization



single logarithms $\alpha \ln \frac{s}{M_\mathrm{W}^2}$







- ultraviolet logarithms ⇔ parameter renormalization at scale $M_\mathrm{W}^2 \ll s$
  ⇒ running of electroweak couplings from M_W to $\sqrt{s}$
  single logarithms $\alpha \ln \frac{s}{M_\mathrm{W}^2}$

⇒ (relatively) simple expression for logarithmic corrections

Summary on electroweak logarithms

- studied by many people

  M. Ciafaloni, P. Ciafaloni, Comelli; Beccaria, Renard, Verzegnassi; Beenakker, Werthenbach; Denner, Pozzorini; Melles; Fadin, Lipatov, Martin; Hori, Kawamura, Kodaira; Jantzen, Kühn, Penin, Smirnov; Chiu, Fuhrer, Golf, Kelley, Manohar, ...
- provide simple estimate for one-loop corrections at level of 5–10%
- useful to estimate electroweak two-loop corrections
- real corrections should be included
  - real photon radiation $\Rightarrow$ large effects
  - real massive vector-boson radiation Baur '06

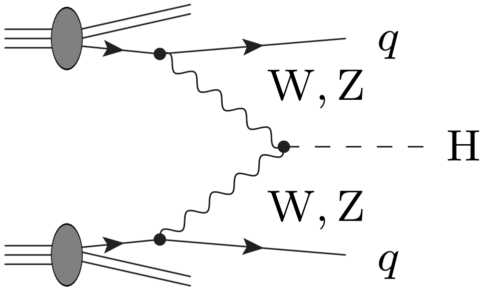
  $\Rightarrow$ partial cancellation of enhanced corrections
- not reliable for processes with other sources of large contributions
  e.g. large logarithms $\log(t/s) \sim 2\log\theta$ for small θ not included (important for processes with large contributions in forward/backward directions: e.g. large M_{ll} (but small t) in Drell–Yan))
- at LHC often sizeable contributions from energies below $1\,\mathrm{TeV}$

$\Rightarrow$ exact calculations of NLO EWRC preferable if possible

Example processes

Higgs production via vector-boson fusion

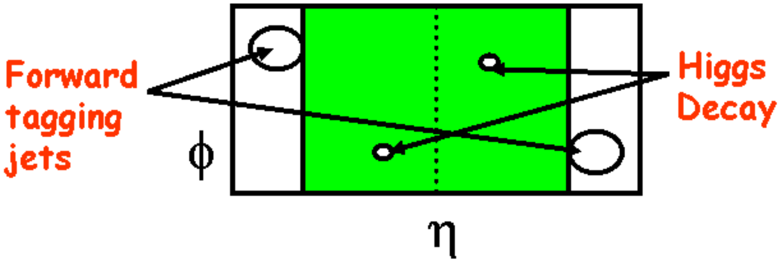
Higgs production via vector-boson fusion (VBF)



process dominated by t- and u-channel diagrams
$\Rightarrow$ t-channel approximation (DIS-like)

dominant contribution has two forward jets $\Rightarrow$ tags

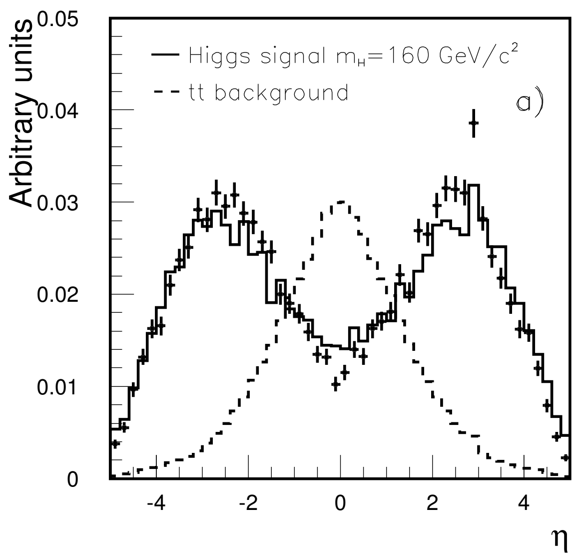
signature $=$ Higgs + 2 jets

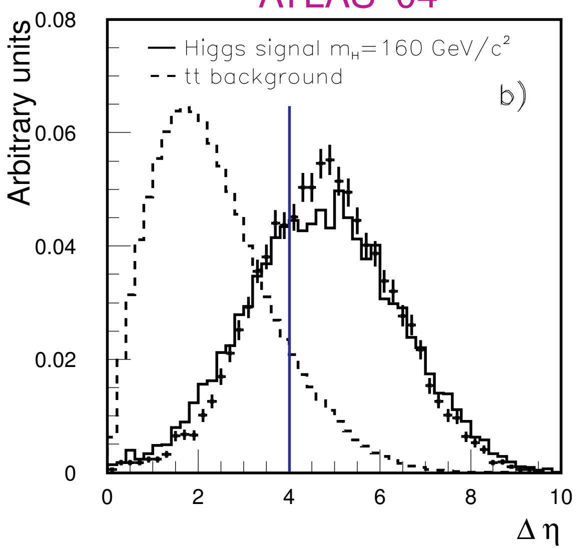


VBF cuts and background suppression:

- 2 hard "tagging" jets demanded:
  $p_{\mathrm{Tj}} > 20\,\mathrm{GeV}, \quad |y_{\mathrm{j}}| < 4.5$
- tagging jets forward–backward directed:
  $\Delta y_{\mathrm{jj}} > 4, \quad y_{\mathrm{j}_1} \cdot y_{\mathrm{j}_2} < 0$

ATLAS '04



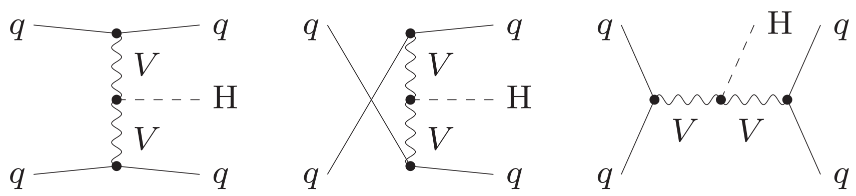


$$\eta = -\ln\tan\left(\frac{\theta}{2}\right) \sim y$$

Details of the NLO calculation for H + 2 jets

EW production of Higgs+2 jets in LO

- many subcontributions from qq, $q\bar{q}$, and $\bar{q}\bar{q}$ channels ($q = \mathrm{u}, \mathrm{d}, \mathrm{c}, \mathrm{s}, \mathrm{b}$)
- each channel receives contributions from one or two topologies ("t, u, s"):

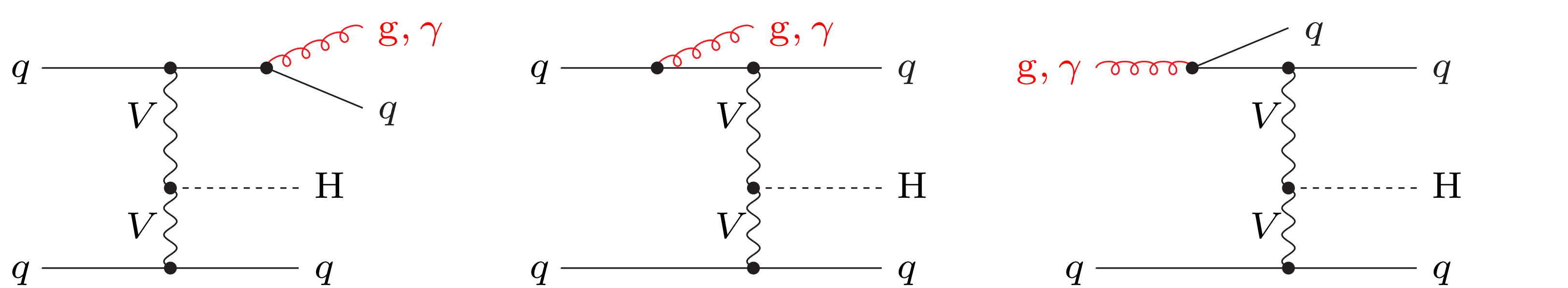


different channels related by crossing

$V = \mathrm{W}, \mathrm{Z}$

EW production of Higgs+2 jets in NLO

- partonic channels for
  - one-loop diagrams: qq, $q\bar{q}$, $\bar{q}\bar{q}$ ($\mathcal{O}(200)$ diagrams per tree diagram)
  - real QCD corrections qq, $q\bar{q}$, $\bar{q}\bar{q}$ (gluon emission), $q\mathrm{g}$, $\bar{q}\mathrm{g}$ (gluon induced)
  - real QED corrections qq, $q\bar{q}$, $\bar{q}\bar{q}$ (photon emission), $q\gamma$, $\bar{q}\gamma$ (photon induced)

