Karlsruhe Institute of Technology

Angular analysis of $B \to \phi(K\pi)^{*0}$ at Belle and search for CP-violation

GK Workshop 2012

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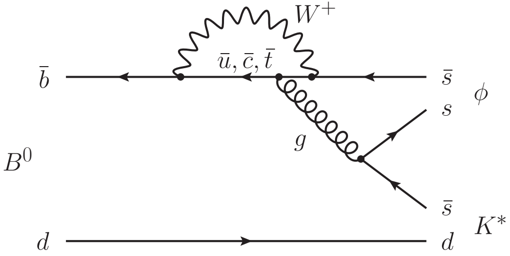
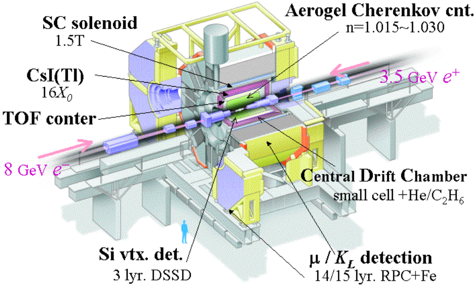
Motivation

- $B \to \phi(K\pi)^{*0}$ is a charmless B decay
- These decays are called rare decays $\mathcal{B} \approx \mathcal{O}(10^{-6})$
- Decay only via penguin loop, no tree level $b \to s$ transition in the SM
- New physics could have a comparable large effect on something that is small in the standard model (e. g. large CP-violation)
- Let's try to measure as much physics as possible!
  - Branching fractions
  - Polarization
  - CP-violation



What is the Belle experiment?

- Detector at the asymmetric $e^- e^+$ collider KEKB in Tsukuba, Japan
- KEKB operates mainly on the $\Upsilon(4\mathrm{S})$ resonance
- $\Upsilon(4\mathrm{S})$ decays almost entirely into $B\overline{B}$ pairs
- KEKB is called a *B*-factory
- $\int \mathcal{L} > 1\,\mathrm{ab}^{-1}$ with > 750 million $B\overline{B}$ pairs
- Great place to study *B*-physics
  - Initial state known
  - Clean event topology
  - No pileup



What does $B \to \phi(K\pi)^{*0}$ stand for?

- Decay of a neutral B meson into the final state $\phi K^{\pm}\pi^{\mp}$
- $(K\pi)^{*0}$ covers two-body decays via different K^* resonances and three-body non-resonant contribution
  - $B \to \phi K^*(892) \to \phi K^{\pm}\pi^{\mp}$ P-wave, Spin 1
  - $B \to \phi K_2^*(1430) \to \phi K^{\pm}\pi^{\mp}$ D-wave, Spin 2
  - $B \to \phi K_0^*(1430) \to \phi K^{\pm}\pi^{\mp}$ S-wave, Spin 0
  - $B \to \phi K^{\pm}\pi^{\mp}$ (non-resonant) S-wave, Spin 0
- Very similar topology for all decays
- We need to separate them somehow

What Belle measures

- $5 - 10$ charged tracks and a few neutral particles
- Use PID system to separate between $K^{\pm}$ and $\pi^{\pm}$
- Combine tracks, compute invariant masses and get a B candidate
  1. $K^{+}K^{-} \to \phi$
  2. $K^{\pm}\pi^{\mp} \to K^{*}$
  3. $\phi K^{*} \to B$

How to measure the $B \to \phi(K\pi)^{*0}$ system?

1. Use the invariant $K^{\pm}\pi^{\mp}$ mass to identify the K^{*} resonance
2. Use the angular distribution to learn about the K^{*} resonance

Modelling resonances

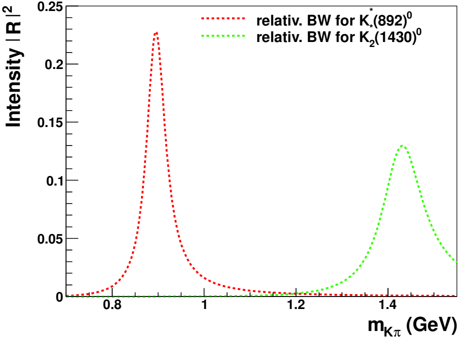
- Resonances can be modelled by relativistic spin-J Breit-Wigners

$$R_J(m) = \frac{m_J\Gamma_J(m)}{(m_J^2 - m^2) - im_J\Gamma_J(m)} = \sin\delta_J e^{i\delta_J} \quad \text{and} \quad \cot\delta_J = \frac{m_J^2 - m^2}{m_J\Gamma_J(m)}$$

- Mass dependent width of spin-J resonance

$$\Gamma_1(m) = \Gamma_1 \frac{m_1}{m} \frac{1 + r^2 q_1^2}{1 + r^2 q^2} \left(\frac{q}{q_1}\right)^3 \qquad \Gamma_2(m) = \Gamma_2 \frac{m_2}{m} \frac{9 + 3r^2 q_2^2 + r^4 q_2^4}{3 + 3r^2 q^2 + r^4 q^4} \left(\frac{q}{q_2}\right)^5$$

- m_J is resonance mass
- Γ_J is resonance width
- r is interaction radius
- q is momentum of daughter particles in the resonance system after 2-body decay
- q_J is q evaluated at m_J



Modelling the S-wave

- LASS collaboration has studied $K^-\pi^+$ scattering in the reaction $K^-p \to K^-\pi^+n$. They found the S-wave can be parametrized as

$$R_0(m) = \sin\delta_0 e^{i\delta_0} \quad \text{and} \quad \delta_0 = \Delta R + \Delta B$$

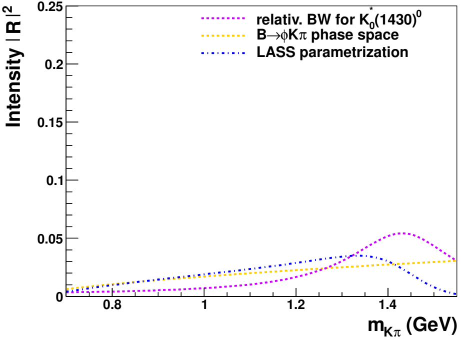
- Resonant part is a relativistic spin-0 Breit-Wigner

$$\cot\Delta R = \frac{m_0^2 - m^2}{m_0\Gamma_0(m)}$$

$$\Gamma_0(m) = \Gamma_0 \frac{m_0}{m}\left(\frac{q}{q_0}\right)$$

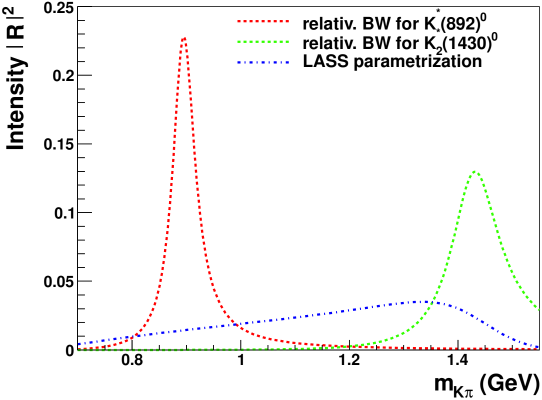
- Non-resonant part given by an effective rage parametrization

$$\cot\Delta B = \frac{1}{aq} + \frac{1}{2}bq$$



- a is scattering length
- b is the effective range

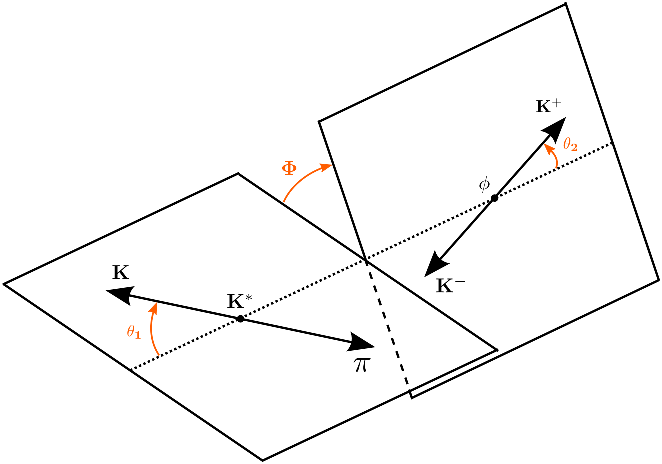
Mass distribution



We need to determine the weights A_J of the coherent sum of amplitudes R_J from data.

$$|\mathcal{M}|^2 = |\sum_J A_J R_J|^2 \neq \sum_J A_J |R_J|^2$$

Angles that define our system



From the reconstructed tracks, we can compute three angles θ_1, θ_2 and ϕ

What about the angular analysis?