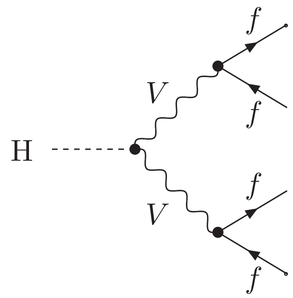


Status of predictions to Higgs+2 jets production

- NLO QCD corrections to VBF in "t-channel approximation" (DIS-like)
  - total cross section — Han, Valencia, Willenbrock '92; Spira '98; Djouadi, Spira '00
  - realistic cuts, distributions — Figy, Oleari, Zeppenfeld '03; Berger, Campbell '04
  - matching with parton shower (POWHEG) — Nason, Oleari '09
- NLO QCD corrections to gluon-initiated channels — Campbell, R.K.Ellis, Zanderighi '06
  - contribution to VBF $\sim\ 5\%$ — Nikitenko, Vázquez Acosta '07
    (NLO scale uncertainty $\sim 35\%$)
  - matching with parton shower (POWHEG) — Ellis, Campbell, Frederix '12
- complete NLO QCD+EW corrections to VBF — Ciccolini, Denner, Dittmaier '07 (HAWK); Figy, Palmer, Weiglein '10 (VBF@NLO)
  $\hookrightarrow$ NLO QCD $\sim$ NLO EW $\sim\ 5{-}10\%$
- NNLO QCD corrections to VBF in DIS-like approximation — Bolzoni, Maltoni, Moch, Zaro '10 (VBF@NLO)
  $\hookrightarrow$ NNLO QCD $\sim\ 1{-}2\%$ for scales $\mathcal{O}(M_\mathrm{W})$
- QCD loop-induced interferences between VBF and gluon-initiated channels
  impact $\lesssim\ 10^{-3}\,\%$ (negligible!) — Andersen, Binoth, Heinrich, Smillie '07; Bredenstein, Hagiwara, Jäger '08
- loop-induced VBF in gg scattering — Harlander, Vollinga, Weber '08
  $\hookrightarrow$ impact $\sim 0.1\%$

Survey of Feynman diagrams for $\mathrm{H} + 2\,\mathrm{jets}$

Lowest order:

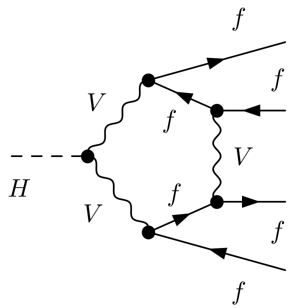


$V = \mathrm{W}, \mathrm{Z}$

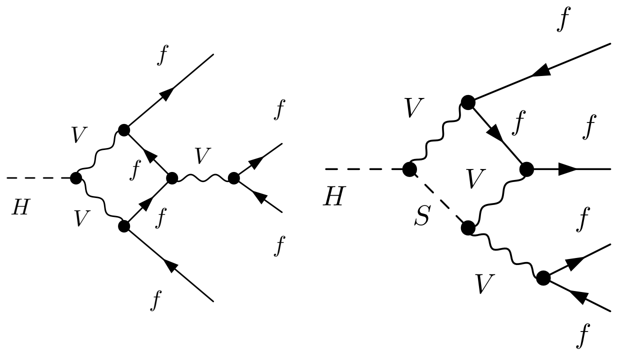
typical one-loop diagrams:

diagrams = $\mathcal{O}(200)$ per tree diagram

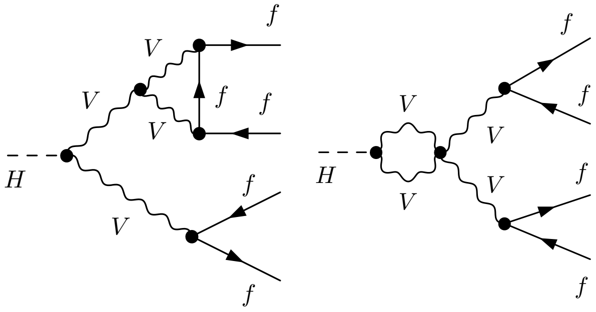
6/8 pentagons



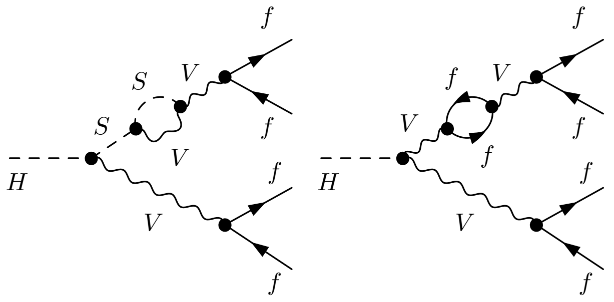
14/24 boxes



vertices



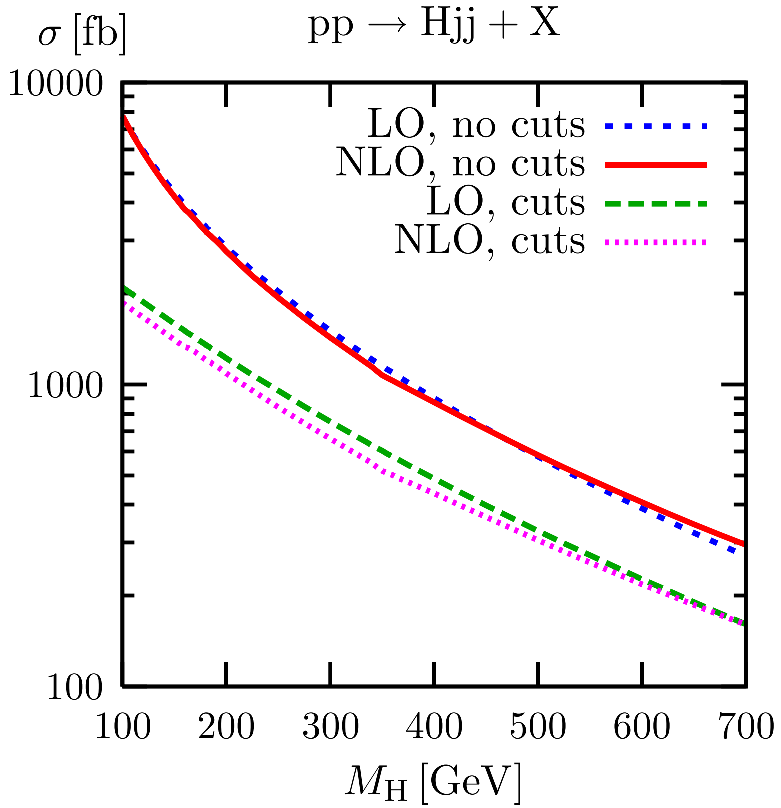
self-energies

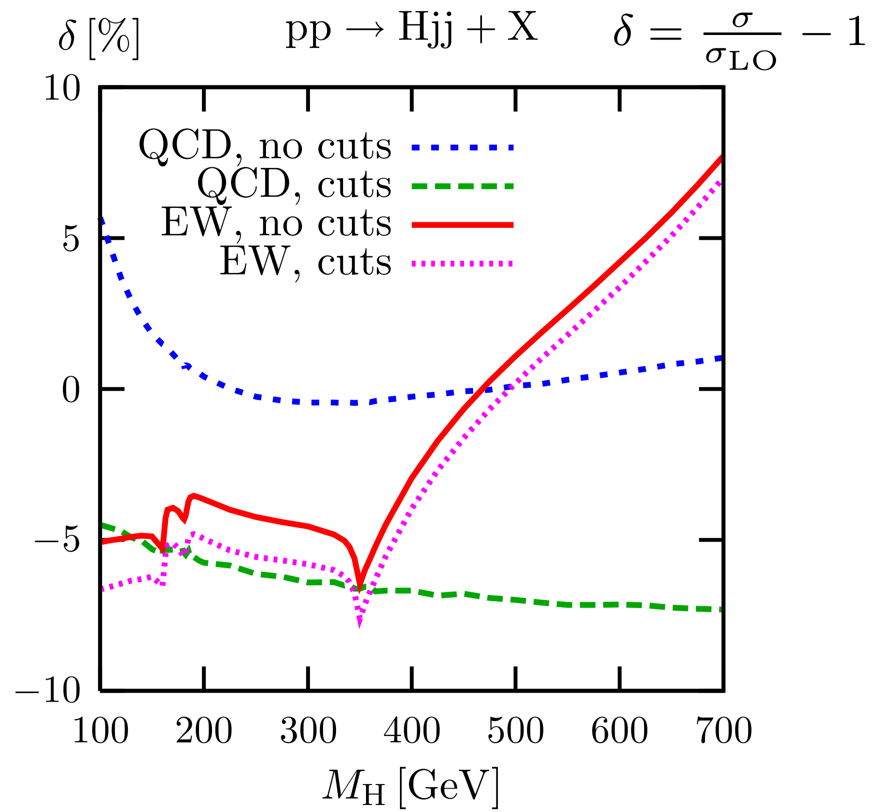


$+$ tree graphs with real photons and gluons

NLO QCD+EW corrections

Ciccolini, Denner, Dittmaier '07 (HAWK)