- B is a pseudoscalar-meson with spin 0
- ϕ is a vector-meson with spin 1
- K^* can be scalar-, vector- or tensor-meson with spin 0,1 or 2
- Decays of $S \to VS$, $S \to VV$ or $S \to VT$ are not isotropic
- The partial angular decay width of B meson decays is given by a sum over the helicity amplitudes H_λ

$$\frac{d^3\Gamma}{d\cos\theta_1 d\cos\theta_2 d\phi} \propto \left|\sum_\lambda A_\lambda Y_{J_1}^{-\lambda}(\pi-\theta_1,-\phi) Y_{J_2}^{\lambda}(\theta_2,0)\right|^2$$

where λ takes all discrete values between $-j$ and $+j$, with $j = \min(J_1, J_2)$ and Y_J^λ are the spheric harmonics

- Thus our system is a sum of 7 different helicity amplitudes

$$S \to VS \text{ with } \lambda = 0 \quad \text{LASS}$$

$$S \to VV \text{ with } \lambda = 0, \pm 1 \quad K^*(892)$$

$$S \to VT \text{ with } \lambda = 0, \pm 1 \quad K_2^*(1430)$$

- We need to determine the weights A_λ for each amplitude from data

Angular distribution in $S \to VV$

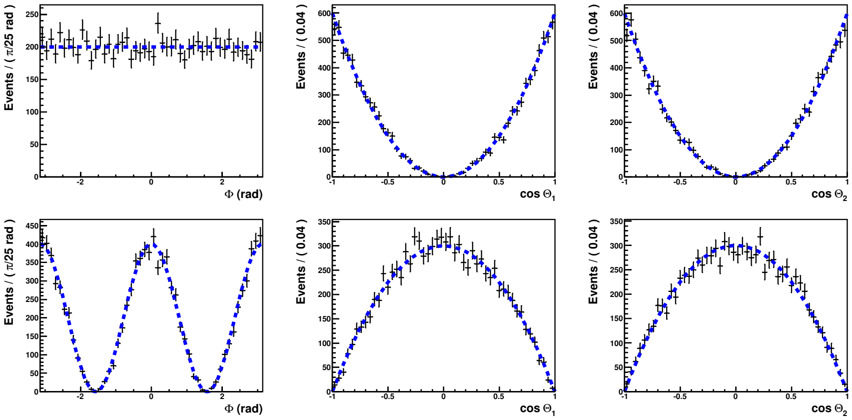


Figure: Angular distribution of a longitudinal (H_0) (**top**) or parallel ($H_\parallel = \frac{H_{+1}+H_{-1}}{\sqrt{2}}$) (**bottom**) polarized $S \to VV$ decay.

Angular distribution in $S \to VT$

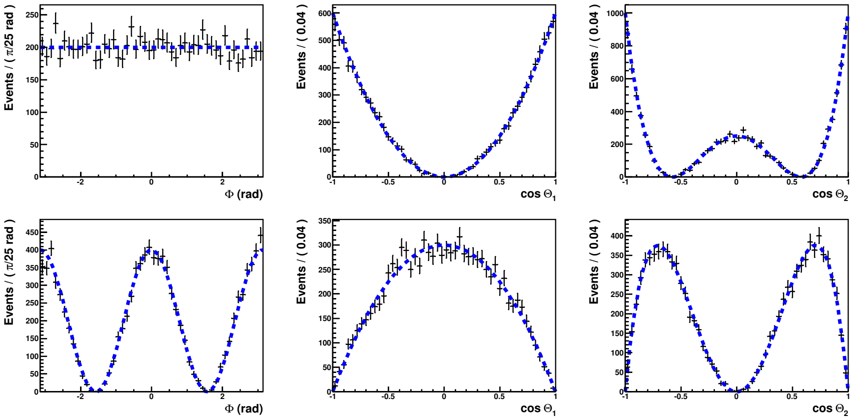


Figure: Angular distribution of a longitudinal (H_0) (**top**) or parallel ($H_{\|} = \frac{H_{+1}+H_{-1}}{\sqrt{2}}$) (**bottom**) polarized $S \to VT$ decay.

Combined 4D mass-angular model

- We need to determine the weights for the resonances A_J
- We need to determine the weights for the helicity amplitudes A_λ
- Combine this to a 4-dimensional mass-angular model

$$\frac{d^4\Gamma}{d\cos\theta_1 d\cos\theta_2 d\phi dm_{K\pi}} \propto |\mathcal{M}|^2 \times F_{m_{K\phi}}(m_{K\pi})$$

  with $F_{m_{K\phi}}(m_{K\pi})$ being a phase space factor to take into account the three body kinematic limits in $B \to \phi(K\pi)^{*0}$
- How does the combined matrix element look like?

4D mass-angular matrix element

$$|\mathcal{M}|^2 = |\mathcal{A}_0(m_{K\pi}, \theta_1, \theta_2, \phi) + \mathcal{A}_1(m_{K\pi}, \theta_1, \theta_2, \phi) + \mathcal{A}_2(m_{K\pi}, \theta_1, \theta_2, \phi)|^2$$

with each amplitude $\mathcal{A}_J$ being a sum over all possible helicity amplitudes, multiplied with its corresponding description of the invariant $K\pi$ mass distribution $R_J(m_{K\pi})$:

$$\begin{aligned}
\mathcal{A}_0(m_{K\pi}, \theta_1, \theta_2, \phi) &= A_{00} Y_0^0(\pi - \theta_1, -\phi) Y_1^0(\theta_2, 0) \times R_0(m_{K\pi}) \\
\mathcal{A}_1(m_{K\pi}, \theta_1, \theta_2, \phi) &= \sum_{\lambda=0,\pm1} A_{1\lambda} Y_1^{-\lambda}(\pi - \theta_1, -\phi) Y_1^{\lambda}(\theta_2, 0) \times R_1(m_{K\pi}) \\
\mathcal{A}_2(m_{K\pi}, \theta_1, \theta_2, \phi) &= \sum_{\lambda=0,\pm1} A_{2\lambda} Y_2^{-\lambda}(\pi - \theta_1, -\phi) Y_1^{\lambda}(\theta_2, 0) \times R_2(m_{K\pi})
\end{aligned}$$

with $A_{J\lambda}$ being the complex weight of the corresponding amplitude $\mathcal{A}_{J\lambda}$.
$\Rightarrow$ We need to determine 7 magnitudes and phases from data.

Can't there be CP-violation?

- Direct CP-violation means $A_{J\lambda} \neq \bar{A}_{J\lambda}$
- Measure the weights for $B^0 \to \phi K^+ \pi^-$ and $\overline{B}^0 \to \phi K^- \pi^+$
- Doubles the amount of parameters
- Probability density function for signal

$$P^{\pm}_{sig}(m_{K\pi}, \theta_1, \theta_2, \Phi) = \frac{|\mathcal{M}^{\pm}(m_{K\pi}, \theta_1, \theta_2, \Phi)|^2 \times F_{m_{K\phi}}(m_{K\pi})}{\mathcal{N}}$$

with the normalization $\mathcal{N}$ being averaged

$$\mathcal{N} = \frac{1}{2}\Big(\int |\mathcal{M}^+|^2 \times F_{m_{K\phi}} d\cos\theta_1 d\cos\theta_2 d\phi dm_{K\pi} + \int |\mathcal{M}^-|^2 \times F_{m_{K\phi}} d\cos\theta_1 d\cos\theta_2 d\phi dm_{K\pi} \Big)$$