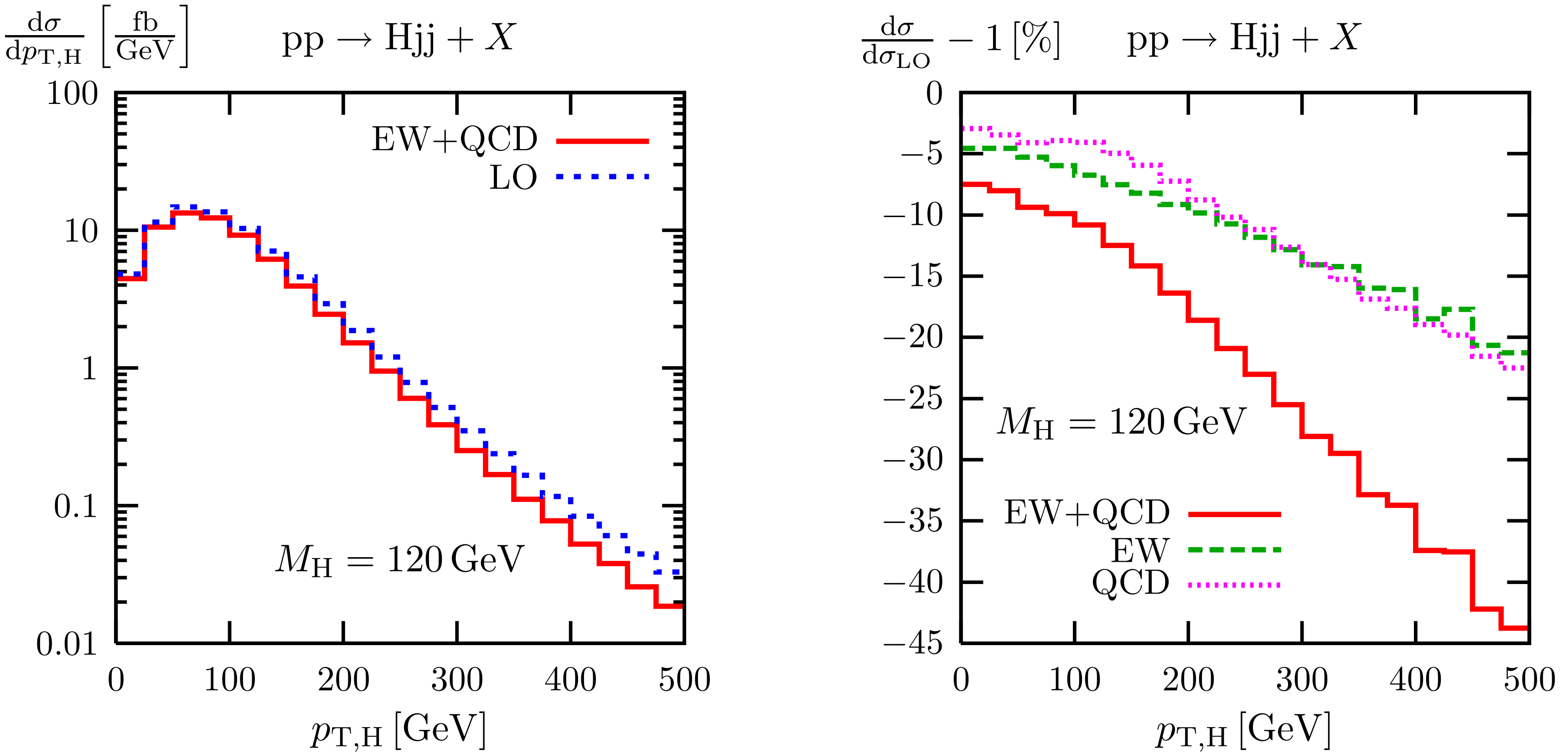


- QCD and EW corrections of same generic size ($\sim 5\%$)
  $M_{\mathrm{H}} = 126\,\mathrm{GeV}$: $\delta_{\mathrm{EW}} = -7\%/-5\%$ with/without cuts
  $\delta_{\mathrm{QCD}} = -5\%/+3\%$ with/without cuts (strongly depending on PDFs)
- scale uncertainty $\sim$ 2–3% within $M_{\mathrm{W}}/2 < \mu_{\mathrm{R/F}} < 2M_{\mathrm{W}}$ in NLO ($\sim 10\%$ in LO)
- corrections $\propto M_{\mathrm{H}}^2$: breakdown of perturbation theory for $M_{\mathrm{H}} \sim 700\,\mathrm{GeV}$

Transverse momentum of Higgs boson

VBF cuts

Ciccolini, Denner, Dittmaier ’07 (HAWK)

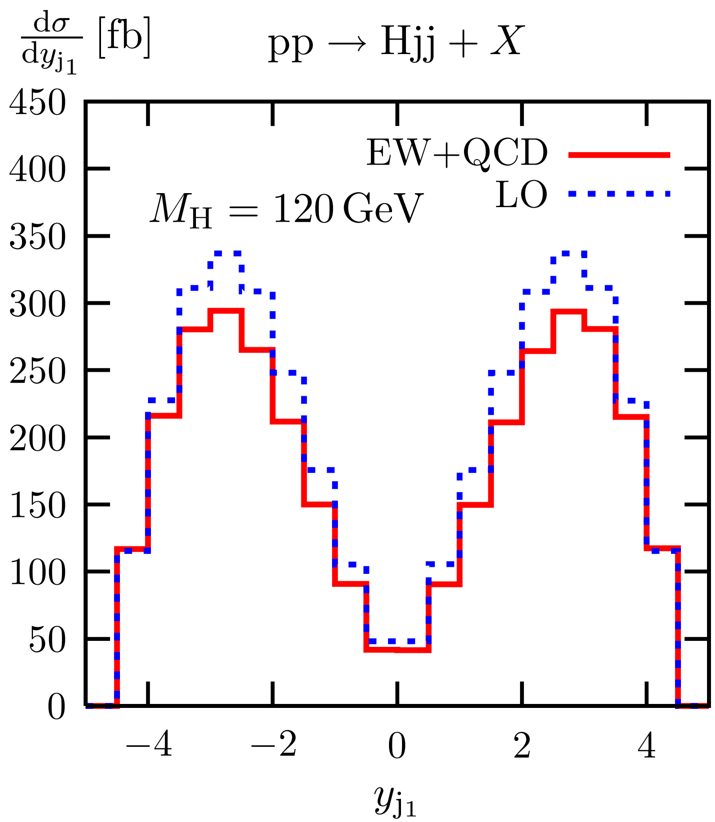


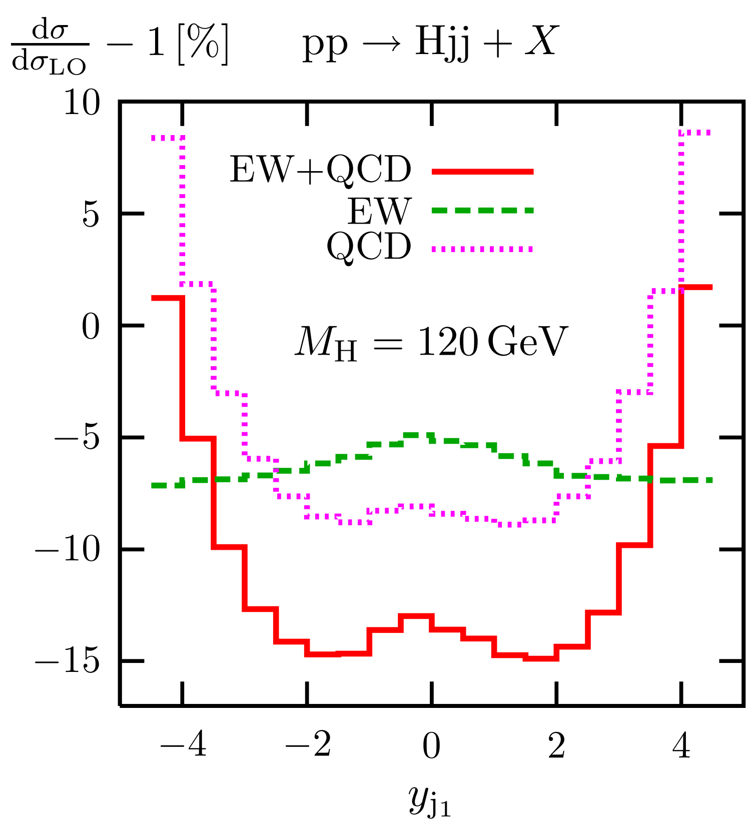
- EW and QCD corrections similar
- both distort shape of distribution
- EW corrections -20% at $p_{T,H} = 500\,\mathrm{GeV}$, from electroweak logarithms!