Combining everything to a maximum likelihood fit

1. Signal model $P^{\pm}_{sig}(m_{K\pi}, \theta_1, \theta_2, \Phi)$
2. Background model $P_{bkg}(m_{K\pi}, \theta_1, \theta_2, \Phi)$ from MC studies
3. 4 additional observables that distinguish between signal and background: $P^{\pm}_{sig}(m_{bc}, \Delta E, m_{KK}, C'_{NB})$ and $P_{bkg}(m_{bc}, \Delta E, m_{KK}, C'_{NB})$

   - m_{bc} is the beam constraint mass of the B candidate
   - ΔE is the difference of E_B to half the beam energy
   - m_{KK} invariant $K^{\pm}K^{\mp}$ mass
   - C'_{NB} neural network to suppress continuum background

- Everything is combined in a simultaneous 8D maximum likelihood fit to determine the signal probability, the angular distribution and CP violation of the $B \to \phi(K\pi)^{*0}$ system
- I. e. we determine all the complex weights $A_{J\lambda}$ and $\bar{A}_{J\lambda}$ in a single fit

8D fit example

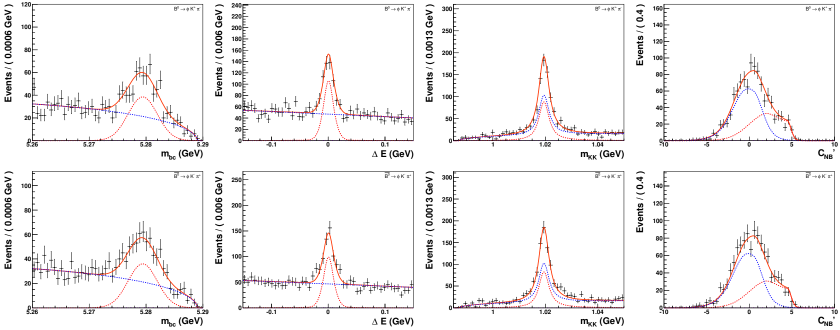


Figure: Exemplary toy MC fit and projection on m_{bc}, ΔE, m_{KK} and C'_{NB} for $B^0 \to \phi K^+ \pi^-$ (**top**) and $\overline{B}^0 \to \phi K^- \pi^+$ (**bottom**) with signal enriching cuts on the other three corresponding dimensions.

8D fit example

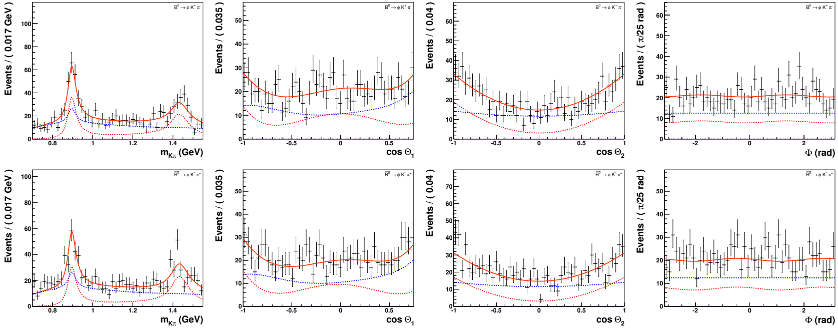


Figure: Exemplary toy MC fit and projection on $m_{K\pi}$, $\cos\theta_1$, $\cos\theta_2$ and Φ for $B^0 \to \phi K^+\pi^-$ (**top**) and $\overline{B}^0 \to \phi K^-\pi^+$ (**bottom**) with signal enriching cuts on m_{bc}, ΔE, m_{KK} and C'_{NB}.

Physics parameters

Parameter	Definition	$\phi(K\pi)^{*0}_0$ $J=0$	$\phi K^*(892)$ $J=1$	$\phi K^*_2(1430)$ $J=2$
$\mathcal{B}_J$	$\frac{1}{2}(\bar{\Gamma}_J+\Gamma_J)/\Gamma_{\text{total}}$	$\mathcal{B}_0$	$\mathcal{B}_1$	$\mathcal{B}_2$
f_{LJ}	$\frac{1}{2}(\lvert\bar{A}_{J0}\rvert^2/\sum\lvert\bar{A}_{J\lambda}\rvert^2+\lvert A_{J0}\rvert^2/\sum\lvert A_{J\lambda}\rvert^2)$	1	f_{L1}	f_{L2}
$f_{\perp J}$	$\frac{1}{2}(\lvert\bar{A}_{J\perp}\rvert^2/\sum\lvert\bar{A}_{J\lambda}\rvert^2+\lvert A_{J\perp}\rvert^2/\sum\lvert A_{J\lambda}\rvert^2)$	–	$f_{\perp 1}$	$f_{\perp 2}$
$\phi_{\parallel J}$	$\frac{1}{2}(\arg(\bar{A}_{J\parallel}/\bar{A}_{J0})+\arg(A_{J\parallel}/A_{J0}))$	–	$\phi_{\parallel 1}$	$\phi_{\parallel 2}$
$\phi_{\perp J}$	$\frac{1}{2}(\arg(\bar{A}_{J\perp}/\bar{A}_{J0})+\arg(A_{J\perp}/A_{J0})-\pi)$	–	$\phi_{\perp 1}$	$\phi_{\perp 2}$
δ_{0J}	$\frac{1}{2}(\arg(\bar{A}_{00}/\bar{A}_{J0})+\arg(A_{00}/A_{J0}))$	0	δ_{01}	δ_{02}
$\mathcal{A}_{CPJ}$	$(\bar{\Gamma}_J-\Gamma_J)/(\bar{\Gamma}_J+\Gamma_J)$	$\mathcal{A}_{CP0}$	$\mathcal{A}_{CP1}$	$\mathcal{A}_{CP2}$
$\mathcal{A}^0_{CPJ}$	$\frac{(\lvert\bar{A}_{J0}\rvert^2/\sum\lvert\bar{A}_{J\lambda}\rvert^2-\lvert A_{J0}\rvert^2/\sum\lvert A_{J\lambda}\rvert^2)}{(\lvert\bar{A}_{J0}\rvert^2/\sum\lvert\bar{A}_{J\lambda}\rvert^2+\lvert A_{J0}\rvert^2/\sum\lvert A_{J\lambda}\rvert^2)}$	0	$\mathcal{A}^0_{CP1}$	$\mathcal{A}^0_{CP2}$
$\mathcal{A}^\perp_{CPJ}$	$\frac{(\lvert\bar{A}_{J\perp}\rvert^2/\sum\lvert\bar{A}_{J\lambda}\rvert^2-\lvert A_{J\perp}\rvert^2/\sum\lvert A_{J\lambda}\rvert^2)}{(\lvert\bar{A}_{J\perp}\rvert^2/\sum\lvert\bar{A}_{J\lambda}\rvert^2+\lvert A_{J\perp}\rvert^2/\sum\lvert A_{J\lambda}\rvert^2)}$	–	$\mathcal{A}^\perp_{CP1}$	$\mathcal{A}^\perp_{CP2}$
$\Delta\phi_{\parallel J}$	$\frac{1}{2}(\arg(\bar{A}_{J\parallel}/\bar{A}_{J0})-\arg(A_{J\parallel}/A_{J0}))$	–	$\Delta\phi_{\parallel 1}$	$\Delta\phi_{\parallel 2}$
$\Delta\phi_{\perp J}$	$\frac{1}{2}(\arg(\bar{A}_{J\perp}/\bar{A}_{J0})-\arg(A_{J\perp}/A_{J0})-\pi)$	–	$\Delta\phi_{\perp 1}$	$\Delta\phi_{\perp 2}$
$\Delta\delta_{0J}$	$\frac{1}{2}(\arg(\bar{A}_{00}/\bar{A}_{J0})-\arg(A_{00}/A_{J0}))$	0	$\Delta\delta_{01}$	$\Delta\delta_{02}$

Table: Definition of 26 real physics parameters in the $B\to\phi(K\pi)^{*0}$ system

Summary

- Studying $B \to \phi(K\pi)^{*0}$ means studying a system of decays
- Understanding the system means studying mass and angular distributions
- Working with complex amplitudes implies to deal with interferences
- Combining everything opens the door to a large amount of physics that can be measured
- Final results on data are expected soon