Rapidity of the harder tagging jet

VBF cuts

Ciccolini, Denner, Dittmaier ’07 (HAWK)



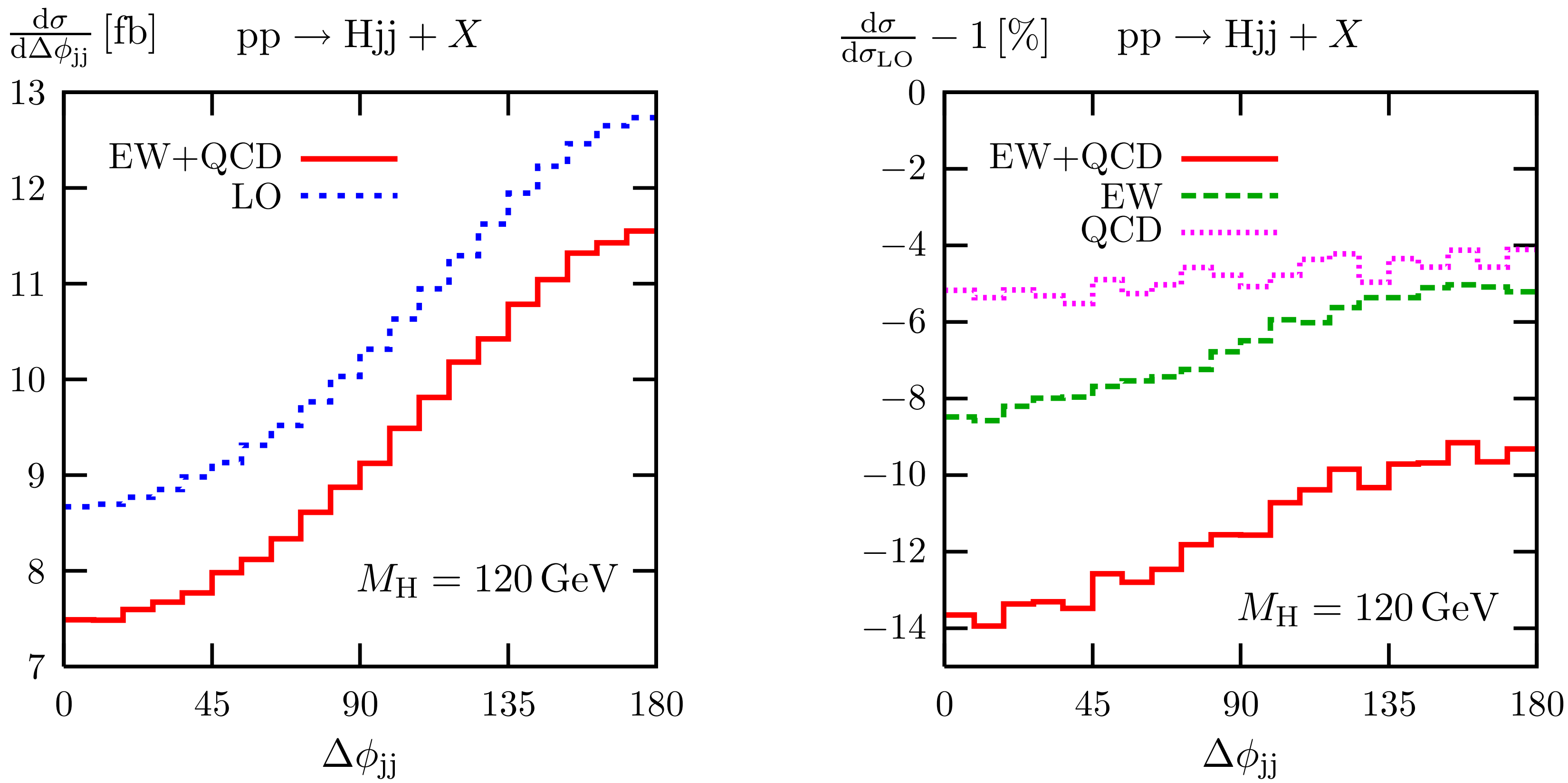


- tagging jets forward–backward
- QCD corrections distort shape significantly
- EW corrections depend only weakly on rapidity y_{j_1} $(-4\% - -7\%)$

Azimuthal angle separation of tagging jets

VBF cuts

Ciccolini, Denner, Dittmaier '07 (HAWK)



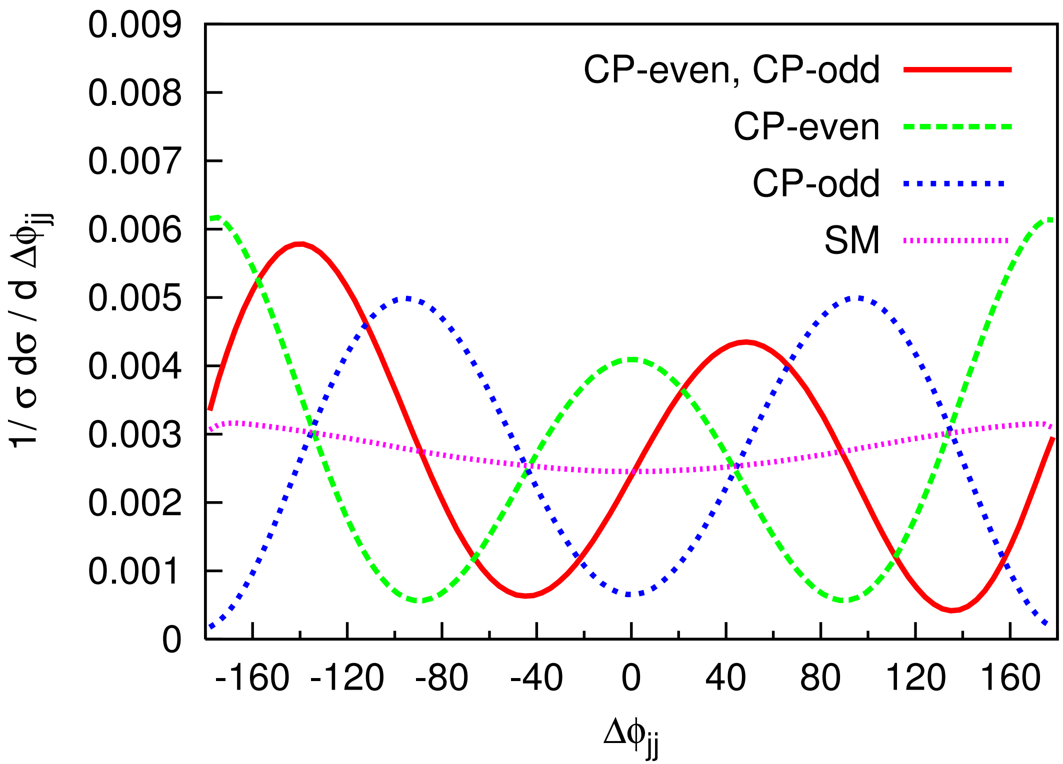
distribution in $\Delta\phi_{jj}$ sensitive to non-standard HVV couplings Figy, Zeppenfeld '04

EW corrections yield distortion of distribution by 4%

WWH and ZZH coupling analyses

- Higgs via VBF plays important role in Higgs-coupling analysis   Dührssen et al. '04
- azimuthal angle difference $\Delta\phi_{\mathrm{jj}}$ of the tagging jets is sensitive to BSM effects

Hankele, Klämke, Zeppenfeld, Figy '06
Ruwiedel, Schumacher, Wermes '07



Individual contributions without SM,
$M_{\mathrm{H}} = 120\,\mathrm{GeV}$
plot from Hankele et al.

CP-even: $\mathcal{L} \propto HW^{+}_{\mu\nu}W^{-,\mu\nu}, \qquad \Gamma^{HW^+W^-}_{\mu\nu} \propto g_{\mu\nu}(k_+k_-) - k_{+,\nu}k_{-,\mu}$

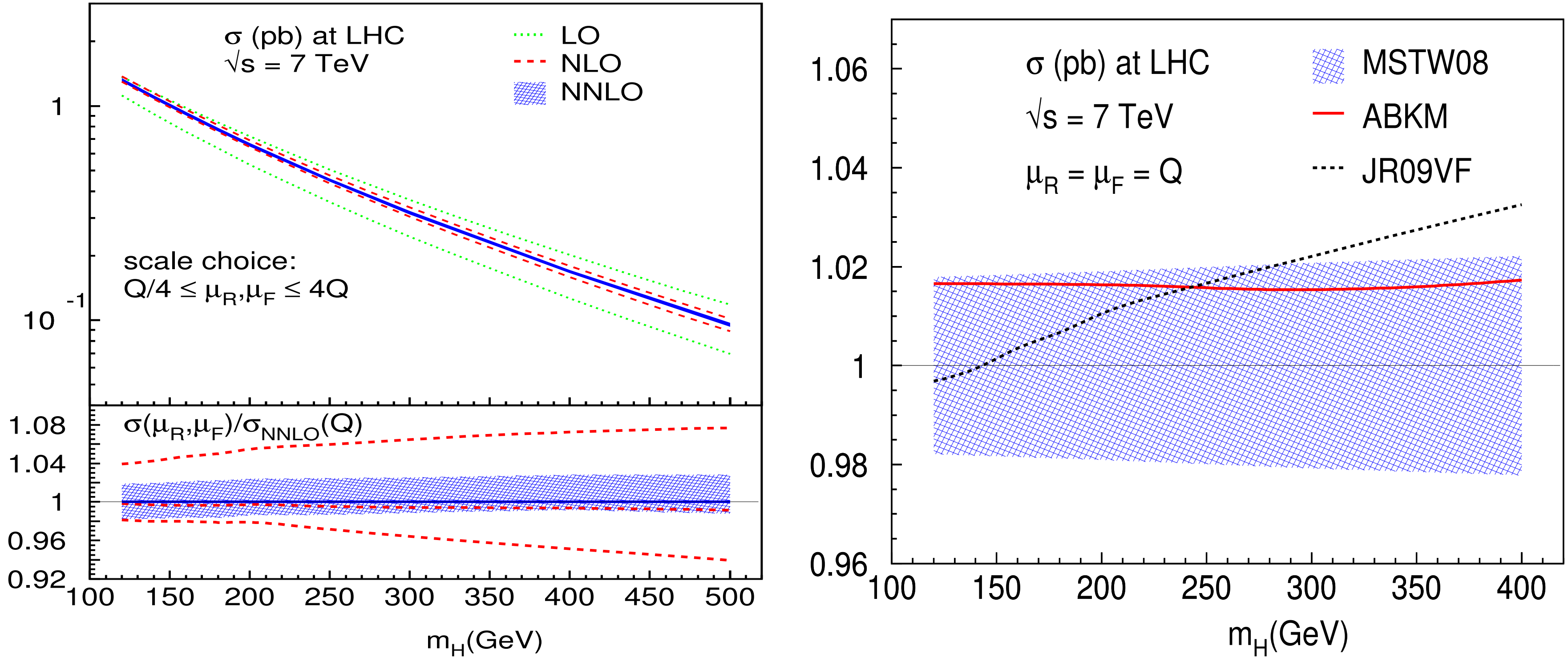
CP-odd: $\mathcal{L} \propto H\tilde{W}^{+}_{\mu\nu}W^{-,\mu\nu}, \qquad \Gamma^{HW^+W^-}_{\mu\nu} \propto \epsilon_{\mu\nu\rho\sigma}k^{\rho}_{+}k^{\sigma}_{-}$

NNLO QCD corrections to total VBF cross section

Bolzoni, Maltoni, Moch, Zaro '10

Scale uncertainty

PDF uncertainty (68% C.L.)

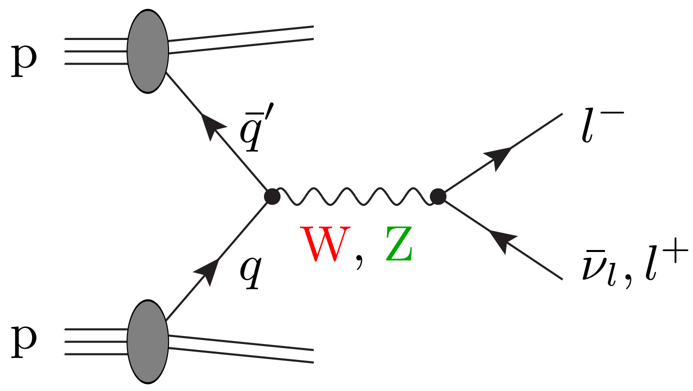


Results for total cross section at LHC:

- NNLO QCD corrections $\sim 1\%$ with scale $Q =$ virtuality of W/Z $= \mathcal{O}(M_\mathrm{W})$
- scale uncertainty $\sim$ PDF uncertainty $\sim 2\%$ (MSTW2008NNLO)

Single gauge-boson production

Drell–Yan-like W and Z production



large cross sections: $\sigma(\mathrm{W}) = 30\,\mathrm{nb}$
$\sigma(\mathrm{Z}) = 3.5\,\mathrm{nb}$

Physics issues:

- σ $\to$ standard candle
- M_Z $\to$ detector calibration by comparing with LEP1 result
- $\sin^2\theta_\mathrm{eff}^\mathrm{lept}$ with $\delta\sin^2\theta_\mathrm{eff}^\mathrm{lept} \sim 0.00014$ $\to$ comparison with results of LEP1 and SLC
- M_W $\to$ improvement to $\Delta M_\mathrm{W} \sim 15\,\mathrm{MeV}(7\,\mathrm{MeV})$, strengthen EW precision tests
  Besson et al. '08
- decay widths Γ_Z and Γ_W from M_{ll} or $M_{\mathrm{T},l\nu_l}$ tails
- search for Z' and W' at high M_{ll} or $M_{\mathrm{T},l\nu_l}$
- information on PDFs, determination of collider luminosity

Known corrections to W/Z production

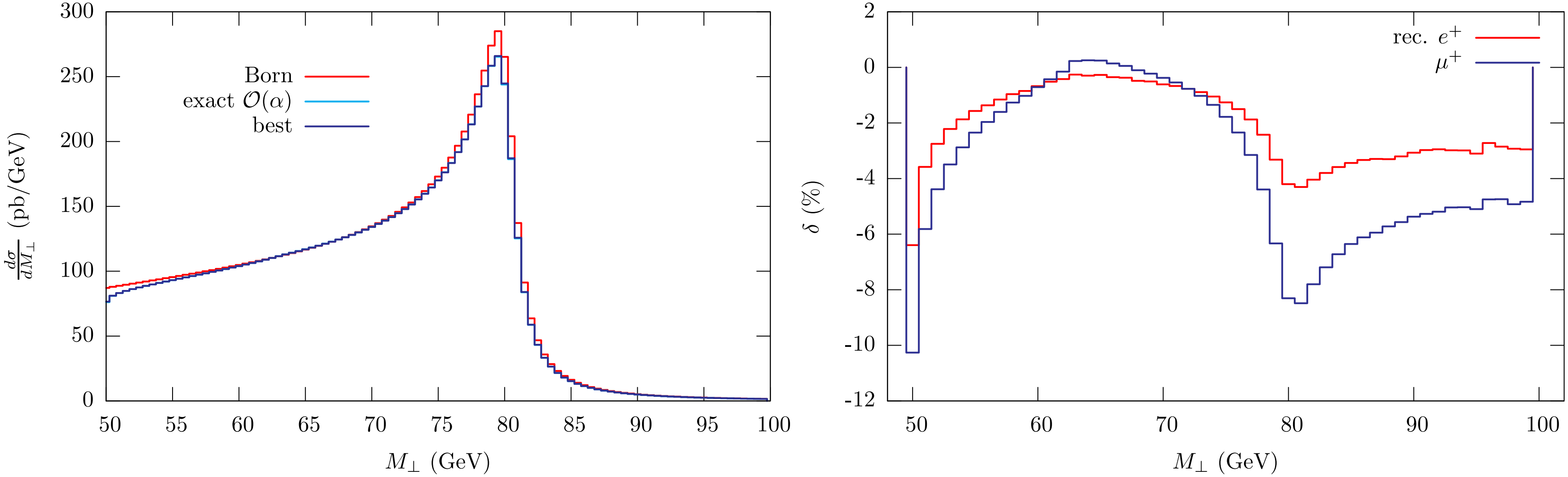
- NLO QCD corrections merged with QCD parton showers MC@NLO, POWHEG — Frixione, Webber ’02; Frixione, Nason, Oleari ’07
- NNLO QCD corrections
  - total cross section — Hamberg, v.Neerven, Matsuura ’91; v.Neerven, Zijlstra ’92; Harlander, Kilgore ’02
  - distributions — Anastasiou et al. ’03; Melnikov, Petriello ’06; Catani et al. ’09
- soft + virtual N^{3}LO QCD corrections — Moch, Vogt ’05; Laenen, Magnea ’05; Idilbi et al. ’05; Ravindran, Smith ’07
- soft gluon resummation — Balazs, Yuan ’97; Ellis, Veseli ’98; Landry et al. ’02; Cao, Yuan ’04; Berge et al. ’05; Bozzi et al. ’08
- NLO EW corrections to W production — Baur, Keller, Wackeroth ’98; Zykunov ’01; Dittmaier, Krämer ’01; Baur, Wackeroth ’04; Arbuzov et al. ’05; Carloni Calame et al. ’06; Brensing et al. ’07
- NLO EW corrections to Z production — Baur, Keller, Sakumoto ’97; Baur, Wackeroth ’99; Brein, Hollik, Schappacher ’99; Baur et al. ’02; Zykunov ’05; Arbuzov et al. ’06; Carloni Calame et al. ’07; Dittmaier, Huber ’09
- multi-photon radiation via leading logs — Baur, Stelzer ’99; Carloni Calame et al. ’03, ’05; Placzek, Jadach ’03; Brensing et al. ’07; Dittmaier, Huber ’09
- photon-induced processes — Dittmaier, Krämer ’06; Arbusov, Sadykov ’07; Brensing et al. ’07; Carloni Calame et al. ’07; Dittmaier, Huber ’09
- POWHEG matching of QCD/EW corr. — Bernaciak, Wackeroth ’12; Barze et al. ’13