BACKUP

Acceptance – 4D numeric integration – Correlations

Acceptance and orthogonality theorem

- Angular acceptance is **not** uniform
- An acceptance function $\varepsilon(m_{K\pi}, \theta_1, \theta_2, \Phi)$ needs to be included

$$P^{\pm}_{sig}(m_{K\pi}, \theta_1, \theta_2, \Phi) = \frac{\varepsilon|\mathcal{M}^{\pm}(m_{K\pi}, \theta_1, \theta_2, \Phi)|^2 \times F_{m_{K\phi}}(m_{K\pi})}{\mathcal{N}}$$

with the normalization $\mathcal{N}$ being averaged

$$\mathcal{N} = \frac{1}{2}\Big(\int \varepsilon|\mathcal{M}^{+}|^2 \times F_{m_{K\phi}} d\cos\theta_1 d\cos\theta_2 d\phi dm_{K\pi} + \int \varepsilon|\mathcal{M}^{-}|^2 \times F_{m_{K\phi}} d\cos\theta_1 d\cos\theta_2 d\phi dm_{K\pi} \Big)$$

Orthogonality theorem from scattering theory

Interference effects between states of different spins cancel out in the projection on e. g. invariant mass during integration over the full phase space if the acceptance is uniform.

4D numeric integration

- Below a simple example of a sum of two complex amplitudes (A_0, A_1) with complex weights ($a_0 e^{i\phi_0}$, $a_1 e^{i\phi_1}$) is shown
- If only the weights are free parameters in the fit, we simplify the calculation of $\int |\mathcal{M}|^2$ by calculating the integrals in advance

$$
\begin{aligned}
\int |\mathcal{M}|^2 = \int |a_0 e^{i\phi_0} A_0 + a_1 e^{i\phi_1} A_1|^2 \\
&= {a_0}^2 \int |A_0|^2 + {a_1}^2 \int |A_1|^2 + a_0 a_1 \int 2\,\mathrm{Re}\{e^{i\Delta\phi} A_0 \overline{A_1}\} \\
&= \cdots + a_0 a_1 \int 2\,\mathrm{Re}\{(\cos\Delta\phi + i\sin\Delta\phi) \times \left(\mathrm{Re}\{A_0\overline{A_1}\} + i\,\mathrm{Im}\{A_0\overline{A_1}\}\right)\} \\
&= {a_0}^2 \int |A_0|^2 + {a_1}^2 \int |A_1|^2 + 2a_0 a_1 \cos\Delta\phi \int \mathrm{Re}\{A_0\overline{A_1}\} - 2a_0 a_1 \sin\Delta\phi \int \mathrm{Im}\{A_0\overline{A_1}\}
\end{aligned}
$$

- During the fit, only the complex weights are free parameters
- The amplitudes are fixed and integrals can be calculated in advance
- Acceptance ε appears below each integral but was omitted here

Correlations in maximum likelihood fits

- Q: How to deal with the correlations in a 8D maximum likelihood fit?
- A: By identifying them and modelling with conditional pdfs

An algorithm for quantifying dependence in multivariate data sets

Michael Feindt, Michael Prim

(Submitted on 4 Jul 2012 (v1), last revised 27 Sep 2012 (this version, v2))

We describe an algorithm to quantify dependence in a multivariate data set. The algorithm is able to identify any linear and non-linear dependence in the data set by performing a hypothesis test for two variables being independent. As a result we obtain a reliable measure of dependence.
In high energy physics understanding dependencies is especially important in multidimensional maximum likelihood analyses. We therefore describe the problem of a multidimensional maximum likelihood analysis applied on a multivariate data set with variables that are dependent on each other. We review common procedures used in high energy physics and show that general dependence is not the same as linear correlation and discuss their limitations in practical application.
Finally we present the tool CAT, which is able to perform all reviewed methods in a fully automatic mode and creates an analysis report document with numeric results and visual review.

Comments: 4 pages, 3 figures
Subjects: **Data Analysis, Statistics and Probability (physics.data-an)**; High Energy Physics - Experiment (hep-ex)
Cite as: **arXiv:1207.0981 [physics.data-an]**
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