W transverse mass distribution

transverse mass $M_{\mathrm{W,T}} = \sqrt{2p_\perp^l p_\perp^\nu (1 - \cos\phi_{l\nu})}$

- Jacobian peak at W mass relatively insensitive to QCD ISR

Carloni Calame et al. '06 LHC14



- final-state photon radiation distorts Breit–Wigner resonance (kinematic effect!)
  logarithmic corrections $\propto (\alpha/\pi)\log(M_V^2/m_l^2)$
  $\Rightarrow$ shift in extracted W mass: $\delta M_{\mathrm{W}} \sim -170(60)\,\mathrm{MeV}$ for $\mathrm{W} \to \mu\nu(\mathrm{e}\nu)$
  partial KLN cancellation for recombined electrons
- full EW $\mathcal{O}(\alpha)$ corrections: $\delta M_{\mathrm{W}} \sim 10\,\mathrm{MeV}$ Baur, Keller, Wackeroth. '99
- multiple final-state photon radiation: $\delta M_{\mathrm{W}} \sim 10(2)\,\mathrm{MeV}$ Carloni Calame et al. '04

Combination of electroweak and QCD corrections

crucial for W and Z precision measurements but difficult beyond NLO

- additive or multiplicative combination of NLO QCD and EW correction
  Balossini et al. '07, '09 (see also Bernaciak, Wackeroth '12)

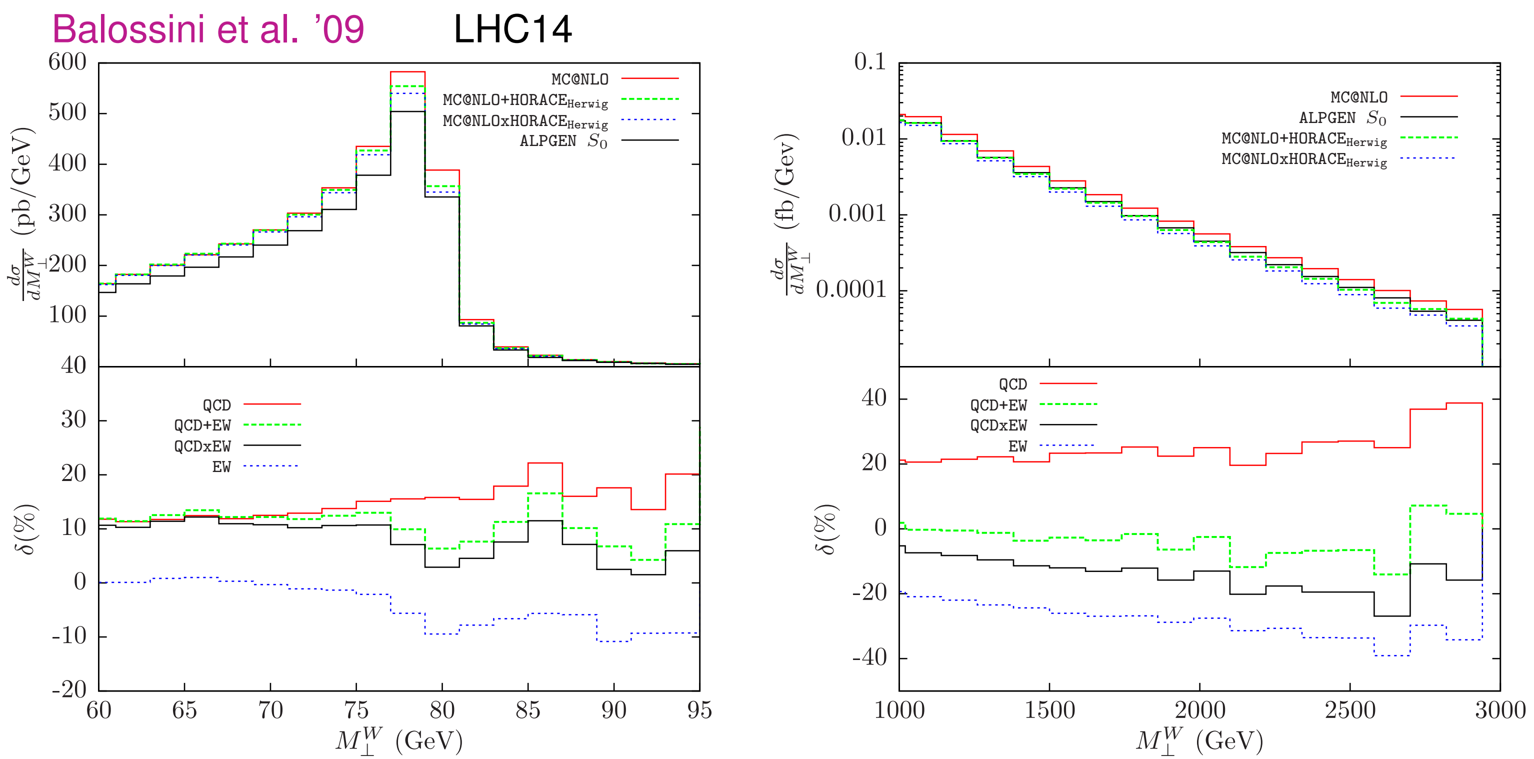
$$\begin{aligned} \mathrm{d}\sigma_{\mathrm{QCD}\oplus\mathrm{EW}} &= \mathrm{d}\sigma_{\mathrm{QCD}} + \{\mathrm{d}\sigma_{\mathrm{EW}} - \mathrm{d}\sigma_{\mathrm{LO}}\}_{\mathrm{HERWIGPS}} \\ \mathrm{d}\sigma_{\mathrm{QCD}\otimes\mathrm{EW}} &= \left(1 + \frac{\mathrm{d}\sigma_{\mathrm{QCD}} - \{\mathrm{d}\sigma_{\mathrm{LO}}\}_{\mathrm{HERWIGPS}}}{\mathrm{d}\sigma_{\mathrm{LO/NLO}}}\right) \{\mathrm{d}\sigma_{\mathrm{EW}}\}_{\mathrm{HERWIGPS}} \end{aligned}$$

  EW = HORACE, QCD = MC@NLO Frixione, Webber '02

  prescriptions agree at $\mathcal{O}(\alpha_\mathrm{s}) + \mathcal{O}(\alpha)$ but differ at $\mathcal{O}(\alpha\alpha_\mathrm{s})$

- beyond additive approximation full two-loop $\mathcal{O}(\alpha\alpha_\mathrm{s})$ analysis needed
  some partial results exist:
  - virtual $\mathcal{O}(\alpha\alpha_\mathrm{s})$ corrections to quark–gauge-boson vertex Kotikov, Kühn, Veretin '07
  - virtual $\mathcal{O}(\alpha\alpha_\mathrm{s})$ corrections to Drell–Yan Kilgore, Sturm '11 (QED $\times$ QCD); Bonciani '11
  - $\mathcal{O}(\alpha\alpha_\mathrm{s})$ corrections to inclusive hadronic W-boson decay Kara '13
  - nonfactorizable $\mathcal{O}(\alpha\alpha_\mathrm{s})$ to Drell–Yan Dittmaier, Huss, Schwinn '13

Transverse W-mass distribution



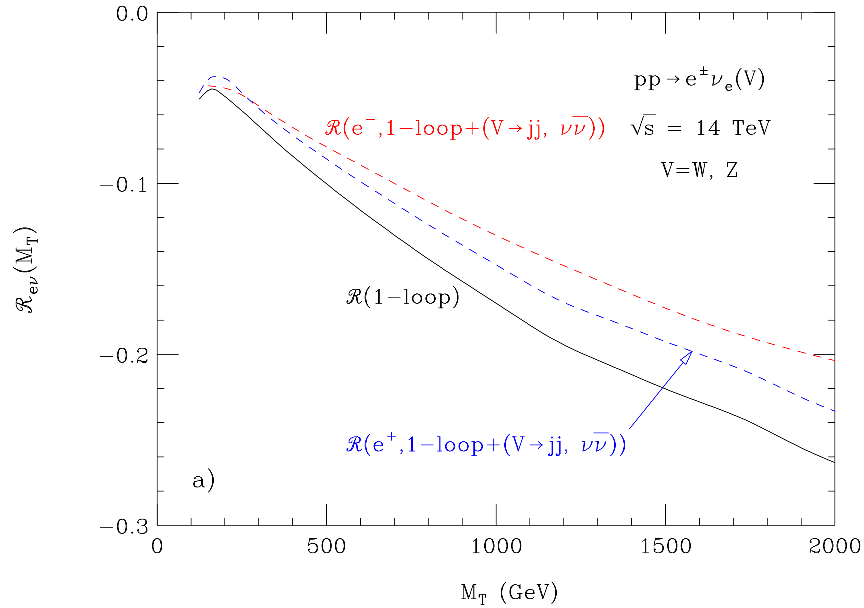
- corrections relative to LO+PS (ALPGEN S0)
- $M_{\mathrm{W,T}} \sim M_{\mathrm{W}}$: negative EW corrections compensate positive QCD corrections
  EW corrections mandatory around Jacobian peak (-10%)
- $M_{\mathrm{W,T}} \gg M_{\mathrm{W}}$: large negative EW corrections (Sudakov logarithms)
  cancel positive QCD corrections
- different ways of combining QCD and EW corrections differ at per-cent level
  $\Rightarrow \mathcal{O}(\alpha\alpha_{\mathrm{s}})$ calculation needed at least in resonance region

Effects of real weak-boson radiation

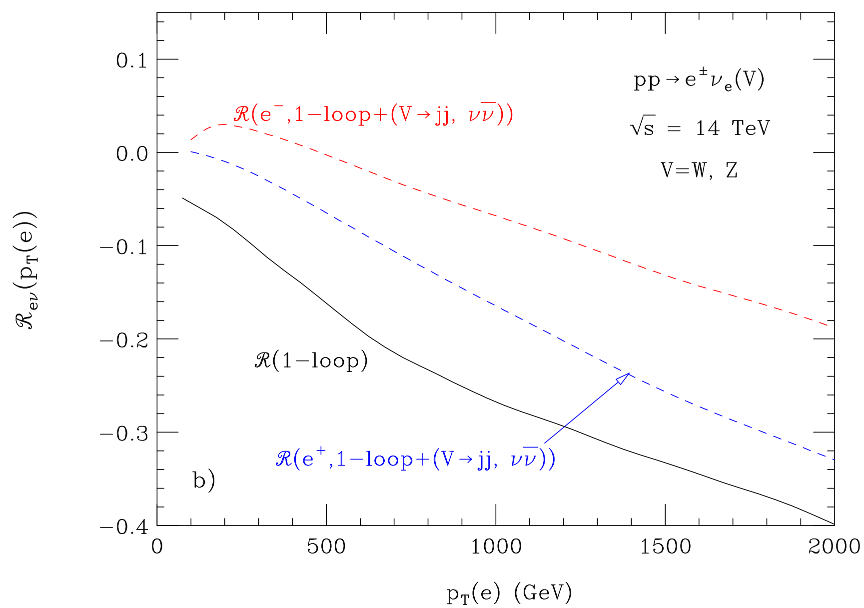
- Virtual EW corrections enhanced by large logarithms at high energy scales
- for realistic cuts real massive gauge-boson radiation partially cancels virtual EW corrections

Baur '06

transverse $e\nu$ mass



transverse electron momentum



- $M_{\mathrm{T}}(\mathrm{e}\nu) = 2\,\mathrm{TeV}$: reduction of corrections from -26% to $-23\%/-22\%$
- $p_{\mathrm{T}}(\mathrm{e}) = 1\,\mathrm{TeV}$: reduction of corrections from -28% to $-17\%/-7\%$

Conclusions

Conclusions

NLO QCD corrections

- crucial for reliable predictions of cross sections and distributions
- typically some 10%, but may be strongly enhanced
- rapid progress in calculational techniques during recent years
  - successes of unitarity-based techniques (first $2 \to 5/6$ results)
  - development of recursive techniques
- $2 \to 4$ calculations state of the art, first $2 \to 5/6$ calculations available
- automation in full progress

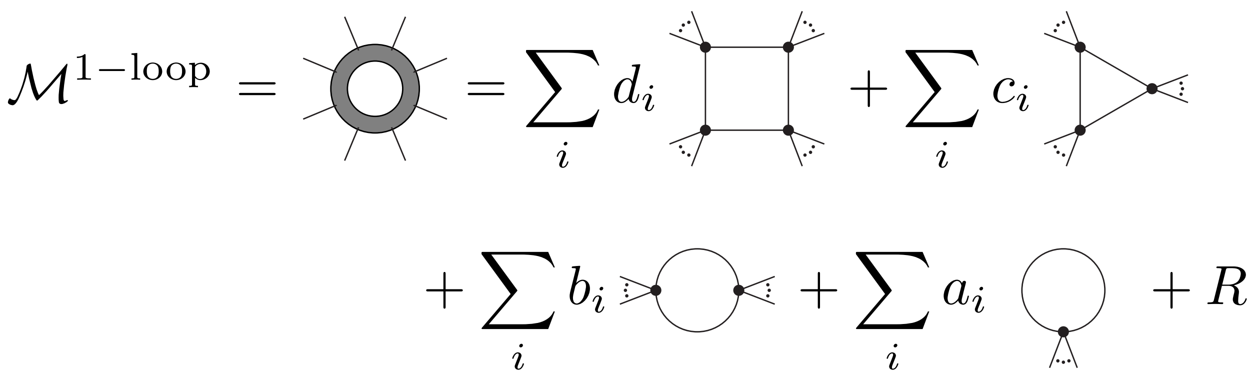
NLO EW corrections

- typically at level of few $\%$ to 10%
  $\hookrightarrow$ important for precise measurements
- strongly enhanced in some kinematic regions
  (high energy scales, resonance regions)
- only few active groups, growing importance, automation started

Backup slides

Structure of one-loop amplitudes

Any one-loop amplitude is a linear combination of scalar one-loop integrals



$$= \sum_l d_l D_0(l) + \sum_k c_k C_0(k) + \sum_j b_j B_0(j) + \sum_i a_i A_0(i) + R$$

- scalar one-loop integrals known: D_0, C_0, B_0, A_0
- R: rational terms = finite terms resulting from dimensional regularization $(D-4)\times 1/(D-4)$ terms
- calculation of amplitude $\Leftrightarrow$ determination of coefficients a_i, b_i, c_i, d_i and R
- representation unique, but becomes nearly degenerate in particular regions of phase space $\Rightarrow$ potential numerical instabilities

Conventional method

- Generate Feynman diagrams
- simplify each diagram (or group of diagrams) algebraically $\Rightarrow$

$$\mathcal{M}^{\sigma_k}_{i_k}(p_k) = \underbrace{\mathcal{C}_{i_k}}_{\text{factorized colour structure}} \sum_m \mathcal{F}_m(\{p_a\cdot p_b\}) \underbrace{\hat{\mathcal{M}}^{\{\lambda_k\}}_m(\{p_k\})}_{\text{standard matrix elements}}$$

- generate Fortran code automatically

features

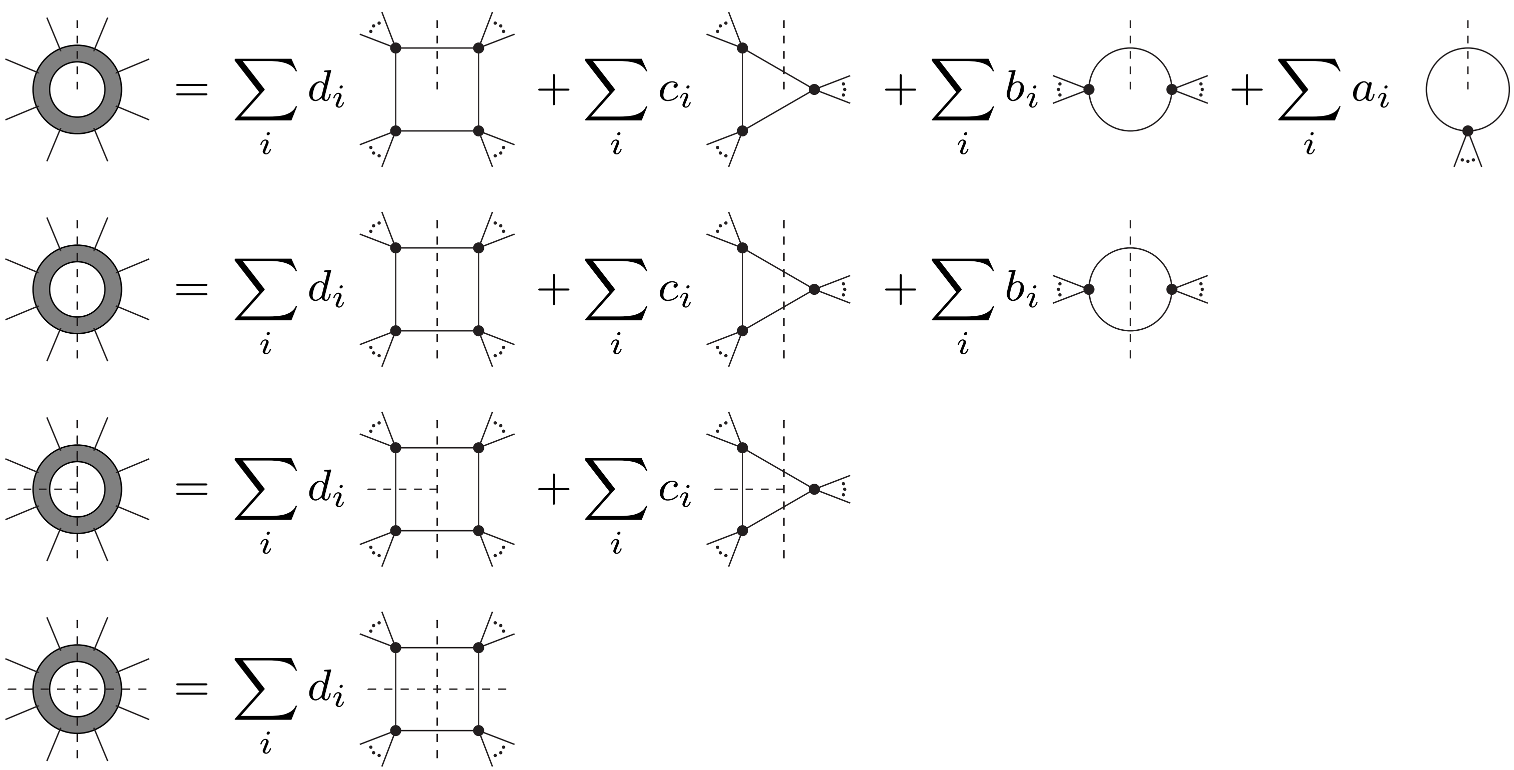
- colour structures $\mathcal{C}_{i_k}$ are factorized $\Rightarrow$ low computational cost
- standard matrix elements $\hat{\mathcal{M}}^{\{\lambda_k\}}_m(\{p_k\})$: purely kinematical
  comprise all tensorial and spinorial objects
- invariant functions $\mathcal{F}_m(\{p_a\cdot p_b\})$:
  linear combinations of tensor integral coefficients
  can be calculated depending on phase-space point using appropriate base functions, no reduction to fixed basis D_0, C_0, B_0, A_0

Generalized unitarity method

determine coefficients a_i, b_i, c_i, d_i via generalized cuts on complete amplitude

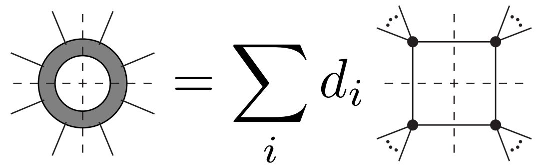
- a unitarity cut is the replacement $\qquad \dfrac{\mathrm{i}}{q^2 - m^2} \to 2\pi\delta^+(q^2 - m^2)$

  cutting a line puts its momentum on-shell
- multiple cuts of (parts of) amplitudes



$\Rightarrow$ recursive determination of coefficients of scalar integrals in terms of tree amplitudes

Box coefficients and quadruple cuts



Britto, Cachazo, Feng ’05

- in a quadruple cut, 4 propagators are replaced by δ-functions

$$\frac{1}{q_1^2-m_1^2}\frac{1}{q_2^2-m_2^2}\frac{1}{q_3^2-m_3^2}\frac{1}{q_4^2-m_4^2} \to \delta(q_1^2-m_1^2)\delta(q_2^2-m_2^2)\delta(q_3^2-m_3^2)\delta(q_4^2-m_4^2)$$

$q_i = q + K_i, \quad q_i^2 = m_i^2,\ i = 1,\ldots,4 \Rightarrow$ 4 constraints
$\Rightarrow$ cut momenta uniquely determined (in 4 dimensions): 2 solutions $q_\pm$
cut momenta $q_\pm$ are complex

- integrand of quadruple cut: product of 4 tree-level matrix elements

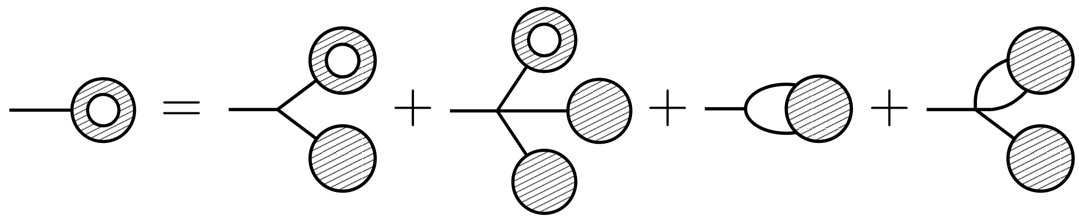
$$I_4(q) = \mathcal{M}_1^{\mathrm{tree}} \times \mathcal{M}_2^{\mathrm{tree}} \times \mathcal{M}_3^{\mathrm{tree}} \times \mathcal{M}_4^{\mathrm{tree}}$$

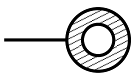
corresponding coefficient

$$d = \frac{1}{2}[I_4(q_+) + I_4(q_-)]$$

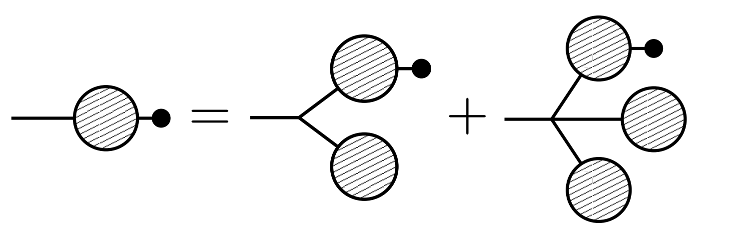
Recursive Methods

Generalize Dyson–Schwinger equations to one-loop level



: off-shell current with exactly one loop

loops are generated by last two diagrams
involve diagrams with two off-shell legs
can be constructed recursively like at tree level, but keeping one additional leg off-shell:



recursion for loop diagrams requires recursion for tensor loop integrals!

Monte Carlo integration

Integration over n-particle phase space $\int \mathrm{d}\Phi_n$ performed by Monte Carlo
($2- > 4$: 8-dimensional phase space, $2- > 5$: 11-dimensional phase space)

importance sampling:

propagators of Feynman diagrams $\Rightarrow$ peaks in matrix element
peaks are flattened with appropriate mappings

$$\mathrm{diagram} \propto \prod_i \frac{1}{p_i^2 - m_i^2}$$

all peaks of individual Feynman diagrams can be flattened with appropriate parametrizations of phase space

multi-channel method:

for each Feynman diagram a set of mappings is introduced
$\Rightarrow$ one integration channel for each Feynman diagram
Monte Carlo samples over all channels

adaptive weight optimization:

weights of different channels are adapted during Monte Carlo integration